

Contents

| | | |
|---|---|-----------|
| 1 | Introduction | 1 |
| 1.1 | Information and Control | 1 |
| 1.2 | Coverage and the Intended Audience | 3 |
| 1.3 | Contents of the Book | 5 |
| 1.3.1 | Part I. Information Structures in a Networked Control System | 5 |
| 1.3.2 | Part II. Stabilization of Networked Control Systems | 6 |
| 1.3.3 | Part III. Optimization in Networked Control: Design of Optimal Policies Under Information Constraints | 7 |
| 1.4 | A Guide for the Reader or the Instructor | 8 |
| Part I Information Structures in Networked Control | | |
| 2 | Networked Control Systems as Stochastic Team Decision Problems: A General Introduction | 11 |
| 2.1 | Introduction | 11 |
| 2.2 | A Mathematical Framework For Static Decision Problems | 12 |
| 2.3 | An Illustrative Example of a Finite Stochastic Team | 17 |
| 2.4 | A Mathematical Framework for Dynamic Decision Problems | 26 |
| 2.5 | An Illustrative Example of a Finite Dynamic Team | 33 |
| 2.6 | Team-Optimal Solutions for Static Teams | 39 |
| 2.6.1 | Teams with Finite Measurement Spaces | 39 |
| 2.6.2 | Teams on Finite-Dimensional Spaces | 49 |
| 2.6.3 | Two Special Cost Structures | 52 |
| 2.7 | Concluding Remarks | 74 |
| 2.8 | Bibliographic Notes | 74 |
| 3 | Characterization and Comparison of Information Structures | 77 |
| 3.1 | Introduction | 77 |
| 3.2 | Comparison of Information Structures | 78 |

| | | |
|----------|--|------------|
| 3.3 | Dynamic Teams with Nonclassical Information: Importance of Signaling | 89 |
| 3.3.1 | Witsenhausen's Counterexample with Discrete Distributions | 90 |
| 3.3.2 | Witsenhausen's Counterexample | 92 |
| 3.3.3 | Generalized Gaussian Test Channel | 95 |
| 3.4 | Dynamic Teams with Classical or Quasi-classical Information Patterns | 97 |
| 3.5 | Probability and Cost-dependent Properties and Expansion of Information Structures | 101 |
| 3.5.1 | Performance-irrelevant Signaling and a Stochastic Interpretation of Nestedness | 101 |
| 3.5.2 | Expansion of Information Structures: A Recipe for Identifying Sufficient Information | 105 |
| 3.6 | Signaling Through Control Actions | 110 |
| 3.7 | Revisiting Witsenhausen's Characterization of Information Structures | 112 |
| 3.8 | Concluding Remarks | 116 |
| 3.9 | Bibliographic Notes | 116 |
| 4 | Topological Properties of Information Structures: Comparison, Convergence, and Optimization | 119 |
| 4.1 | Introduction | 119 |
| 4.2 | Measurement Channels as Information Structures | 120 |
| 4.3 | Concavity on the Space of Channels and Blackwell's Comparison of Information Structures | 121 |
| 4.4 | Topological Characterization of Measurement Channels | 124 |
| 4.5 | Single Stage: Continuity of the Optimal Cost in Channels | 127 |
| 4.6 | Single Stage: Existence of Optimal Channels | 132 |
| 4.7 | Quantizers as a Class of Channels | 134 |
| 4.8 | The Multistage Case | 139 |
| 4.9 | Multi-agent Setting | 142 |
| 4.10 | Revisiting Nonclassical Information Structures and Lack of Convexity Due to Signaling | 143 |
| 4.11 | Conditions for Continuity Under Weak Convergence and Empirical Consistency | 145 |
| 4.12 | Appendix: Proofs | 147 |
| 4.12.1 | Proof of Lemma 4.4.1 | 147 |
| 4.12.2 | Proof of Theorem 4.5.1 | 148 |
| 4.12.3 | Proof of Lemma 4.6.3 | 148 |
| 4.12.4 | Proof of Theorem 4.7.3 | 149 |
| 4.13 | Concluding Remarks | 150 |
| 4.14 | Bibliographic Notes | 151 |

