Preface

Simply, quantum calculus is ordinary classical calculus without the notion of limits. It defines *q-calculus* and *h-calculus*. Here h ostensibly stands for Planck's constant, while q stands for quantum. A pioneer of q-calculus in approximation theory is the former Professor Alexandru Lupas [117], who first introduced the q-analogue of Bernstein polynomials. Ten years later Phillips [133] introduced another generalization on Bernstein polynomials [113] based on q-integers. Ostrovska [125, 127] studied q-Bernstein polynomials. After that several researchers have estimated the approximation properties of several operators. This book is an attempt to compile and present some papers on q-calculus in approximation theory.

We divide the book into seven chapters. In Chap. 1, we mention some notations and basic definitions of q-calculus, which will be used throughout the book. We also present the generating functions of some of the important q-basis functions. In Chap. 2, we present some discrete q-operators, which include the q-Bernstein polynomials, q-Baskakov operators, q-Szász operators, q-Blemian–Butzer–Hahn operators, and q-Meyer–König and Zeller operators. We present the approximation properties of such operators.

In Chap. 3, we present the q-analogue of integral operators which include q-Picard and q-Weierstrass-type singular integral operators and study their rate of convergence and weight approximation. We also discuss error estimation and global smoothness preservation property of such operators. In the last section of this chapter, we study generalized Picard operators and pointwise convergence, order of pointwise convergence, and norm convergence of the generalized operators. In the last section, we study the q-Meyer–König–Zeller–Durrmeyer operators and estimate the moments and some direct results.

In Chap. 4, we study the integral modifications of Bernstein operators using the q-beta functions of the first kind. We present the approximation properties of the q-Bernstein–Kantorovich operators, q-Bernstein–Durrmeyer polynomials, discretely defined q-Durrmeyer-type operators, and genuine q-Bernstein–Durrmeyer operators. We mention the moment estimation, direct results, and the limiting convergence of such operators. We have also included a section on fuzzy approximation and applications.

In Chap. 5, we discuss some other recently introduced q-integral operators on the positive real axis. To tackle such operators, we generally use q-beta functions of the second kind. This chapter includes q-Baskakov–Durrmeyer operators, q-Szász-beta operators, q-Szász–Durrmeyer operators, and q-Phillips operators. We present moments, recurrence relations for moments, asymptotic formula, and weighted approximations for such operators.

In Chap. 6, we study the statistical convergence of the q-operators. We mention results for a general class of positive linear operators and present statistical approximation properties in weighted space. We also present the results for q-Szász–King-type operators and q-Baskakov–Kantorovich operators and the study rate of convergence.

In the last chapter, we present the quantitative Voronovskaja-type estimate for certain q-Durrmeyer polynomials. In this way, we put in evidence the overconvergence phenomenon for these q-Durrmeyer polynomials, namely, the extensions of approximation properties (with quantitative estimates) from the real interval $[0,1]$ to compact disks in the complex plane. Also, we study the complex q-Gauss–Weierstrass integral operators. We show that these operators are an approximation process in some subclasses of analytic functions giving Jackson-type estimates in approximation. Furthermore, we give q-calculus analogues of some shape-preserving properties for these operators satisfied by the classical complex Gauss–Weierstrass integral operators.

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http://www.springer.com/978-1-4614-6945-2

Applications of q-Calculus in Operator Theory
ARAL, A.; Gupta, V.; Agarwal, R.P.
2013, XII, 262 p., Hardcover
ISBN: 978-1-4614-6945-2