Chapter 2
Making Progress in U.S. Mathematics Education: Lessons Learned—Past, Present, and Future

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Abstract Critics have deplored the quality of U.S. mathematics education for over 50 years. Schemes to improve it disappoint in their outcomes. At the same time, much more is now known about the challenges of effective mathematics education and about what it takes to tackle them. The U.S. mathematics education community stands at a threshold where it could help the country take substantial steps forward if it deliberately learned from the past, clarified its best ideas, and developed strategies for moving those ideas into the public debate. This chapter characterizes the challenge and argues for action informed by current practice and past reforms.

Americans have long complained about the quality of mathematics education. This discontent was evident in the wake-up calls that spawned the "New Math" of the Sputnik era and in the warnings of *A Nation at Risk* (National Commission on Excellence in Education, 1983). It has grown as American students appear to fall further and further behind the students of other countries. Although complaints about schools differ, concerns about mathematics education consistently trouble the public. Common schemes for improving mathematics education (e.g., new curricula, high stakes assessment, and teacher incentives) have been overused with little lasting impact.

This chapter offers a redefinition of what might be referred to as the "mathematics education problem" and articulates a solution. If the discourse 10 years from now is to be something other than a refrain about why U.S. mathematics education does not work, a different strategy is needed. This chapter begins by clarifying the problem before drawing on lessons from past failures to propose a plan for improvement.

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Framing the Problem of Mathematics Education in the United States

In a recent National Science Foundation special report, *Math*: *What*'*s the Problem*? (Zacharias, 2009), William Schmidt traces the poor mathematics achievement of U.S. students to the simple fact that the United States has not adequately taught its children mathematics for generations. This travesty has led to a situation in which it is acceptable for adults in U.S. society to say, "I'm not good at math," as if it were a joke or a badge of honor. Schmidt suggests that Americans have routinely communicated to their children that a few people have a "math gene," but most do not—a notion he claims is completely wrong and profoundly damaging. He says that while everyone may not excel in math, everyone can develop a strong mathematical foundation. In addition, the problem in the United States has a new urgency with features notably different from those of the past.

The new urgency stems from four pressing realities. First are the persistent gaps in achievement gains among different groups. African-American and Hispanic students in this country consistently score lower and exhibit lower achievement gains than their white and Asian-American counterparts, even when taking social class into account (Fryer & Levitt, 2006; Kao & Thompson, 2003; KewelRamani, Gilbertson, Fox, & Provasnik, 2007; Reardon & Galindo, 2009; Riegle-Crumb & Grodsky, 2010; Strutchens, Lubienski, McGraw, & Westbrook, 2004). Similar gaps are evident for students when comparisons are based on family income, again even when taking social class into account (Lubienski & Crane, 2010). And these gaps, associated with social class and race, are not shrinking. Likewise, there is a gap in achievement between U.S. students and their counterparts in similar countries. See, for instance, results of the Trends in International Mathematics and Science Study 2007 (Gonzales et al., 2008) and of the Programme for International Student Assessment 2009 (Organisation for Economic Co-operation and Development, 2010). It is appalling that a country with the resources and strengths of the United States, and built on principles of freedom, equality, and justice, has a system that educates its young people so poorly and so unevenly.

The second point contributing to the urgency complements the first. In this same system, in which the education of a diverse population of students is already a challenge and in which mathematics achievement can be predicted based on students' race and family income, the school population is changing dramatically. Drawing from the U.S. census, the Federal Interagency Forum on Child and Family Statistics (2010) reported the following: In 1972 about 80 % of the students in the United States were white and about 20 % were underrepresented minorities; currently the country is about 55 % white; and by 2023, according to its projections, white students in U.S. schools will be a minority. Although there is little change in the proportion of African-American students, there are large changes in the Hispanic population and in non-Asian-American Asian populations, a group whose achievement patterns are similar to Hispanics and African-Americans (Federal Interagency Forum on Child and Family Statistics, 2010; Zhao & Qiu, 2009).

A third source of urgency is language diversity. The Federal Interagency Forum on Child and Family Statistics (2010) reported that in 1979 about 9 % of U.S. students spoke a language other than English in the home and that this is now about 21 % of U.S. students. These language differences lead to a variety of challenges for teachers and students, but require careful consideration.[1] It is worth noting that many of these children learn to speak English well in school—only 5 % both speak a language other than English at home and have difficulty speaking English (Federal Interagency Forum on Child and Family Statistics, 2010). Having so many students come from homes where English is not the language used in the home creates a challenge for teachers in communicating with parents, made all the more difficult because, while the children often speak English, the parents often do not. The relationship of school to home is crucial to children's success, so the challenge of communicating with parents is rapidly becoming culturally and linguistically more complex. This, too, adds to the immediate needs for reconsidering the problem of mathematics education and for designing a system for improving it.

In addition to imperatives resulting from who is in school and how well they are being served, the country is also expecting more complex academic outcomes of all students than ever before. These increased expectations increase the demand on the education system and increase the need to find solutions to the mathematics education problem. For instance, state curriculum frameworks now specify goals that are considerably more challenging than in the past. As an example, the State of Michigan recently decided that in order to graduate from high school all students must pass a state-certified Algebra 2 course (this in a context in which 25 % of the entering ninth graders in Detroit, Lansing, Pontiac, Flint, and several other Michigan cities drop out before completing high school). What will happen over the next few years as the system expects students who are inadequately prepared and who typically have not taken this course to begin suddenly to not only take it but also to pass it? It may be a good idea to expect students to take Algebra 2, but a number of issues deserve both thoughtful consideration and public debate. States across the country are setting higher expectations though they have been unable to meet current expectations. In short, schools that are not doing well with their students are being asked to teach more mathematics to more students—this dramatic double rise in demands (of what is taught and who is taught) greatly adds to the urgency of the problem.

To point out the nature of the problem, below is a short "pretest" that highlights a number of prevailing myths about the condition of schools in the United States, myths that color common views of the problem (see Fig. 2.1). This simple pretest is

[1] We do not mean to imply that language diversity should be viewed as an impediment to teaching. Indeed, different languages provide additional resources for learning mathematics that often are not used well. For example, in Spanish some mathematical terms are much more comfortably related to the targeted mathematical meaning than are the English terms, yet programs often require that students go through awkward English terminology as they move from Spanish to English to mathematical language. Smarter instruction would make better use of the resources that Spanish-speaking children bring.

1. The U.S. mathematics education system used to educate our nation's young people much better than it does now.
2. The number of mathematics courses that a teacher has taken is a good predictor of how effective he or she will be.
3. Societal problems (e.g., inequality, poverty, the eroding family unit) are so overwhelming that schools cannot do their job.
4. Teacher education and mathematics curricula are similar to those taught 50 years ago.
5. College and university programs prepare teachers better than alternative routes into teaching.

Fig. 2.1 Pretest for identifying prevailing myths about the conditions of U.S. schools

meant to provide a sense of how these myths operate. It consists of five common statements, some supportable with evidence, some not. Which are myths and which are true?

The four pressing problems discussed above also provide an important metric for judging progress. Looking forward 10 years, a better education system would not produce achievement gaps between, on the one hand, underrepresented minority students and students living in poverty and, on the other hand, their white and middle-class counterparts. That students differ in their achievement is to be expected because people differ, but those differences should not be predictable based on ethnicity or family wealth. Ours is not an argument that everyone be treated the same or come out looking the same. It is that in an acceptable, equitable education system social identifiers would not be predictors of achievement gains. That is what we mean by eliminating the gap.