Part II Stabilization of Networked Control Systems

| | | |
|----------|--|-----|
| 5 | Coding for Control and Connections with Information Theory | 155 |
| 5.1 | Introduction | 155 |
| 5.2 | Quantization and Real-Time Coding | 155 |
| 5.2.1 | Real-Time Coding | 155 |
| 5.2.2 | Information Structures for Real-Time Encoders and Controllers: Policies, Actions and Measurability | 157 |
| 5.3 | Information Theoretic Preliminaries and Performance of Quantizers | 162 |
| 5.3.1 | Information Theoretic Preliminaries | 162 |
| 5.3.2 | Fixed or Variable Rates of a Quantizer/Encoder | 164 |
| 5.3.3 | Rate-distortion Theory | 165 |
| 5.3.4 | Channel Coding and Shannon Capacity | 166 |
| 5.4 | Infinite-Dimensional Coding Versus Finite-Dimensional Coding | 168 |
| 5.5 | Noncausal Coding for Stationary and Nonstationary Sources | 170 |
| 5.6 | Fundamental Bounds on Information Rates for Real-time Stabilization Over Noiseless Channels | 172 |
| 5.7 | Appendix: Proof of Theorem 5.6.1 | 173 |
| 5.8 | Concluding Remarks | 175 |
| 5.9 | Bibliographic Notes | 176 |
| 6 | Stochastic Stability and Drift Criteria for Markov Chains in Networked Control | 179 |
| 6.1 | Introduction and Motivation: Why Stochastic Drift Criteria? | 179 |
| 6.2 | Stochastic Stability and Drift Criteria | 180 |
| 6.2.1 | One-stage Foster–Lyapunov Drift Criteria | 180 |
| 6.2.2 | State-dependent Drift Criteria | 181 |
| 6.2.3 | Random-time State-dependent Stochastic Drift Criteria | 181 |
| 6.3 | Appendix: Proofs | 184 |
| 6.3.1 | Proof of Theorem 6.2.4 | 184 |
| 6.3.2 | Proof of Theorem 6.2.8 | 186 |
| 6.4 | Concluding Remarks | 187 |
| 6.5 | Bibliographic Notes | 188 |
| 7 | Stochastic Stabilization Over Noiseless Channels | 189 |
| 7.1 | Introduction | 189 |
| 7.2 | Control and Communication Models | 189 |
| 7.3 | Stochastic Stability Analysis for a Scalar System | 190 |
| 7.3.1 | Adaptive Quantizers and a Zooming Scheme | 190 |
| 7.3.2 | Stochastic Stability Analysis | 191 |
| 7.3.3 | Application of the Theory of Random-time State-dependent Stochastic Drift | 193 |
| 7.3.4 | Simulation | 194 |

