Chapter 2
Vibrations 1

Sound begins with vibrations. For example, the sound of a guitar begins with the vibration of the strings, and the sound of a trumpet begins with the vibrations of the player's lips. Perhaps less obvious, the sound of the human voice begins with the vibration of vocal folds (vocal cords), and sounds that are reproduced electronically are ultimately converted into acoustical waves by the vibration of a loudspeaker cone. Because sound starts with vibrations, it's natural that the study of acoustics begins with the study of vibrations.

Fundamentally, a vibration is a motion that is back and forth. Vibrations can be simple, like the swinging of a pendulum, or they can be complicated, like the rattle of your car's glove box when you drive on a bumpy road. Here we follow a path that is traditional in science; we consider first the simplest possible system and try to describe it as completely as possible. This vibration is called *simple harmonic motion*.

2.1 Mass and Spring

2.1.1 Definitions

The most fundamental mechanical vibrator is a single mass hanging from a spring. If there is no motion (no vibration) the force of gravity pulling the mass down is just balanced by the force of the spring pulling up. The mass is at rest. It is said to be at *equilibrium.* If the mass moves away from its equilibrium position it has a *displacement*. If the mass moves up above the equilibrium position, the displacement is positive, if it moves down, the displacement is negative. When the mass is at equilibrium the displacement is zero (Fig. 2.1).

If the spring and mass system is allowed to vibrate freely, the mass will undergo simple harmonic motion. As the mass vibrates up and down, its displacement is

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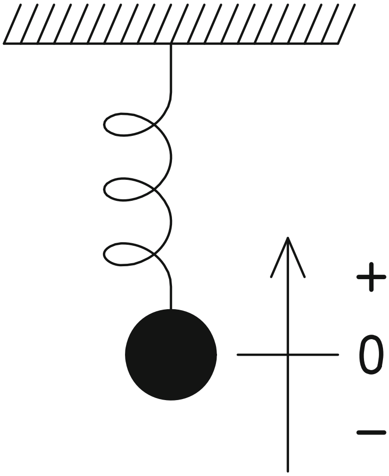

Fig. 2.1 The spring and mass system. In equilibrium the displacement is zero. When the mass moves above the equilibrium point the displacement is positive. When the mass moves below the equilibrium point the displacement is negative

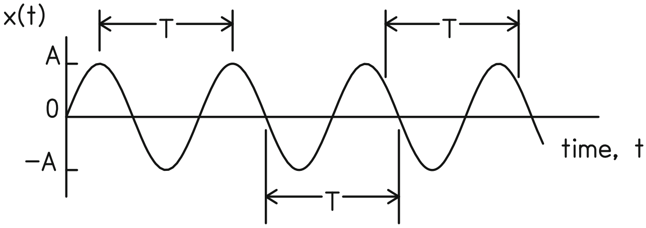

Fig. 2.2 The sine wave has extreme values $\pm A$, called the amplitude. It has a characteristic waveform shape as shown here. The sine wave is periodic, so that any two equivalent points on the waveform are separated in time by the time interval T known as the "period." Theoretically, the sine wave extends indefinitely to infinite positive and infinite negative time, having no beginning or end

alternately positive and negative. The displacement (x) can be plotted as a function of time (t) on a graph like Fig. 2.2.

The maximum displacement is called the *amplitude*. The most negative displacement is the same number with a minus sign in front of it. Simple harmonic motion is an example of *periodic* motion. That means that the motion is repeated indefinitely. There is a basic pattern, called a *cycle*, that recurs. The duration of the cycle is called the *period*. It is a time and it is measured in seconds, or milliseconds (1 one-thousandth of a second).

2.1.2 How It Works

To start the vibrations of a mass and spring system, you displace the mass from its equilibrium position and let it go. Let us suppose that you gave the mass a negative (downwards) displacement, stretching the spring. Because the spring is stretched, there is a *force* on the mass trying to bring the mass back to its equilibrium position. Therefore, the mass moves toward equilibrium. Soon the mass arrives at the equilibrium position, and at that point there is no more force. You might think that the mass would just stop at equilibrium, but this is not what happens. The mass doesn't stop because in the process of coming back to equilibrium, the mass has acquired some speed and, precisely because it is massive, it has acquired some *momentum*. Because of the momentum, the mass overshoots the equilibrium point, and now it has a positive displacement. The positive displacement causes the spring to be compressed, which ultimately forces the mass to move back down again toward equilibrium. As the mass passes the equilibrium point for a second time, it now has momentum in the opposite direction. That momentum keeps the mass going down until it has reached the negative amplitude that it had at the very start of this story. The mass and spring system has now executed one complete cycle. The amount of time needed to complete the cycle is one period.