In addition, all students (every student) would have reliable access to high-quality mathematics instruction, no matter who they are or where they live (every year). The United States is far from this goal right now. Currently, in the United States, the likelihood that a child's teacher understands mathematics and can teach it skillfully to every student is low—and it is even lower in schools that serve underrepresented, poor communities. No other occupation in the country is handled in this way. When people go to the dentist for a root canal, they expect the dentist to know what he or she is doing. If a hairdresser does not cut hair well, clients do not go back. The situation for teaching is different. The target for teaching needs to be high levels of achievement gains by all students, and high levels need to be maintained across transitions—from preschool to elementary, elementary to middle, middle to high school, and high school to college. In other words, the slippages and gaps so evident now need to be replaced with significant learning across social groups and across transitional points.

Commitment by the country to the importance of the mathematics education of all students needs to be demonstrated by the allocation of adequate human, fiscal, social, and political resources. It's easy for people to say that the situation needs to improve, but the current will and allocation of resources are insufficient for the goals and challenges described above. This is not simply an issue of money. It is about understanding the goals and challenges, and their implications, well enough to act effectively. Without changes in attitudes and understanding, U.S. citizens are unlikely to choose improvement strategies wisely or to rally the necessary will if a promising strategy were launched. To help mobilize citizens, the mathematics education community needs to develop strategic formulations of the key issues and effective ways to engage the broader society in their solution. Those hoping to contribute to improvement need to invest in developing a well-tuned campaign that helps educate people about the problem. Without such a campaign, little progress is likely to be made.

The metrics discussed above omit other reasonable criteria, such as adequate mathematical literacy for good citizenship or satisfaction among leaders in business and industry with the mathematical skills of people entering the job market. However, the metrics above are critical indicators of dynamics associated with the overall health of an education system. For this reason they deserve immediate attention. Our argument about the importance of these indicators is central to this chapter. However, to understand the problem fully, it is important first to have a clear sense of the goal. The next section lays out a vision of mathematical literacy and of the nature of quality mathematics instruction—because these factors are key to building effective improvement strategies.

A Vision of Mathematical Literacy

At the center of the problem is a vision of mathematical literacy, or of mathematical proficiency. Namely, what should a mathematically educated person know and be able to do? Experimenting with a vision of mathematical literacy that could be used with the general public, we propose that it would involve being able to:

- Understand and be able to use mathematical ideas and procedures
- Frame and solve problems involving quantity, space, and probability
- Interpret and reason about quantitative, probabilistic, and spatial information
- Use representations to model situations and communicate about mathematical ideas
- Think with and use data

Somewhat buried in this list is the idea of being able to use and manipulate the symbolic languages crucial to mathematics. Not being taught to use these languages fluently is a significant disservice to people. Perhaps this aspect of mathematical literacy is best embedded inside the five elements listed above, but perhaps it deserves its own place on the list.

Drawing from established mathematics education literature, another way to think about mathematical literacy is to use the concept of proficiency developed in *Adding It Up* (National Research Council, 2001). This National Research Council report suggests five strands for thinking about mathematical proficiency (see Fig. 2.2). It argues that being good, or skillful, at mathematics does not rely on any one of these strands alone, but relies on all five of them and on the interrelationship among them.

Procedural fluency can be thought of as computational skill in the lower grades, but it also involves being able to manipulate equations, use algorithms, and work

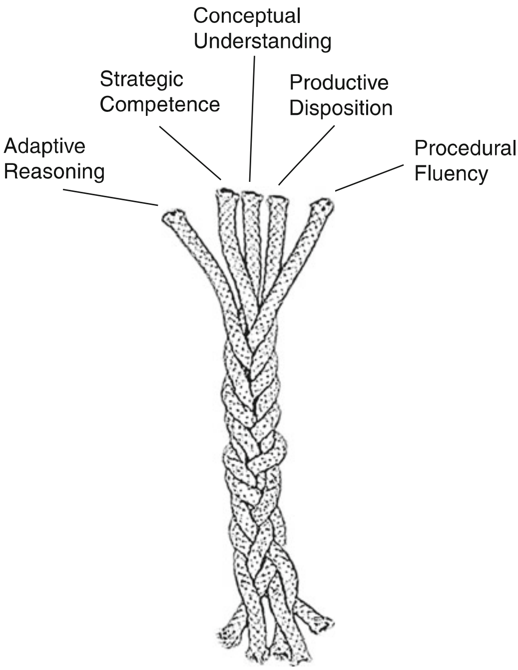


Fig. 2.2 Intertwined strands of mathematical proficiency (from Kilpatrick et al., 2001, p. 117)

quickly with things expressed in condensed form, with fluency and understanding. *Conceptual understanding* can be thought of in several related ways, but focuses on central ideas instead of procedures. *Adaptive reasoning* is the reasoning, proving, and explaining that are central to building mathematical knowledge. *Strategic competence* is skill in thinking about the way one decides to formulate problems, choose and use representations, or set up and manipulate equations or other symbolic forms. *Productive disposition* is the only strand that deals with the person, for instance with whether one sees oneself as someone who is capable of doing mathematics. Productive disposition also includes, perhaps even more importantly, seeing mathematics itself as a rational domain in which effort, learning, and work make it possible to be successful. The National Research Council report argued for the combination of these five strands based on summaries of a wide range of studies in the field. Taken together, these five strands offer a relatively succinct, yet accurate, way to think about mathematical literacy.

The point here is not simply about a collection of strands taken individually, but about the intertwined nature of the strands that make the rope, about the idea that if one only works on a single strand, one is unlikely to become fully mathematically literate. This feature of a vision of mathematical literacy is important because the history of mathematics education in this country has been dominated by pendulum swings back and forth between procedural fluency and conceptual understanding, *Two people are discussing the weather forecast.*

Saturday: 50% chance of rain
Sunday: 50% chance of rain

One says, "Darn, what a bummer! I was planning to play golf on the weekend, but now there's a 100% chance of rain on the weekend."

Is this right?

Fig. 2.3 A "simple" probability problem

with little attention to the intertwined nature of these strands and to the importance of adaptive reasoning.

Schools in the United States do a poor job of preparing young people to reason about mathematics. This is true in the teaching of proof at the university level, but earlier versions exist in the lower grades, where few students are taught what constitutes a mathematical explanation. Instead, many students think that reporting the steps they used to solve a problem is an explanation, and they readily propose taking a vote if a debate ensues about a mathematical claim. Thus, a curriculum that would grow students' capacity to reason about and justify mathematical claims would look very different from current practice in schools. In other words, while members of the mathematics education community are busy arguing about the relative weight of procedural fluency and conceptual understanding, other key ingredients of mathematical literacy are being ignored. And, because these students become people who participate in and shape the public debate, their miseducation carries over into policies and perceptions of the broader society.

To illustrate these ideas about mathematical literacy, consider the problem in Fig. 2.3. People often laugh upon reading it, but consider both the answer and what people are likely to answer. Assuming that rain on the 2 days are independent events, what is the chance of rain? Why? And, how would you represent the problem in order to reason about it and communicate your answer clearly?

There are two reasons we give this problem. One is that it captures some of the sense of mathematical literacy described above, and a second is to propose it as an example one might give to people who need help appreciating the nature of the broader problem in mathematics education. Most people who read this chapter will be concerned about mathematics education, but many in the larger society do not think that mathematics education is a concern. We argue that this example provides a good start for raising key issues with people of different backgrounds and convictions.