| | | |
|----------|---|------------|
| 7.4 | The Multidimensional Case | 195 |
| 7.5 | The Partially Observed Case | 198 |
| 7.6 | Appendix: Proofs | 199 |
| 7.6.1 | Proof of Theorem 7.3.1 | 199 |
| 7.6.2 | Proof of Theorem 7.3.2 | 200 |
| 7.6.3 | Proof of Theorem 7.3.3 | 203 |
| 7.6.4 | Proof of Theorem 7.3.4 | 206 |
| 7.6.5 | Proof of Theorem 7.3.5 | 210 |
| 7.6.6 | Proof of Theorem 7.3.6 | 211 |
| 7.6.7 | Proof of Theorem 7.4.1 | 212 |
| 7.7 | Concluding Remarks | 213 |
| 7.8 | Bibliographic Notes | 213 |
| 8 | Stochastic Stabilization Over Noisy Channels | 215 |
| 8.1 | Introduction | 215 |
| 8.2 | Stabilization Over Noisy Channels with Noiseless Feedback and a Converse Theorem | 217 |
| 8.2.1 | Control and Communication Model | 217 |
| 8.2.2 | Converse Theorem on Stochastic Stability Over a Discrete Memoryless Channel | 218 |
| 8.3 | Stochastic Stabilization Over Erasure Channels with Feedback .. | 219 |
| 8.3.1 | Connections with Random-time Drift Criteria | 223 |
| 8.3.2 | Simulation | 224 |
| 8.4 | Stochastic Stabilization Over DMCs with Feedback | 225 |
| 8.5 | Channels with Memory and Multidimensional Sources | 230 |
| 8.6 | Stabilization with Noisy Forward and Feedback/Reverse Channels | 233 |
| 8.6.1 | Formulation | 233 |
| 8.6.2 | Necessary Conditions for Stabilization | 235 |
| 8.6.3 | Stabilization Over Discrete Channels and State-dependent Sampling | 237 |
| 8.6.4 | Stabilization Over Continuous-Alphabet Channels | 243 |
| 8.7 | Appendix: Proofs | 245 |
| 8.7.1 | Proof of Theorem 8.5.2 | 245 |
| 8.7.2 | Proof of Proposition 8.5.1 | 246 |
| 8.7.3 | Proof of Proposition 8.5.3 | 249 |
| 8.7.4 | Proof of Proposition 8.3.1 | 249 |
| 8.7.5 | Proof of Theorem 8.3.1 | 255 |
| 8.7.6 | Proof of Theorem 8.3.3 | 257 |
| 8.7.7 | Proof of Theorem 8.4.1 | 259 |
| 8.7.8 | Proof of Theorem 8.4.2 | 269 |
| 8.7.9 | Proof of Theorem 8.4.3 | 270 |
| 8.7.10 | Proof of Theorem 8.4.4 | 271 |
| 8.7.11 | Proof of Theorem 8.4.5 | 276 |
| 8.7.12 | Proof of Theorem 8.5.4 | 276 |

| | | |
|----------|---|------------|
| 8.7.13 | Proof of Theorem 8.6.1 | 278 |
| 8.7.14 | Proof of Theorem 8.6.2 | 279 |
| 8.7.15 | Proof of Theorem 8.6.3 | 280 |
| 8.7.16 | Proof of Theorem 8.6.4 | 281 |
| 8.7.17 | Proof of Theorem 8.6.5 | 281 |
| 8.7.18 | Proof of Theorem 8.6.6 | 283 |
| 8.7.19 | Proof of Theorem 8.6.7 | 284 |
| 8.8 | Concluding Remarks | 285 |
| 8.9 | Bibliographic Notes | 287 |
| 9 | Stabilization of Decentralized Systems | |
| | Over Communication Channels | 293 |
| 9.1 | Introduction | 293 |
| 9.2 | Problem Formulation | 294 |
| 9.3 | Existence of Decentralized Stabilizing Controllers and Time-Varying Linear Feedback Laws | 295 |
| 9.4 | Decentralized Stabilization over Communication Channels | 300 |
| 9.5 | Multi-Sensor Structure with a Centralized Controller | 305 |
| 9.6 | Multi-Sensor and Multi-Controller Systems Driven by Noise | 306 |
| 9.6.1 | Multi-Sensor Systems Driven by Unbounded Noise | 306 |
| 9.6.2 | Multi-Controller Systems Driven by Unbounded Noise | 308 |
| 9.7 | Illustration of Binning and Its Use in Decentralized Stabilization | 308 |
| 9.8 | Appendix: Proofs | 311 |
| 9.8.1 | A Supporting Lemma | 311 |
| 9.8.2 | Proof of Theorem 9.4.1 | 311 |
| 9.8.3 | Proof of Lemma 9.4.1 | 313 |
| 9.8.4 | Proof of Lemma 9.4.2 | 315 |
| 9.9 | Concluding Remarks | 315 |
| 9.10 | Bibliographic Notes | 315 |