The workings of a simple harmonic vibrating system like the mass and spring illustrate some very general principles. First, it seems clear that if we have a very stiff spring, then any displacement from equilibrium will lead to a large force and that will accelerate the mass back toward equilibrium faster than a weak spring. Therefore, we expect that making the spring stiffer ought to make the motion faster and decrease the period. Second, it also seems clear that if the mass is very massive, then it will respond only sluggishly to the force from the spring. (Did you know that the English-system unit for mass is the *slug*? How appropriate!) Therefore, we expect that making the mass heavier ought to make the motion slower and increase the period. As we will see in Chap. 3, that is exactly what happens.

2.2 Mathematical Representation

Mathematically, simple harmonic motion is described by the sine function of trigonometry. Therefore, simple harmonic motion is sometime called "sine wave motion," and the waveform shown in Fig. 2.2 is called a *sine* wave. It is described by the equation

$$x(t) = A \sin(360\, t/T + \phi) \tag{2.1}$$

where A is the amplitude, T is the period in seconds, and ϕ is the *phase* in degrees. [Note: Symbol ϕ is the Greek letter *phi*. The names of all Greek letters are in Appendix H.] A review of trigonometry, particularly the sine function, is given in Appendix B.

We will spend some words dissecting equation (2.1). The units of the amplitude A are the same as the units of x, in our case, mechanical displacement. It might be measured in millimeters. If the sine wave $x(t)$ describes some other physical property, such as air pressure or electrical voltage, the units of the amplitude change accordingly. By convention, A is always a positive real number. Amplitude A can be zero, but in that case there is no wave at all, and there is nothing more to talk about. The amplitude multiplies the sine function—**sin**—which has a maximum value of 1 and a minimum value of -1. It follows that the sine wave $x(t)$ has a maximum value of A and a minimum value of $-A$.

The sine function is a function of an angle, i.e., whenever you see an expression like sin(...), the quantity..., which is the *argument* of the sine function, has to be an angle. Therefore, "$360t/T + \phi$" is an angle. The special phase ϕ in the equation is the value of the angle when $t = 0$. In some cases we are interested in the function only for positive values of time t, it being presumed that the wave starts at time $t = 0$. Then phase ϕ is called the "starting phase."

The sine function is periodic; it repeats itself when the angle (or argument) changes by 360°. Equation (2.1) shows that if the running time variable t starts at some value and then increases by T seconds, the function comes back to its starting point. That is why we call T the period.

The reciprocal of the period leads us to the definition of the frequency (f) of the sine wave,

$$f = 1/T, \tag{2.2}$$

and its units are cycles per second, or Hertz, (abbreviated Hz). Therefore, one can write the sine wave in another form,

$$x(t) = A \sin(360\, ft + \phi). \tag{2.3}$$

Example: As an example, we consider the sine-wave vibration of an object whose position in millimeters is given by

$$x(t) = 4 \sin(360 \cdot 60\, t + 90). \tag{2.4}$$

From Eq. (2.1), the amplitude of the vibration is 4 mm, which means that the maximum positive and negative excursions are ±4 mm. The frequency is 60 Hz, and the starting phase is 90°. See Fig. 2.3.

The argument of the sine function, here given by $360ft + \phi$, is an angle called the *instantaneous phase.* We will give it the symbol Θ. It is assumed that angle Θ is always expressed in units of degrees. In a complete circle there are 360°.