Many people answer that they think 100 % is wrong and that an answer of 100 % is "funny," but then they often say, "Oh, obviously it's 50 %." If you then ask them to explain, they then provide one of a variety of explanations. Below are two different ways of explaining a correct answer for this problem. For each, there are two questions worth considering, one about the mathematical reasoning and the other about the representations used.

a

	Sunday: No Rain	Sunday: Rain
Saturday: No Rain		25%
Saturday: Rain		

b

	Sunday: No Rain	Sunday: Rain
Saturday: No Rain	25%	75%
Saturday: Rain		

Fig. 2.4 Two-by-two table representations of possible events indicating the chance of (**a**) no rain on Saturday and rain on Sunday; (**b**) rain on the weekend

The situation in the problem gives a weather forecast for each day and a person who wants to play golf on the weekend. The question is: What is the chance it will rain on the weekend? Different things could happen. It could not rain on Saturday, or it could rain on Saturday. And we know that the chances are 50/50. Likewise, it could not rain on Sunday, or it could rain on Sunday. This information can be represented in a two-by-two matrix of possible events for what could happen when you combine the 2 days. One possibility for the weekend would be that it could not rain on Saturday and then, after that, it could rain on Sunday. And, there are one in four chances of that happening, or 25 % (see Fig. 2.4a). Likewise, it could rain on Saturday and not rain on Sunday, with one in four chances of that happening. It could not rain on either day, or it could rain on both days, also with chances one in four. The question of what are the chances of rain on the weekend is really a question of whether it will rain at any time on the weekend. The only way it does not rain on the weekend is if it does not rain on either day. Hence, there is a 75 % chance of rain on the weekend (see Fig. 2.4b).

Next, consider a second representation and explanation, paying attention to its features and ways in which its features are similar or different from the previous representation. Remember that the purpose is to illustrate how to help a general audience to understand that the aims of mathematics education ought to be the ability to think well in everyday life using the tools of mathematics and to communicate and debate effectively with others. The second representation breaks down the problem in a different way. One could say that it could rain or not rain on Saturday, and that it could rain or not rain on Sunday, with a 50/50 chance for each. Thinking chronologically, the possibilities can be arrayed out as in Fig. 2.5a, where the first possibility is that it rains on Saturday and also on Sunday, and the second is that it rains on Saturday but not on Sunday, and so on. Then, if one reorganizes one's thinking to ask "What are the chances of rain on the weekend?" these different trees, or pathways, can lead to understanding that there is only one of four possible ways that it would not rain at all (see Fig. 2.5b).

These are two different ways, among many, of representing this problem. The underlying mathematical structure for both is the same, and in that sense they are not so different, but the very competency of recognizing them as the same is indeed a crucial part of the mathematical literacy we need in the United States. For many, these two explanations are quite different. Different ways of thinking lead to them, and the representations support different kinds of thinking.

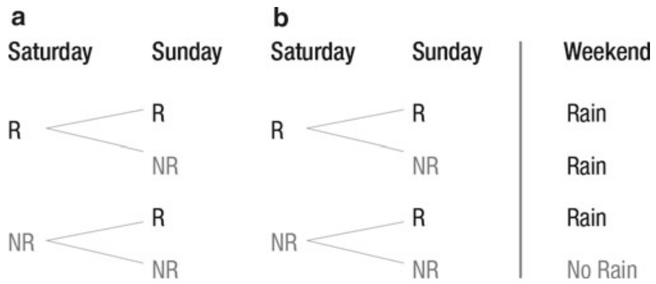


Fig. 2.5 Tree representations of (**a**) possible 2-day events; (**b**) possible 2-day events with associated outcomes

Returning to what it means to be mathematically literate, people need to be able to solve problems like this one, but they also need to be able to explain their answers, choose and use representations for addressing such problems, follow approaches different from their own, and communicate about and across approaches. They need to be able to recognize what it means to reason about something that is ordinary, such as questions that have to do with probabilistic reasoning about the weather. This probability problem illustrates that one has to understand some basic probabilistic concepts about the space of different possible outcomes for a particular problem, but that one must also understand how one represents the problem, how one frames the question, how one uses representations, and how one thinks with data.

The field of mathematics education needs problems such as this, ones that a very math-phobic or math-uninterested adult would appreciate and that might help to make more clear, in serious and nontrivial ways, what the consequences are that so many people think that the probability of weather, or of coin tosses, or of other problems with the same structure, is 100 or 50 %. Having compelling ways to represent the problem of mathematics education to the general public is crucial for moving from a concern held by a small minority to a widely shared concern necessary for real change. The mathematics education community needs to become better at communicating with the majority of people in this country, people who are not mathematically inclined or well educated, people beyond the immediate community. This is challenging for the very reason that the current system does not work well. Efforts to improve mathematics education suffer from the fact that most educated adults are not very well oriented to the problem of creating a mathematically literate society.

Thus, the problem of mathematics education in the United States is characterized by (a) severe underperformance and inequality in educating our nation's youth in mathematics; (b) no shared sense in the country—maybe even within the mathematics and mathematics education communities—about what mathematical literacy is and its importance; and (c) an enormous challenge of building a strategy for improvement in a country where so many people, including leaders and policy makers, are themselves not mathematically well educated. This is an unusual and extraordinary problem. No such problem exists when it comes to language literacy. In contrast to mathematics, leading policy makers can read and do appreciate that it is important for young people to read. Many people in positions of authority do not understand why mathematics educators think it is important for students to be better at mathematics—both our most able children and all of our children.

Before proposing ways to address the problem of mathematics education, it is essential that we are clear about what most influences student learning and how that influence is achieved. We move now to consider mathematics teaching in the United States.

Constituents of High-Quality Mathematics Instruction

Evident in the comments above is the idea that instruction in the classroom is key. With the word instruction, we don't mean quite the same thing as teaching. Instead we mean to suggest a systemic point—where the alignment of components of the system affords opportunities for students to learn. As described by Cohen, Raudenbush, and Ball (2003), it is in the interactions in the classroom that alignment occurs and it is these interactions that result in student learning (see Fig. 2.6). An instructional system is one in which teachers are interacting with the content and are representing it to students and where students are listening to one another, even in lecture classes, hearing answers from other students, and engaging with others in learning the content. Students hear and interact with their teachers and peers, and they interact with the content.

All of these dynamics shape what any particular presentation of content produces. One can produce a well-designed sequence of lessons on fractions, for

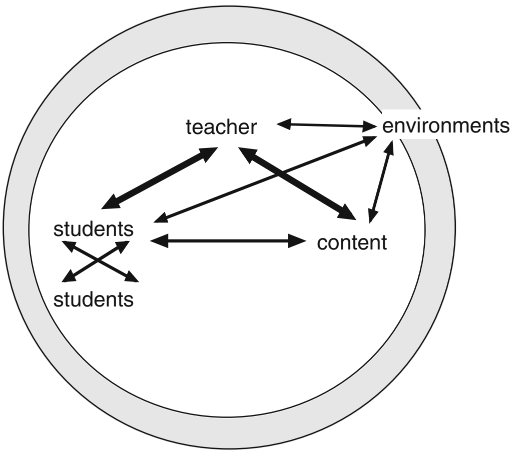


Fig. 2.6 Instruction as interaction of teacher, students, and content, in environments

instance, but one must also understand that there is nothing about that sequence of well-designed lessons that will predict instruction because it will depend on what particular students bring to the lessons, how the students interpret the teacher's use of those lessons, and how the teacher understands the lessons. Even a slightly different statement of a problem given on a worksheet or of a definition written on the board can change what students are thinking and learning. A student could ask a question in one classroom that is not asked in another, either creating a favorable opportunity or throwing everyone off. All of the interactions down the line can vary just slightly—like an accumulation of measurement errors—and result in very different lessons. Thus, it is impossible to determine from any specification of a lesson what instruction will actually be.

However, teachers are the ones who are charged with increasing the probability that the lessons one hopes will get taught do get taught and that students learn what they are supposed to learn. Primary responsibility cannot be assigned to the curriculum, and it cannot be assigned to students. Professionals should be held, and should expect to be held, accountable because they are the people with the skill to raise the probability that the lesson produces the outcomes it was designed to produce.

Of course, these interactions do not happen in a vacuum. All of them are influenced by the surrounding environment—by the values of the community, by the policy in the context, by testing, and by the principal in the school—but the double-edged arrows in Fig. 2.6 are intended to suggest that the environment does not simply bear down on schools. The environment is interpreted, and not uniformly. Two teachers working in the same school often interpret the pressures from the school board differently. A principal who says, "You must be on page 358 on April 11th," will not have every teacher in the school on page 358 on April 11th, because some teachers will say, "I know how to handle that principal. I can do it this way—I can teach and make sure I'm covering the content," and another will say, "I feel completely intimidated about what I'm being told, and will be sure to be on page 358 on April 11th." Regarding instruction, all of the interactions that constitute it are bidirectional and none fully determines any of the others.