Part III Optimization in Networked Control: Design of Optimal Policies Under Information Constraints

| | | |
|-----------|--|------------|
| 10 | Optimization of Real-Time Coding and Control Policies: | |
| | Structural and Existence Results | 319 |
| 10.1 | Introduction | 319 |
| 10.2 | Policies and Action Spaces for Encoding | 320 |
| 10.3 | Single Terminal Case: Optimal Causal Coding of a Partially Observed Markov Source | 322 |
| 10.3.1 | Single Terminal, Fully Observed Case | 322 |
| 10.3.2 | Partially Observed Markov Source | 323 |
| 10.3.3 | Structural Results for Systems with Control | 326 |

| | | |
|-----------|---|------------|
| 10.4 | Existence of Optimal Zero-Delay Quantizers | 327 |
| 10.5 | Multiterminal (Decentralized) Setting | 329 |
| 10.5.1 | Memoryless Sources | 329 |
| 10.5.2 | Markov Sources: Nonclassical Information Structure and a Counterexample Under Signaling | 330 |
| 10.6 | Simultaneous Optimization of LQG Coding and Control Policies: Optimal Quantization and Control | 332 |
| 10.6.1 | Application to the LQG Setup: Separation of Estimation and Quantization | 333 |
| 10.6.2 | Optimal LQG Coding and Control Policies and Separation Results | 334 |
| 10.6.3 | Existence of Optimal Quantization Policies | 338 |
| 10.6.4 | Partially Observed Case | 339 |
| 10.7 | Case with Noisy Channels and Noiseless Feedback | 341 |
| 10.8 | Appendix: Proofs | 342 |
| 10.8.1 | Proof of Theorem 10.3.1 | 342 |
| 10.8.2 | Proof of Theorem 10.3.2 | 344 |
| 10.8.3 | Proof of Theorem 10.3.3 | 347 |
| 10.8.4 | Proof of Theorem 10.3.4 | 350 |
| 10.8.5 | Proof of Theorem 10.3.6 | 352 |
| 10.8.6 | Proof of Theorem 10.4.2 | 353 |
| 10.8.7 | Proof of Theorem 10.5.1 | 359 |
| 10.8.8 | Proof of Lemma 10.6.1 | 363 |
| 10.8.9 | Proof of Theorem 10.6.3 | 364 |
| 10.8.10 | Proof of Theorem 10.6.4 | 366 |
| 10.9 | Concluding Remarks | 369 |
| 10.10 | Bibliographic Notes | 369 |
| 11 | Optimal Coding and Control for Linear Gaussian Systems Over Gaussian Channels Under Quadratic Cost | 373 |
| 11.1 | Introduction | 373 |
| 11.2 | Gaussian Source-Channel Pairs and Optimality of Linear Policies | 374 |
| 11.2.1 | Optimality of Linear Coding Policies over a Gaussian Channel with Matching Between the Source and the Channel | 374 |
| 11.2.2 | The Gaussian Pair: Gaussian Sources and Channels ... | 375 |
| 11.2.3 | Multi-Dimensional Source and Channels | 376 |
| 11.3 | Joint Optimization of Encoder and Controllers for Linear Systems Controlled Over Gaussian Channels | 377 |
| 11.3.1 | Problem Setup | 377 |
| 11.3.2 | Optimality of Linear Policies | 379 |
| 11.4 | Stabilization over Gaussian Channels and Sufficiency of Shannon Capacity Conditions | 382 |