The role of the sine function as a periodic function of a time-dependent instantaneous phase is emphasized by separating the aspects of periodicity and time dependence. One simply writes the sine wave in the form

$$x(t) = A \sin(\Theta) \tag{2.5}$$

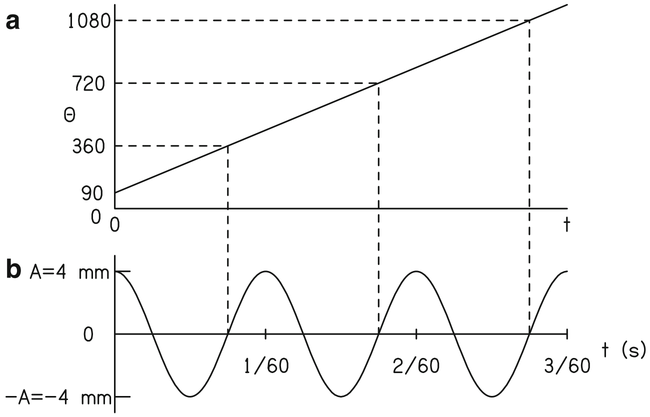

Fig. 2.3 Panel (**a**) shows the instantaneous phase angle Θ, increasing from an initial value $\phi = 90$ as time goes on. Over the duration shown, this angle advances through three multiples of 360°. Panel (**b**) shows what happens when one takes the sine of angle Θ and multiplies by the amplitude A. Here the amplitude A was chosen to be 4 mm

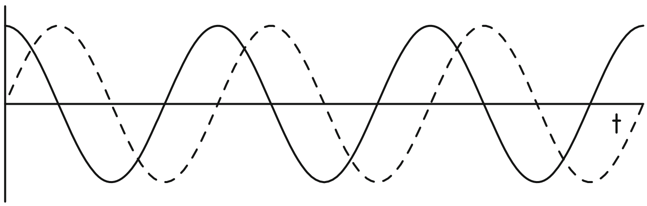

Fig. 2.4 The wave shown by the *solid line* is said to lead the wave shown by the *dashed line* because every waveform feature—peak, positive-going zero crossing, etc.—occurs at an earlier time for the solid line. Alternatively the dashed line wave can be said to lag the solid line wave. Both waves have the same frequency and amplitude, but their starting phases are different

where Θ is the instantaneous phase angle, measured in degrees, as shown in Appendix B. Here, angle Θ is a function of time

$$\Theta = \Theta(t) = 360ft + \phi \tag{2.6}$$

As t increases, the angle Θ increases linearly, and the sine function goes through its periodic oscillations, as shown in Fig. 2.3. Figure 2.3 shows the special case where the phase is $\phi = 90°$.

Phase Lead–Phase Lag When two sine waves have the same frequency but different starting phases, one of them is said to lead or to lag the other. Figure 2.4 is an example. The lagging wave is given by $x(t) = A\sin(360ft)$ and the leading wave is given by $x(t) = A\sin(360ft + 90)$. According to Eqs. (2.5) and (2.6) the starting phase angle for the leading wave is $\phi = 90°$. This angle is positive and less than 180°, which corresponds to a condition for leading. It should be evident that if ϕ had been equal to 180°, then neither wave would lead or lag the other.

2.3 Audible Frequencies

The spring and mass system that we used to introduce simple harmonic motion lets you see the vibration but the frequency is too low to hear. Such vibration is said to be "infrasonic." Vibrations of a few Hertz can sometimes be felt, and they can do damage too (an earthquake would be an example), but they cannot be heard. The audible range of frequencies is normally said to be from 20 to 20,000 Hz, or 20 Hz to 20 kHz. That statement of the range is easy to remember, but the practical range tends to be smaller. Without a special acoustical system it is not possible to hear 20 Hz. A frequency of 30 Hz is a more practical lower limit. Many college students cannot hear 20,000 Hz either. A more realistic upper limit of hearing is 17,000 Hz. We are quite accustomed to doing without the lowest and highest frequencies. Many loudspeakers that claim to be high quality cannot reproduce sounds below 100 Hz. FM radio stations are not even allowed to transmit frequencies higher than 15,000 Hz, and telephone communication in the USA is normally limited to the band between 300 and 3,300 Hz.

Octave Measure

To change a frequency by an octave means to multiply it or divide it by the number 2. Starting with a frequency of 440 Hz, going up one octave gets you to 880 Hz and going up another octave gets you to 1,760 Hz. The sequence is continued in Fig. 2.5. Again starting at 440 Hz, going down an octave leads to 220 Hz and going down another octave leads to 110 Hz. The measure of an octave is a basic element in the music of all cultures.