From our experiences studying teaching and from teaching, we have developed a working hypothesis that four elements of high-quality mathematics instruction lie at the heart of the premise that teachers can raise the probability that the dynamics of instruction will produce the desired outcomes. The first has to do with having a *coherent mathematics curriculum*—the curriculum must be focused in a balanced way across the features of mathematics literacy, or strands of proficiency (see Fig. 2.2). The second element is a *supportive learning environment*. This includes characteristics both of the classroom itself, such as a careful use of language and the availability of public space for recording mathematical resources, and of the situation beyond the classroom, such as high-quality homework that connects the home to school and meaningful connections to students' out-of-school lives. The third element is *educational infrastructure*. This includes alignment among the curriculum, the assessment, the training of teachers, and the policy environment, as well as structural features that support successful instruction.

Unfortunately, right now the education system lacks infrastructure specifically for the support of instruction. Beginning teachers enter schools that provide little or no support for them to improve their skills. This makes teaching an anomaly among professional occupations. For instance, nurses are put into hospitals where it is assumed that the charge nurse and the other nurses on the shift will assist the beginning nurse in knowing which cases he or she is ready to handle alone and for which help is warranted. Inexperienced nurses are not assigned all of the difficult cases at the outset, as often happens in schools. Even in an occupation such as nursing, with similar scale and preparation to that of teaching, there is a system for taking novices and building their skill. In education, beginners are asked, haphazardly, to do work at all different levels of complexity. Infrastructure is lacking, but desperately needed.

The last element of high-quality mathematics instruction is *skilled teaching*, which is characterized by five features. Skilled teaching focuses on core concepts and skills. It is also culturally and linguistically sensitive. If we take as a given that a teacher's success is a matter of whether or not children learn, then teaching has to deal with the students who are in class. Because teaching is about relating content to students, teachers have to be sensitive linguistically and culturally to who their students are. They have to know which example will work best, where language matters, and so on. Third, students need to be active and engaged. This does not mean that students need to be talking or be in small groups. A student can be engaged and be in a lecture. Engagement is about whether students' minds are actively interacting with the content, and there are different ways to accomplish this (Dewey, 1965/1904). Perhaps more to the point, if a teacher is giving an elegant lecture and no one is following it, then that does not count as active engagement, or skilled teaching, but small group work where students are fooling around and not working on math problems also does not qualify as active engagement. Engagement does not rely on the physical organization; it depends on an intellectual connection.

The phrase equitable engagement is meant to point out that if you call on students, whether in a university lecture hall or in an elementary school classroom, and you only call on the people with the answers, this would not count as skilled teaching. Teaching a few people who already know what you are teaching does not constitute skilled practice. Skilled teaching requires the complexity of having people who do not understand the content actively thinking about and learning that content. That work is harder than simply coming up with a good explanation. (For an extended discussion of this issue, see Cohen, 2011.)

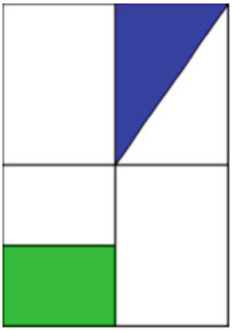
In addition, skilled teaching involves attention to mathematical language and reasoning. It is our impression that the role of mathematical language has been woefully underestimated in practical guidance given to teachers by the mathematics education community. Language is a key medium for teaching and learning. A number of theorists (e.g., Cazden, 1988; Vygotsky, 1986; Wittgenstein, 1958; and others) have focused on the importance of language in teaching and learning. Given that teaching and learning are language intensive, disciplinary language practices can offer resources for teaching mathematics. That mathematics educators express such affection for students' invented language while giving little attention to disciplinary language means that they are poorly positioning teachers to make good judgments about when it is okay for students to speak in a mathematically sloppy way and when it is not. This is not to say that first graders should always speak precisely. They cannot and should not be expected to do so. However, a teacher needs to know when it matters that students are implicitly defining something in their speech in a way that is going to lead to distorted understanding of mathematics within a year or two, and when what they are saying is an intermediate step in becoming competent and is not going to cause problems. Being able to recognize this difference is critically important.

As mentioned earlier, reasoning is important for student learning and is central to skillful teaching. As Ball and Bass (2003, p. 29) argue, mathematical reasoning is as fundamental to knowing and using mathematics as comprehension of text is to reading. As with the importance of disciplinary language practices, given that teaching and learning center on students' knowledge building, disciplinary knowledge-building practices offer resources as well. In addition to attending to students' mathematical language and mathematical reasoning as part of students' growing mathematical proficiency, attending to mathematical language and reasoning can support teachers in the language-intensive and reasoning-intensive work of instruction.

Last, it is important to understand that teaching is diagnostic work, whether teaching 500 students or 12. Given that teachers are responsible for the content and students, the skill of understanding whether students are "getting it" is at the heart of being able to teach skillfully. If a teacher flies blind, hoping that students are understanding, he or she is likely to stray far from where students actually are and is likely to greatly reduce the chances that students learn. For instance, in a mathematics departmental seminar at the University of Michigan, faculty did interviews of students who had received A's and B's in honors calculus courses, students whom faculty believed had done well in the calculus sequence, and they found that even with basic questions about the meaning of the derivative, students routinely gave wrong, curious, or even remarkable answers. One implication is that skillful teaching requires exam questions and methods of assessment that allow for finding out whether even very good students are missing major concepts and skills essential to mathematics literacy. Teaching that does not do that is effectively abdicating its defining responsibility.

The Work of Teaching

To get a clearer sense of skilled teaching, we next provide a short vignette of a lesson on fractions from the 2007 Elementary Mathematics Laboratory at the University of Michigan (adapted from Thames, 2009). The teaching described here is not meant to be good or not good, but is meant to illustrate in more detail the dynamics of instruction and to do so in a way that conveys the fact that there are



- What fraction of the big rectangle is shaded blue?
- What fraction of the big rectangle is shaded green?
- What fraction of the big rectangle is shaded altogether?

Fig. 2.7 *Blue–green rectangle* fraction problem (The *small shaded rectangle* is *green* and the *shaded triangle* is *blue*)

many teachers in the United States who know a great deal about how to teach. What happens in this vignette is not exceptional; events like this occur routinely across the country in many classrooms, where teachers manage emotions, coordinate activities, and focus student attention on mathematics. Our purpose is to point out the specialized work teachers do, and the extensive knowledge and skill they demonstrate. If mathematics education is to be improved, the image this country holds of skilled teaching cannot make teaching precious. The United States cannot put itself in a position where there are just a few people who can teach well. It needs to have four million people who can teach well, a fact that has major policy and practical implications.

The mathematical task given to students in this episode (see Fig. 2.7) was developed by researchers at the University of California, Berkeley, as part of their work at the Elementary Mathematics Laboratory and as part of a larger investigation of upper elementary students' learning of fractions. In using this problem, there are several questions a teacher would need to consider: What fundamental mathematical issue is the task designed to address? For the first question, what answer is a fourth grader who is just beginning to understand key ideas about fractions likely to produce? For instance, they might say one-half. Why? They might say one-sixth. Why?