| | | |
|-----------|---|------------|
| 11.5 | Two Counterexamples on Sub-optimality of Linear Policies | 385 |
| 11.5.1 | Gaussian Relay Channels with Two Encoders: Person-by-Person-Optimality of Linear Policies and Lack of Convexity of the Team Problem | 385 |
| 11.5.2 | A Decentralized Sensing Problem Over Vector Gaussian Channels | 387 |
| 11.6 | Looseness of Information Theoretic (Cut-Set) Bounds for Gaussian Networks | 389 |
| 11.7 | Appendix: Proofs | 390 |
| 11.7.1 | Proof of Theorem 11.3.1 | 390 |
| 11.7.2 | Proof of Theorem 11.3.2 | 391 |
| 11.7.3 | Proof of Theorem 11.5.1 | 394 |
| 11.8 | Concluding Remarks | 396 |
| 11.9 | Bibliographic Notes | 396 |
| 12 | Agreement in Teams and the Dynamic Programming Approach Under Information Constraints | 399 |
| 12.1 | Introduction | 399 |
| 12.2 | Common Knowledge and Agreement | 400 |
| 12.2.1 | Common Knowledge | 400 |
| 12.2.2 | Asymptotic Agreement with Common Priors but Different Posteriors | 401 |
| 12.2.3 | Inconsistent Priors (Probability Models), Lack of Agreement and Merging | 402 |
| 12.2.4 | Agreement in Finite Time Over Noisy Channels | 404 |
| 12.3 | Common Knowledge as Information State and the Dynamic Programming Approach to Team Decision Problems | 405 |
| 12.4 | k -Stage Periodic Belief Sharing Pattern and Communication Requirements | 406 |
| 12.4.1 | k -Stage Periodic Belief Sharing Pattern | 406 |
| 12.4.2 | Minimum Communication for the Belief Sharing Pattern | 412 |
| 12.5 | A Team Cost-Rate Function | 416 |
| 12.6 | Concluding Remarks | 419 |
| 12.7 | Bibliographic Notes | 419 |
| A | Topological Notions and Optimization | 423 |
| A.1 | Sets | 423 |
| A.2 | Vector Spaces | 424 |
| A.3 | Matrices | 429 |
| A.4 | Convex Sets and Functionals | 430 |
| A.5 | Optimization of Functionals | 431 |
| A.6 | Contraction Mappings and Fixed-Point Theorems | 433 |

| | | |
|----------|---|-----|
| B | Probability Theory and Stochastic Processes | 435 |
| B.1 | Probability..... | 435 |
| B.1.1 | Measurable Spaces..... | 435 |
| B.1.2 | Integration..... | 437 |
| B.1.3 | Probability Spaces and Random Variables..... | 438 |
| B.2 | Convergence of Probability Measures..... | 441 |
| B.3 | Conditional Expectation and Estimation..... | 443 |
| B.4 | Stochastic Processes..... | 444 |
| C | Markov Chains, Martingales, and Ergodic Processes | 447 |
| C.1 | Markov Chains..... | 447 |
| C.2 | Discrete-Time Martingales..... | 451 |
| C.3 | Stochastic Stability of Dynamical Systems and Random Processes..... | 452 |
| C.3.1 | Stationary, Ergodic, and Asymptotically Mean Stationary Processes..... | 452 |
| D | Markov Decision Theory and Optimality of Markov Policies | 455 |
| D.1 | Controlled Markov Models..... | 455 |
| D.1.1 | Fully Observed Markov Control Problem Model..... | 455 |
| D.1.2 | Classes of Control Policies..... | 456 |
| D.1.3 | Optimality of Markov Policies and Elimination of Irrelevant Information..... | 457 |
| D.1.4 | Markov Decision Processes (MDPs) and Optimality of Markov Policies..... | 457 |
| D.1.5 | Dynamic Programming and Measurable Selection Criteria..... | 458 |
| D.1.6 | Partially Observable MDPs (POMDPs)..... | 459 |
| D.2 | Kalman Filter and Linear-Quadratic-Gaussian Optimal Control Problem..... | 460 |
| | References | 463 |
| | Index | 481 |



<http://www.springer.com/978-1-4614-7084-7>

Stochastic Networked Control Systems
Stabilization and Optimization under Information
Constraints

Yüksel, S.; Başar, T.

2013, XVIII, 482 p., Hardcover

ISBN: 978-1-4614-7084-7

A product of Birkhäuser Basel