

Contents

1	Embedded Graphs	1
1.1	Embedded Graphs and Their Representations	1
1.1.1	Abstract Graphs	1
1.1.2	Surfaces	2
1.1.3	Cellularly Embedded Graphs	5
1.1.4	Ribbon Graphs	5
1.1.5	Band Decompositions	7
1.1.6	Ribbon and Arrow Marked Graphs (Ram Graphs)	8
1.1.7	Arrow Presentations	9
1.1.8	Signed Rotation Systems	10
1.1.9	A Note on Terminology	10
1.2	Further Properties of Embedded Graphs	11
1.2.1	Subgraphs of Embedded Graphs	11
1.2.2	Genus and Loops	12
1.3	Petrials of Embedded Graphs	14
1.4	Geometric Duality	14
1.5	Medial Graphs, Tait Graphs, and Duality	16
1.5.1	Medial Graphs	17
1.5.2	Vertex States and Graph States	18
1.5.3	Tait Graphs	19
2	Generalised Dualities	23
2.1	Partial Petrials	23
2.2	Partial Duals	24
2.2.1	Partial Duality with Respect to an Edge	25
2.2.2	Other Constructions of Partial Duals	27
2.2.3	Basic Properties of Partial Duality	33
2.3	Twisted Duality	34
2.3.1	Sequences of Partial Duals and Petrials	34
2.3.2	Twisted Duals	36

2.4	The Ribbon Group and its Action	39
2.4.1	Defining the Group Action	40
2.4.2	Recovering Dualities from Actions of Subgroups of the Ribbon Group	41
3	Twisted Duality, Cycle Family Graphs, and Embedded Graph Equivalence	43
3.1	Characterising $\text{Orb}(G)$	44
3.1.1	Extending Tait Graphs to Cycle Family Graphs	45
3.1.2	Twisted Duality and Cycle Family Graphs	47
3.2	A Structural Hierarchy and Corresponding Dualities	52
3.2.1	Forms of Equivalences	52
3.2.2	Induced Dualities	53
3.3	Properties of Some Special Orbits	58
4	Interactions with Graph Polynomials	61
4.1	Classical Graph Polynomials	61
4.2	Deletion, Contraction, and Medial Graphs	63
4.3	Twisted Duals and the Topological Transition Polynomial	65
4.3.1	The Topological Transition Polynomial	66
4.3.2	The Topological Transition Polynomial and the Ribbon Group Action	68
4.4	The Penrose Polynomial	70
4.4.1	The Penrose Polynomial of an Embedded Graph and Its Relation to the Transition Polynomial	71
4.4.2	Identities for the Topological Penrose Polynomial	73
4.4.3	k -Valuations and the Penrose Polynomial	76
4.4.4	Graph Colouring and the Penrose Polynomial	78
4.5	Topological Tutte Polynomials	80
4.5.1	The Ribbon Graph Polynomial and the Topochromatic Polynomial	80
4.5.2	Relation to the Topological Transition Polynomial	88
4.5.3	Duality Relations for Topological Tutte Polynomials	91
4.5.4	Polynomials of Signed Embedded Graphs	92
4.6	Relating the Penrose and Topochromatic Polynomials	95
5	Applications to Knot Theory	101
5.1	Knots and Links	102
5.1.1	Links in a 3-Manifold	102
5.1.2	Link Diagrams	103
5.2	Virtual Links	105
5.2.1	Virtual Link Diagrams	105
5.2.2	Virtual Links as Links in Thickened Surfaces	106
5.3	Presenting Links as Embedded Graphs	108
5.3.1	Signed Tait Graphs	108
5.3.2	Ribbon Graphs and Link Diagrams	111

5.4	The Jones Polynomial and Graph Polynomials	116
5.4.1	The Jones Polynomial and the Kauffman Bracket	117
5.4.2	The Jones Polynomial as a Graph Polynomial	120
5.4.3	The Kauffman Bracket and the Transition Polynomial	123
5.5	The HOMFLY-PT Polynomial and Graph Polynomials.....	125
5.5.1	The HOMFLY-PT Polynomial	126
5.5.2	Graph Polynomials from the HOMFLY-PT Polynomial	128
References		133
Index		137

Graphs on Surfaces

Dualities, Polynomials, and Knots

Ellis-Monaghan, J.A.; Moffatt, I.

2013, XI, 139 p. 82 illus., 41 illus. in color., Softcover

ISBN: 978-1-4614-6970-4