An interesting point here is that U.S. school curricula do not always do a good job of helping students make the transition from a counting model to an area model. When considering the fraction of people in a room who are male, the size of the individual males is irrelevant, with attention given to just the *number* of males. In this problem about shaded regions of the rectangle, an answer of one-sixth is consistent with such a counting model. Saying that the blue part is one-sixth is a common, and sensible, initial answer at this point in children's learning about fractions. Unfortunately, as instruction begins to move into area models, it often fails to signal to students that the crucial issue now has to do with equal areas and not with equal numbers of parts. Furthermore, the phrase "equal parts" is often used in teaching fractions in the United States, pervasively, even in work on area models, which surfaces an important language issue, reinforcing the point we made earlier about the crucial role of language in teaching. Given the common use of “equal parts,” it should not be surprising that many students would answer one-sixth, because one-sixth of the parts is shaded. In addition to the language demands of teaching that are implicated by this problem, we can also glimpse the kind of work involved in choosing well-designed problems that target key ideas, in this case the meaning of equal parts in an area model and the importance of paying careful attention to the whole.

This problem was used in a summer laboratory class with students who had finished fourth grade the previous spring and had been identified as struggling in mathematics (by their teachers and schools). The problem was used as a warm-up, written on the board for students to work on as they entered class. The episode represented here is from a whole-class discussion that occurred after discussing the previous day’s work and before beginning the next major work. The 4-min interaction is between the teacher and a student who has an answer to the problem after having worked on it for a few minutes at the start of class. Consider what is involved in the work of teaching—in being responsible for designing and enacting instruction to support student learning. The point in describing the episode is to make clear why the strategy for improving mathematics education in the United States has to pay much more attention to this level of the work. Our argument is that the United States will not make improvements for students without worrying more about what instruction requires and about what the dynamics of instruction mean for improving the system.

Early in the class, the teacher asks for a volunteer to explain his or her thinking about the first question. A number of students raise their hands, but she lingers, encouraging more students to consider volunteering, in particular someone who has not spoken in whole group yet on this day. Mahluli volunteers for the first time. She calls on him, and he says the answer is one-half. When asked to explain, he says, “Because they both equal—they both equal—and one, one, half of it is shaded in and the other half is not.” The teacher then asks him to go to the board to explain his answer. As he makes his way to the front of the class she engages another student, Doran, in explaining what Mahluli has said:

> Okay. Can you come up to the board and point and show us what you're looking at? Just—there's a diagram right there. Can you come up and show? Did everyone hear what Mahluli said? You should be thinking already about his reason. Who can repeat what Mahluli said? Okay. Well if you're listening carefully, you should always be able to tell what someone just said. Doran, what did he say?

Doran says that Mahluli is just looking at the rectangle and saying it is one-half—that Mahluli is not looking at the whole. As Doran starts to go on to explain what Mahluli has *not* done (to explain a “correct” solution to the problem) the teacher interrupts him, asking that he not go on to explain it yet, and she turns back to Mahluli to have him explain his thinking using a large poster of the figure that is stuck to the blackboard. Mahluli repeats his explanation, quickly pointing to the two triangles and saying that they are equal, so the shaded one is one-half. The teacher then suggests looking back at the working ideas the class generated the day before. On the board is a list labeled "working ideas" about fractions. It contains three points:

- Identify the whole
- Equal parts
- How many parts of the whole

She asks Mahluli what he is calling the whole and has him run his finger around the part he is calling the whole. Mahluli indicates that he is using the upper right rectangle as the whole. In keeping with the working ideas, the teacher has him indicate the equal parts and say how many of the parts are shaded. She then works with another student to reiterate Mahluli's explanation, checking with Mahluli that they are understanding him correctly, and then asks the class, "If Mahluli calls this the whole, is he right that that's one half?" Getting affirmatives, she then says, "Now the question asks you something a little bit different. So who can tell everybody what question we're trying to answer? What Mahluli did is right, but he used something different to be the whole." Referring back to the first question, the teacher asks Avery what she thinks is meant by "the big rectangle." Avery ventures, "The whole rectangle." The teacher responds, "What whole rectangle? You want to come up and show us? Mahluli, are you watching?" Avery uses her finger to trace around the outside of the full figure. The teacher then reiterates the first question, emphasizing the intended whole: "If you use the whole big rectangle to be the whole, how much is shaded blue?" Before inviting students to answer the intended question, she checks in with Mahluli, "Do you see the difference between the question you answered and this question?"

Mahluli: You gotta try to figure out of the whole square.
Teacher: Out of the whole rectangle. And you used what?
Mahluli: And I did half of the rectangle.
Teacher: You did a smaller part of the rectangle. Okay?

The class goes on to discuss why the answer is one-eighth.

This episode suggests some of what is involved in teaching—beforehand, during the class, and possibly later. It shows the importance of teachers' listening, investigating, and drawing-out skills. One thing is that teachers must try to figure out what students are saying and what they do and do not understand at the current point in instruction. A student remark can seem completely incorrect, but a teacher needs to figure out what exactly the student is thinking, and a teacher's precision in reading a student's remark informs the quality of the response. The episode above reveals the fluency a teacher needs in being able to hear the correct thinking in Mahluli's seemingly errant answer and then deciding whether this is the right time to make a point of it and what sort of correction can be made so that at least the public version of things is not incorrect. Many people would just hear Mahluli's answer as wrong and would not even understand what might have led him—and very likely other students—to misinterpret or fail to attend adequately to what whole is intended. The ability to hear students' broken speech about their emerging ideas in real-time instruction demands a degree of speed and fluency that are routinely underestimated in deliberation on teaching. In contrast to professional mathematics, where people may spend years on a single problem, mathematical problems of teaching often require quick judgment and prompt action. Teachers cannot spend years figuring out what a student is thinking, not even minutes. And mathematics educators responsible for teacher education have not thought very much about how to train teachers for mathematical fluency. If a teacher cannot think mathematically on his or her feet, the teacher is quite impaired in his or her ability to teach. Some things are predictable, others are not, and if a teacher cannot quickly say, "I think this is what's going on," then teaching becomes an awkward enterprise. Having to interview students every time something unexpected comes up is cost expensive in a way a teacher often cannot afford.

In sizing up a student's answer, a teacher also has to coordinate hearing a student's thinking with a sense of whether or not the mathematical point is crucial at the moment. In the class just described, students are just developing clarity about the notion of the whole and the meaning of equal areas (as area and not necessarily congruent). In this situation, it is probably important to invest in Mahluli's thinking, but in another situation it might not be wise to invest in Mahluli's use of the word "square" in place of "rectangle" or his statement about considering "half" of the big rectangle when it was one-fourth. In another situation, in response to his incorrect answer of one-half, it might be better to quickly say, "no, not one half," and move on. It depends. In the episode above, it is important to take up his thinking because it is on point for the lesson, represents a misunderstanding likely shared by other students, and provides an opportunity to develop explicit language about the key idea of the whole. This does not mean that taking up a wrong answer like this is always the right thing to do in teaching. Instead, the point here is to draw attention to the sort of diagnostic nature of the work—that Mahluli's answer represents a crucial mathematic hinge moment for opening up the topic and achieving the goals of the lesson.

Another thing to notice in this episode is that the teacher invited other students to think actively about Mahluli's reasons for saying that the blue triangle is one-half. Teachers need to maintain interest and engagement. They need to productively fill the few moments taken while Mahluli walks to the front of the class. Additionally, teachers are responsible for teaching all of the students in the class, even when probing the thinking of a single student. By setting an expectation that, after a student has spoken in a whole-class discussion, all of the other students should be able to repeat what was said, a teacher directs attention, sidesteps misbehavior, maintains mental engagement, and teaches students skills for productive participation in public discussions about mathematical ideas and issues.

The selection and design of tasks themselves, whether from a textbook or self-made, is another important domain of the work of teaching, as is setting up or framing the task. The task in the example above has students think not only about equal areas but also specifically about distinct shapes whose areas are not readily comparable. The shapes in the figure are deliberately different, which then means that there is some work to do to establish that each shaded region is one-eighth, or that the two of them together are one-fourth of the overall shape. The need to rely on a deduced equality of their areas maintains an intellectual honesty about an area model for fractions. This is a key mathematical feature of the task. Figuring out how to put the task in play so that the students get to the mathematical point is an important piece of mathematics teaching. A teacher can have a problem, can get all kinds of interesting things to come up, and can raise an interesting side trip during the lesson, but more importantly, the teacher must keep an eye on the goals for the lesson and must coordinate decisions with whatever students are doing to get to that goal. If a teacher does not accomplish this, and do so routinely, then the teacher is well off the mark of skillful teaching, whether in a college course or a first-grade classroom.

