

# Preface

A functional differential equation (FDE) describes the evolution of a dynamical system for which the rate of change of the state variable depends on not only the current but also the historical and even future states of the system. FDEs arise naturally in economics, life sciences, and engineering, and the study of FDEs has been a major source of inspiration for advancement of nonlinear analysis and infinite-dimensional dynamical systems. Therefore, FDEs provide an excellent theoretical platform for developing an interdisciplinary approach to understanding complex nonlinear phenomena via appropriate mathematical techniques.

Unfortunately, the study of FDEs is difficult for newcomers, since a background in nonlinear analysis, ordinary differential equations, and dynamical systems is a prerequisite. On the other hand, the novelty and challenge of fundamental research in the field of FDEs has often been underappreciated. This is especially so in our effort to describe the qualitative behaviors of solutions near equilibria or periodic orbits: these qualitative behaviors can be derived from those of finite-dimensional (ordinary differential) systems obtained through a center and center-unstable manifold reduction process, and hence the (local) bifurcation theory that deals with significant changes in these qualitative behaviors is in principle a consequence of the corresponding theory for finite-dimensional (ordinary differential) systems. The highly nontrivial and often lengthy calculation of center manifold reduction, however, not only leads to enormous duplication of calculation efforts, but also prevents us from discovering simple and key mechanisms behind observed bifurcation phenomena due to the infinite-dimensionality of FDEs. This, in turn, makes it difficult to express bifurcation results explicitly in terms of model parameters and to compare and validate different results. Another challenge is the study of the birth and global continuation of bifurcation of periodic solutions and the coexistence of multiple periodic solutions when the parameters are far from the bifurcation/critical values. There has been substantial progress dedicated to this global bifurcation problem, and remarkably, the presence of a delayed or advanced argument in the nonlinearity can sometimes facilitate the application of topological methods such as equivalent degrees to examine the global continua of branches of periodic solutions, and this has inspired interesting developments in the spectral analysis of circulant matrices.

On the other hand, the study of dynamical systems with symmetries has become well established as a major branch of nonlinear systems theory. The current interest in the field dates mainly to the appearance of the equivariant branching lemma of Vanderbauwhede and Cicogna and the equivariant Hopf bifurcation theorem of Golubitsky and Stewart, both of which are reviewed in the book by Golubitsky, Stewart and Schaeffer. Since then, important new theories have been developed for more complex dynamical phenomena, including the existence, stability, and bifurcations of systems of heteroclinic connections, and the symmetry groups and bifurcations of chaotic attractors.

To a large extent, the phenomenal growth in the subject has been due to its effectiveness in explaining the bifurcations and dynamical phenomena that are seen in a wide range of physical systems including coupled oscillators, reaction–diffusion systems, convecting fluids, and mechanical systems. A local symmetric bifurcation theory for FDEs can be derived from that of but since some special properties of spatiotemporal symmetry of FDEs may be reflected generically in the reduced finite-dimensional systems, one can and should make general observations about the particular bifurcation patterns of symmetric FDEs.

The purpose of this book is to summarize some practical and general approaches and frameworks for the investigation of bifurcation phenomena of FDEs depending on parameters. The book aims to be self-contained, so the reader should find in this book all relevant materials on bifurcation, dynamical systems with symmetry, functional differential equations, normal forms, and center manifold reduction. This material was used in graduate courses on functional differential equations at Hunan University (China) and York University (Canada). We want to thank all students in these courses for their careful reading and some helpful comments. We would like especially to thank Dr. Jing Fang and Dr. Xiang-Sheng Wang for their careful reading of an early version of the manuscript and for their critical comments.

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