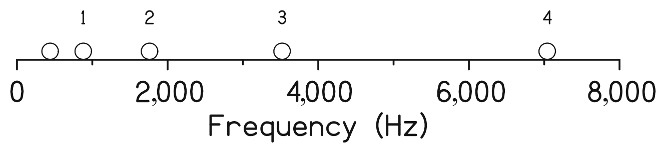

Fig. 2.5 The horizontal axis is a linear frequency scale. The *circles* show 440 Hz and octaves above 440, namely 880, 1,760, 3,520, and 7,040 Hz. With increasing octave number the frequencies become more widely spaced on the linear scale

Exercises

Exercise 1, Human limits

Nominally, the limits of human hearing are 20 and 20,000 Hz. Find the periods of those two waves.

Exercise 2, A low frequency

The second hand of a clock takes 60 s for one cycle. What is the frequency in Hertz?

Exercise 3, Time conversions

A millisecond is 1 one-thousandth of a second, and a microsecond is 1 one-millionth of a second. (a) How many milliseconds is 2 s? (b) How many milliseconds is 30 μs?

Exercise 4, Frequency conversions

A kilohertz (kHz) is 1,000 Hz. (a) How many kilohertz is 16,384 Hz? (b) How many hertz is 10 kHz?

Exercise 5, Period and frequency

(a) If the period is 1 ms, what is the frequency in kHz? (b and c) If the frequency is 10,000 Hz, what is the period in milliseconds and in microseconds?

Exercise 6, Conditions for phase lag

The mathematical conditions for phase leading are described in the text. From this, can you infer the conditions for lagging?

Exercise 7, Optical analogy

If an "infrasonic" sound has a frequency that is too low to hear, what is "infrared" light?

Exercise 8, Telephone bandwidth

The telephone bandwidth is from 300 to 3,300 Hz. How many octaves is that bandwidth?

Exercise 9, Sine waves

The sine wave in Fig. 2.2 has a starting phase of zero. It is reproduced in Fig. 2.6. (a) On the same set of axes, draw a sine wave with the same frequency and same amplitude but with a starting phase of 180°. (b) Draw a sine wave with the same amplitude but twice the frequency. (c) Draw a sine wave with the same frequency but half the amplitude. (d) The sine wave is said to be a "single valued function," meaning that for every point in time there is one and only one value of the wave. Sketch a function that is not single valued for comparison.

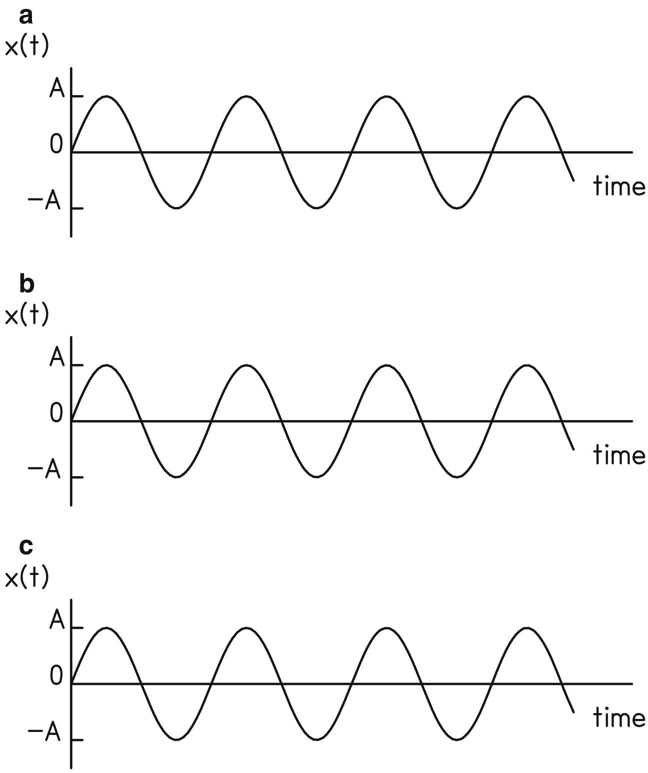

Fig. 2.6 Three practice waves for Exercise 9

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