The pedagogical issues that can be mined from almost any 4-min episode of skillful teaching are nearly boundless. Teachers manage interpersonal dynamics, even in lectures, where a skillful choice of jokes, seriousness, and emphasis can greatly enhance student learning. They need to recognize and construct what is going on mathematically, where their use of silence can matter as much as their talk. There is much that could be discussed about the 4-min episode described above. Here is one possible list, only some of which have been mentioned:

- Selecting/designing tasks
- Teaching students what counts as “mathematics” and mathematical practice
- Making error a fruitful site for mathematical work
- Deciding what to clarify, what to make more precise, and what to leave in students’ own language
- Attending to the ambiguity of “big rectangle”
- Listening to and interpreting students’ responses
- Identifying and working toward the mathematical goal of the lesson

The point in describing skillful teaching in some detail is that, if mathematics teaching and learning are to be improved significantly, the mathematics education community must address the fact that the current system does not prepare people to teach at a reasonable level of skill. Our argument is that for any change to matter, be it curriculum, school finance, or high standards, it must necessarily change interactions in classrooms among teachers and students around content. Short of that, nothing changes. And teachers have primary responsibility for managing instructional interactions. Thus, skillful teaching and a system that adequately prepares a large number of people to teach skillfully are requirements of a system that adequately educates children.

Learning from the Past

Consider now the question of what can be done so that the story will be different in the near future. It is important to note that in the United States a great deal is expected of schools and calls for improvement are business as usual. The United States, as well as the U.S. mathematics education community, keeps gravitating to the same reform strategies, perhaps because those strategies represent a commonsense point of view (e.g., fix the curriculum; hold students, teachers, and schools accountable; overhaul the administration). Improving education is a big order and many past decrees have languished. Thus, it is important to ask what can be learned from past attempts so that the pattern may be changed.

Here are some of the most widely touted strategies for improving mathematics education in the United States:

- Teacher-proof instruction
- Install a more challenging curriculum
- Increase accountability
- Reorganize schools
- Pay teachers more
- Recruit talented teachers by lowering the barriers for entry

Notice that the word "mathematics" does not appear in any of these strategies, yet they dominate both conversations and policies aimed at improvement. They have been used for other subjects as well, but mathematics is arguably the subject that has received the most attention. That makes it the saddest story because more work has been done on mathematics, done on the part of the U.S. society and the professional community, than has been done with science, social studies, or other school subjects, yet the payoff for that investment has been small. The point in listing these six strategies is not to say that none of them is worthwhile; each has merit, but taken alone, none has accomplished much, or is likely to.

We argue that one reason for this is simple, and goes back to what we said about instruction. None of these strategies guarantees that instruction will be different because none gets directly at instruction, changing what happens between teachers and students in schools. For example, a teacher-proof instructional program could be implemented, but only those naïve about teaching could think there is a way to control completely what a teacher says to students. It is probably a good idea to give teachers more guidance, but the notion of teacher-proof instruction underestimates the complexity of the work. Likewise, the introduction of a challenging curriculum or increased accountability or reward, without attention to teacher capacity, is unlikely to change instructional dynamics. Schools are regularly reorganized without changing what happens in classrooms, and talented recruits would only be more effective if their presence systematically altered patterns of interaction—the nature and process of such a change are unclear. Taken individually and apart from a plan for impacting teaching and learning, it is not clear why any of the most common approaches to sweeping reform would change basic classroom interactions.

We can also look at past attempts to improve education, see what has impeded progress, and consider which factors are the ones we can effect. One impediment to progress is endless arguments about which matters more, skills or concepts, when they both matter. This is an unproductive argument. Nobody who knows much about mathematics or mathematics teaching believes that only one matters. Such debates in the United States need to stop and the focus needs to be turned to learning to teach both skills and concepts better and to subtler issues about their order within a pedagogical approach.

Another, perhaps more controversial, lesson from the past is that the lack of a central or a common curriculum is a major impediment. It is popular to talk about what goes on in Japan, China, Hong Kong, Taiwan, or Singapore, but one of the main differences is that each of these countries has a coherent, uniform curriculum. When children move within the United States, they go from one curriculum to another, often with quite different goals. Think what it is like to be a third grader whose parents move frequently. You are already having trouble with math. You show up at a new school district—it is a different book, with a different vocabulary, and a different set of representations. The situation is absurd. Fractions are no different in Idaho than they are in Utah, yet they are treated as though they were and students often experience them as though they were. Every state and every one of the 15,000 school districts in the United States does not need a different curriculum. Currently, each school district, to a large extent, makes its own decisions about what gets taught. Efforts such as the Common Core State Standards Initiative are meant to address this problem, but the United States, with its history of political commitment to local control of education, is still a long way from establishing a common curriculum coherently used throughout the country. It is precisely for a public debate like the one shaping up around the Common Core State Standards that the mathematics education community needs to develop tools to frame and clarify the nature of the problem.

In addition to the focus on local control of education in the United States, there is also a tendency toward frontier individualism with regard to teacher autonomy. Even within a single school, teachers hold different convictions about what is important to teach, what formulation of a concept is best to use, and how best to sequence instruction for a given topic. Even if an individual teacher makes wise choices, as students pass from grade to grade, the curricular confusion can be profound. This is not a formula for a coherent system for teaching mathematics.

Another impediment to progress is the inclination to persist with outdated and refuted ideas about "teacher quality," especially with respect to content knowledge. (See the National Math Panel Report (2008) for an appraisal of the issue of teacher content knowledge.) The focus tends to be on *teacher* quality, particularly when it comes to teachers' inadequate content knowledge. However, the issue is not teacher quality, but *teaching* quality (Gitomer, 2009). If teachers could be selected in ways that were predictive of how well those teachers would teach, then teacher quality could be taken as indicative of the quality of teaching, but because there are currently no effective ways of identifying the characteristics of teachers that will predict whether their teaching will be good, the focus should be on assessing the quality of teaching. Maybe someday, when more is known about which teacher characteristics account for shaping interactional dynamics and for variance in student learning, then measures of teacher characteristics could be useful, but in the end it is not the characteristics of a group of teachers that matter. Instead, it is what children do with mathematics that is of high quality, and that is more immediately related to teaching than it is to characteristics of teachers. The field needs more studies that focus on instructional dynamics and needs to develop tools for evaluating it.

Similarly, the United States continues to engage in pendulum swings from teacher-proofing schooling to presuming that, if obstacles are removed and professional community provided, teachers will grow improvement on their own. The focus should be on building an understanding of, and a capacity for, skillful teaching. What needs to be changed in the United States is the way teaching is done.

A final lesson to draw from that past has to do with the education of teachers in the United States. Teacher education, both preservice and in-service, persistently emphasizes things other than practice (e.g., reflection, beliefs, propositional knowledge, experience) (Hiebert, Morris, Berk, & Jansen, 2007). Poor teaching is inevitable in a system that teaches knowledge remote from the actual doing of teaching, strives to change what teachers believe, and spends time on having people reflect. Again, these things do matter. Teachers need to learn to be analytic about their teaching, and they need to learn to talk clearly about teaching. However, teaching is primarily a practice, something done. Thus, a professional training system that does not hold itself accountable for whether people can do the work is a bankrupt system. That is the current situation in the United States.

What to Do About the Problem of Mathematics Education in the United States

So, what do these lessons from the past imply for making real progress—for making it possible to design a strong instructional system? With an aging workforce, the next 3–4 years will see the largest incoming group of new teachers that this country has seen in a long time, even with the downturn in the economy. The notion of greater accountability for beginning practice is more acute than it has ever been. In a sense, this is an opportunity—if the country can figure out how to capitalize on it. From our understanding of what it would take to support the improvement of teaching, we argue that there are five strategic elements for designing a strong instructional system:

- Build a common mathematics curriculum
- Develop valid and reliable assessments coordinated to the curriculum
- Build a system of supplying skilled teachers to every school to teach that curriculum
- Center teacher licensure and training on practice
- Organize schools to support beginning teachers

In recognizing these five critical elements for designing a strong instructional system, it is important to note that instruction is foundational to the endeavor and that the country needs strategies that address this foundation. The dynamics of instruction are the educational core. They are what affect the children engaged in learning. Consequently, an effective strategy has to build a system that changes that set of transactions. If the strategy does not affect that core, then it is playing at the edges of the problem. Tinkering with the curriculum only improves learning if the tinkering increases the chances of lessons getting taught well in classrooms by teachers. It is good for programs to foster student confidence, as confidence may increase their motivation. However, by themselves such programs will not necessarily change the transactions in schools. What is needed are strategies that ensure that programs actually change the transactions of teaching and learning.

The five elements above offer promising tools for building a system capable of creating change. Establishing a common mathematics curriculum is at the heart because it provides coherence that enables everything else. There is, for example, evidence that, when professional education for teachers is situated in the curriculum that they have to teach, teachers are demonstrably more skillful than when professional development is based on guesses or generic versions of what teachers will teach (Cohen & Hill, 2001). Adopting a common curriculum would not only provide children with a more coherent experience as they move geographically between schools and vertically through the grades, but it would also provide critical infrastructure for high-quality teacher training. Currently, when prospective teachers are taught either about content or ways of teaching that content, instruction must be designed without knowledge of the books they will be using when they get a job, which severely handicaps what can be done. A system built with knowledge about what teachers will be teaching would be able to get much closer to assuring that teachers know mathematics and pedagogical practice well enough to deliver mathematics to their students. The fact that this information is missing leads to programs for teacher education that must constantly stretch, guess, and talk in generalities. As long as agreement on a common curriculum is presumed to be unobtainable, teacher education is likely to continue to be a story of eclectic improvisation. It may be that the United States is not ready for a common curriculum.[2]

In addition to a common curriculum, in a coherent education system common assessments are needed to track students' progress and these assessments need to be coordinated to the common curriculum. It is a direct result of not having a common curriculum that assessments in the United States are not coordinated to a curriculum but are, of necessity, curriculum-less (Cohen, 2011). Historically, the testing systems used in this country do not test the curriculum taught (because no shared curriculum exists across the contexts that assessment must serve). The prime example of this phenomenon is college admission testing established by the College Board, historically called the "Scholastic Aptitude Test" and specifically designed to create

[2] Other countries that have a common curriculum, and build around it, do not decide on the curriculum by political means, such as is the case in the United States with its politically mandated state curriculum standards. In other countries, governing bodies have professional authority and oversight for determining the common curriculum. Of course, achieving this necessary first step in the United States would require a great deal of discussion.

an evaluation tool that would work across the uninterpretable information about high school course-taking and grades. The choice to assess aptitude, however ambiguous the notion, was a result of not being able to assess actual achievement in a fair way—in a way that adequately accounted for variation in local curricula. However, this is only one extreme example. The Iowa Test of Basic Skills, the Stanford Achievement Test, TerraNova, and many others, because they are designed to be administered across schools, districts, and states in the U.S. context, make choices about what content to assess, in what amounts, and at what level. These decisions require some smoothing of the curricular terrain. As these tests are used for high stakes accountability, it is no wonder that, because they do not match the curriculum being taught, teachers feel pressed to teach to the test—in other words, to take the test as the curriculum. However, these tests are not curricula and are a poor replacement. Every major assessment of student achievement in the United States reveals this dilemma in one way or another, and this feature contributes to the overall weakness of the current system.

Even if there were some degree of agreement in some group about the content that students should know and even if that group developed assessments aligned to that content, those assessments would still not be coordinated with what is actually being taught in the current nonsystem of U.S. education. This is yet another example of the effects of a very decoupled, nonsystemic approach to teaching in the United States. Current assessment technologies, however, permit some new designs for how assessments can be delivered and for the reliability and validity of those assessments. For example, a progress-variable approach that views learning as progress toward higher levels of competence, instead of acquisition of more knowledge and skill, can be conceptualized, assessed, and informatively displayed (Wilson & Scalise, 2006). With recent psychometric advances, the possibility exists for building better assessments than ever before. If the country is to make progress on improving mathematics education, then the all-too-common aversion to assessment among professional educators and mathematics educators is untenable. Testing (in some form) is critical to education. The fact that current assessments are not ideal does not mean that good ones cannot be designed, ones that attend to cultural and linguistic equity, that assess what is taught and what matters, and that readily inform teaching and learning.

The third strategic area to target has to do with the teacher supply problem. Many people are fond of thinking of ways to get rid of teachers who do not teach well, but the United States does not have a system for supplying people to replace the teachers who would be fired. Much of the accountability rhetoric and the push for value-added measures are motivated by an agenda of finding the teachers whose students are not making gains and getting rid of them. Instead, the country needs to figure out how to supply schools with skilled teachers.

This goal will require several things. Ideally, the training system would need to be coordinated to the curriculum that teachers would teach. Physicians are not certified, for example, without being prepared to use the tools central to their practice. No medical school says, “You are going to be a surgeon and we cannot tell you what tools your hospital will have, but if they have this tool you do this, and maybe you will make your own, because we want you to be resourceful and creative." A common assignment in teacher education is to have prospective teachers develop their own lesson or unit. This activity is analogous to having surgeons make scalpels. Curriculum is the central tool of teaching. Teaching itself, in large part a matter of using a curriculum, is a skillful work. Attention needs to be given to training people in the demanding work of using a curriculum, not spent on a wilderness camp experience of making all of one's own tools from scratch.

Mathematics teacher educators need to agree that effective teacher preparation is about preparing teachers to use a relatively well-specified curriculum. Then, as professionals with standing in the larger society, they need to insist that a common curriculum is not optional and explain to others that it is essential to professional preparation for skillful teaching. Learning to teach with such a curriculum would include knowing when it is working and knowing when it needs to be supplemented. Blind use of a curriculum is not good, but teachers need to be prepared for professional and serious use of what is in fact the central tool of teaching and learning. For teacher education programs to effectively teach future teachers to recognize, for example, how objectives are addressed in specific lessons and how topics are developed across a textbook, it would require having a curriculum to teach people to use in the first place, one that would be used wherever a teacher is hired.

Related to this need for building a system for supplying skilled teachers to every school—teachers who are prepared to teach the school's curriculum—mathematics teacher educators need a concept of "safe to practice." It is irresponsible to put untrained people into classrooms to teach. Students are real—not the manikins used by nurses and doctors for learning practice. Other professionals responsible for people do not just watch for some time and then jump in to see how it goes, but this is common for teachers. With regard to teacher preparation, the expression "sink or swim" is more often used for what it means to learn professional practice than are analogies from professional domains more closely resembling teaching. Novices should not be put in classrooms to teach without supervision. No other occupation functions in such a way. There are performance standards for cutting hair and for plumbing a building, and there ought to be at least comparable levels of standards for teaching children.

In addition, we argue that licensure needs to be associated with performance. Current forms of teacher training are not stellar, but if agreement were established on some core practices that teachers should be able to perform, then a common licensure system could be built, which would position teacher educators to build a better system to train teachers. Mathematics teacher educators need to establish some minimal threshold—agreeing that no one should be in the classroom unless they know X and can do Y. Such a step would help to build multiple pathways into teaching. The country needs a diverse teaching workforce, and a large one. Having different ways for people to enter teaching would be good, but multiple pathways are not helpful when there is no agreement about what qualifies someone to teach children. Less worry needs to be given to which pathways, or how many pathways, and more worry needs to be given to establishing a standard that says whether someone is safe to practice or not. Then, a standard for continued professional education could be set so that once people meet the initial licensure standard, a second license would be required in order to remain or to specialize.

At present, people are leaving teaching in droves. Two points are worth making about this problem. One is that the relationship between teacher age and their leaving follows a U-shaped curve (Ingersoll, 2001). In any occupation for which people enter in their twenties, a high proportion of them leave by the time they are thirty. People in their twenties are often still seeking their niche, and some change course. That should be expected and it would be good to accept this reality and to design for it. A path could be planned for people who are willing to teach for a few years in ways that ensured they did a good job for their time teaching in the classroom before moving on to other roles and occupations. Indeed, it would be good to have more adults in U.S. society who had taught for a few years, who understand something about schools from the perspective of having actually taught. This might help to counteract the apprenticeship of observation that Lortie (1975) argued profoundly distorts people's view of classroom teaching, creating a society ill prepared to engage in informed public discourse about teaching and its improvement. In addition, these journeymen and -women could then carry knowledge about children and teaching into other roles, for instance as parents, managers, or voters. Likewise, it would be good to have business leaders who are oriented toward the teaching and learning of people in their corporation, and connected to and invested in schools. Teaching and learning have applicability broader than the classroom and having more people in our society who are familiar with classroom teaching and learning would be helpful.

At present, people often leave teaching because they do a bad job (Ingersoll, 2001; Smith & Ingersoll, 2004). On the one hand, we might want such teachers to leave, but on the other hand, it is frustrating to have so many leave because they were not adequately prepared and feel badly about the role they have played. Leaving because while they like the profession, they haven't been prepared to be successful, is bound to leave a bad taste for schools and for education, further eroding public sentiment toward mathematics education.

At present, there is no evidence that teacher training or certification actually produces people who are more skilled in teaching than anyone else (National Mathematics Panel, 2008). This is not a popular view in the education community, but admitting it is the first step toward building training systems and assessments that are effective. Otherwise, the message is, "anyone can teach," even though that is not what anyone in the field thinks. It is not the case that anyone who knows mathematics well is prepared to teach. Teaching mathematics is a mathematically specialized endeavor. The current system does not equip people with specialized training. The fact that training mechanisms do not work now does not mean that good training and assessments cannot be developed. They just do not exist now. It would be better to acknowledge that the current system is inadequate and to start building training and assessment that is adequate. This would be a better investment of time and energy than defending the status quo.

A fourth strategic idea for designing a strong instructional system would be to focus mathematics teacher licensure and teacher training on practice. When we say practice here, we mean to include content as it relates to practice. Included here is the work of our own research group at the University of Michigan on mathematical knowledge for teaching, that is, a practical use of mathematics in teaching (Ball, Hill, & Bass, 2005). This has been written about extensively elsewhere, but the point here is to be clear that a focus on practice includes attention to practice-based content knowledge. There is evidence that such professional knowledge and skill are positively related to student achievement (Hill, Rowan, & Ball, 2005; Rockoff, Jacob, Kane, & Staiger, 2008) and that it can be effectively taught (Hill & Ball, 2004). What we still need, however, is research that further builds this knowledge base and builds systems to scaffold people's capacity to know this mathematics and to know how to use it to teach students.

A more direct focus on practice, and a much-needed focus, concerns finding ways to distill teaching practice into a strategic and manageable set of high-leverage practices and designing ways of integrating these practices into teacher development. Several scholars have begun exploring issues of closely modeling, training, and coaching teaching practice (Ball, Sleep, Boerst, & Bass, 2009; Grossman et al., 2009; Grossman & McDonald, 2008; Lampert, 2010; Lampert & Graziani, 2009). There is increasing evidence that teaching practice can be taught and scaffolded and that doing so addresses the unpredictability of learning from experience and the problem of building capacity at scale, especially as it reshapes the culture around teaching in schools.

The final key target area for designing a healthy instructional system, one discussed earlier, is the need for a system to support the early years of teaching so that people go from an initial, basic, safe-to-practice stage to full membership in a professionally skillful staff. That would require different licensure levels for the same school (and differentiated staffing) and that would require that teachers be able to continue to develop their skill as they become more accomplished. For example, it might be good if first-year teachers were able to identify some of the most frequent difficulties children have when learning a specific topic, and then, as beginning teachers become more accomplished, they could learn about some of the more subtle difficulties that students have. They do not need to grasp all of the nuances in the first year. It is important to recognize that beginning teachers who have two or three things that they know about frequent student difficulties for each topic are in a noticeably better place to teach than those who do not. A teacher who does not know the most frequent things that come up and has to puzzle at every turn should probably not be teaching. Qualified beginning teachers need to know the most common and central student difficulties and, as they become more advanced, they need to be exposed to more difficult and complex issues that students face as they learn specific content. In addition, they should be supported in developing the more complex practices of teaching. In short, teacher learning of practice needs to be more staged across time than it currently is.

Early career support is assumed in most other occupations in our country, from unskilled or blue-collar occupations to professional fields, such as architecture, nursing, or social work. It is quite astounding that early-career support is not built into teacher induction and development. Instead, the work of beginners is undifferentiated from the work of more experienced experts. One might ask why. As we mentioned before, teaching is by far the largest occupation in this country. There are roughly 3.75 million teachers, and the number is rising (Keigher, 2010). This is not surprising because children are the largest demographic group. Hence, the scale of the problem is significant. The improvement of mathematics education will require preparing a large number of ordinary people to do important and skilled work. That is why our point about systemic-ness is important. Little progress will result from recruiting a few talented people. It is great to have some retired engineers, and others, turning to teaching as a second career. It is great that Math for America recruits mathematically well-trained people and that Teach for America recruits able, elite, college students. Adding these people to the teaching workforce is a boon, but these additions do not come close to obtaining the millions of quality teachers that are needed. The United States must add to the teaching workforce without putting teachers in the classroom who do not yet know what they are doing and do not yet know the mathematics in ways that would allow them to teach it effectively. Mathematics teacher educators in the United States are responsible for supplying the training and licensure components of the system and they have a vital role to play in advancing national agendas that address key features of a revamped instructional system.

Moving Beyond Myths

Returning to the pretest at the beginning of this chapter (see Fig. 2.1), the issues raised are interpretative and are meant to be engaging and provocative. It may be surprising to know, for example, that the teacher education curriculum that is taught in most institutions of higher education is the same as it was in 1940. That is telling when one thinks about how much the situations that teachers have to deal with in schools have changed. The third statement is particularly important, and worth emphasizing. As people engage in the profession of education, it is important that mathematics teacher educators be spokespeople for the fact that there is extensive evidence that individual teachers have an enormous impact on student learning and that one good teacher can make a big difference in children's gains in a school year. Studies consistently indicate that, adjusting for student characteristics, roughly one-tenth of the variance in student achievement gains is associated with teachers, that cumulative effects are even larger, and that effective teaching substantially lessens differences in achievement predictable by student characteristics, in particular differences predictable by race/ethnicity and social class (Goldhaber, 1999; Nye, Konstantopoulos, & Hedges, 2004; Rivkin, Hanushek, & Kain, 2005; Rockoff, 2004; Sanders & Rivers, 1996). These effects are as significant within a single school building as they are between schools. For instance, two teachers can work side by side with the same population of students in the same context, with one teacher producing, year after year, more than a year's worth of growth in students while the next-door teacher does not. The problem, of course, is that no one has yet completely figured out what differentiates the skill of those two teachers. Education has an effect, even at the level of the individual teacher. And this is the positive note, one about the profound importance of teaching, which we have argued is central to making progress on the problem of mathematics education in the United States.

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