

Contents

1	Introduction	1
1.1	Aim of the Book	1
1.2	Analogies in Physics	9
1.3	Role of a Classification	13
1.4	Role of Geometry in Physics	14
1.4.1	Topological Concepts	15
1.5	Algebraic Formulation of Fields	17
1.6	Summary	17
1.6.1	Notation Used in Book	18
 Part I Analysis of Variables and Equations		
2	Terminology Revisited	21
2.1	Why Do We Need Proper Notation and Terminology?	21
2.2	Many Meanings of Δ	22
2.3	Use and Misuse of Symbol ‘d’	23
2.4	Rates and Derivatives	26
2.5	Definite and Indefinite Integrals	27
2.6	Multiplication Symbols \times and \wedge	27
2.7	Many Meanings of “Force”	30
2.8	Many Meanings of “Flux”	30
2.9	Many Kinds of Equality	32
2.10	Irrotational and Solenoidal Vector Fields	34
2.11	Reversal of Motion Is Not Time Reversal	35
3	Space and Time Elements and Their Orientation	39
3.1	Space Elements	39
3.2	Orientation of Space Elements	40
3.3	Combinatorial Side of Orientation	41
3.4	Orientation in Geometry	42

3.5	Two Kinds of Orientation: Inner and Outer	44
3.5.1	Inner and Outer Orientation of Lines	45
3.5.2	Inner and Outer Orientation of Surfaces	45
3.5.3	Inner and Outer Orientation of Volumes	47
3.5.4	Inner Orientation of Points	49
3.5.5	Outer Orientation of Points	51
3.6	Role of Space of Immersion	53
3.6.1	Induced Orientation	54
3.6.2	Holes	54
3.7	Historical Note on Orientation	56
3.8	Time Elements	57
3.8.1	Primal Time Elements and Their Orientation	58
3.8.2	Dual Time Elements and Their Orientation	59
4	Cell Complexes	63
4.1	Coordinate Systems and Cell Complexes	63
4.1.1	Coordinate Cell Complexes	65
4.1.2	Families of Cells	66
4.1.3	Simplicial Cell Complexes	67
4.2	Dual Cell Complexes	69
4.2.1	Duals of Simplicial Complexes	72
4.2.2	Voronoi Dual	72
4.2.3	Barycentric Dual	74
4.3	Inner and Outer Oriented Cell Complexes	75
4.3.1	Outer Orientation	76
4.4	Role of Dual Complex in Mathematics	78
4.5	Role of Dual Complex in Physics	79
4.6	Global Variables and Computational Physics	81
4.6.1	Finite Difference Method Reinterpreted	82
4.7	Classification Diagram for Space Elements	84
4.8	Classification of Time Elements	85
5	Analysis of Physical Variables	89
5.1	Role of Mathematics in Physics	89
5.2	Material and Spatial Descriptions	90
5.2.1	Material Description	92
5.2.2	Spatial Description	93
5.2.3	Material and Spatial Descriptions: An Overview	94
5.3	Physical Quantities	96
5.3.1	Physical Constants	96
5.3.2	Physical Parameters	96
5.3.3	Physical Variables	97

5.4	Configuration, Source and Energy Variables	99
5.4.1	Source Variables	100
5.4.2	Configuration Variables	101
5.4.3	Energy Variables	102
5.5	Fundamental Problem of a Physical Theory	103
5.6	Set Functions	104
5.7	Global Variables and Field Variables	105
5.8	Global Variables	106
5.8.1	Extension of the Notion of Density	108
5.8.2	Global Variables in Space	109
5.8.3	Continuity of Global Variables in Space	112
5.8.4	Space Association	113
5.9	Genesis of + and – Signs	118
5.10	Sign of Physical Variables	118
5.11	Oddness Principle	122
5.11.1	Global Variables in Time	125
5.12	Time Association	126
5.12.1	Primal or Dual Time Elements?	130
5.12.2	Global Variables in Space and Time	133
5.13	Field Variables: Inherited Association	135
5.14	How to Find the Space and Time Association	137
5.15	Physical Variables Can Be Grouped into Families	139
5.16	Identification Criteria	141
5.16.1	Possible Ambiguities	143
5.17	Differential Formulation and Orientation	145
5.18	What Vector Calculus Ignores	147
5.18.1	Energy as a Potential of Constitutive Equations	148
5.19	Conjugated Variables	149
5.20	Phase, Angular Frequency and Wave Vector	152
6	Analysis of Physical Equations	155
6.1	Introduction	155
6.2	Phenomenological Equations	160
6.3	Constitutive Equations	160
6.3.1	Reversible and Irreversible Constitutive Equations	161
6.3.2	Interaction Equations	165
6.4	Topological Equations	168
6.4.1	Balance Equations	170
6.4.2	Circuitual Equations	171
6.4.3	Space Differences	173
6.5	Invariance of Equations Under Inversion of Orientation	174
6.6	Defining Equations	176
6.7	Equations of Behaviour	177

7	Algebraic Topology	181
7.1	Why Algebraic Topology?	181
7.2	Topology and Algebraic Topology	184
7.3	Role of Cell Complexes	185
7.3.1	Faces of a p -Cell and Boundary	185
7.3.2	Cofaces of a p -Cell and Coboundary	185
7.3.3	Incidence Numbers and Incidence Matrices	186
7.4	The Notion of a Chain	190
7.4.1	Boundary of a Line, Surface and Volume	193
7.4.2	Boundary of a Chain	193
7.4.3	The Boundary of a Boundary Is a Null Chain	194
7.5	Notion of Discrete Form	196
7.5.1	Value of a Discrete Form on a Chain	198
7.5.2	Coboundary of a Discrete p -Form	200
7.5.3	Coboundary of a Discrete 0-Form	200
7.5.4	Coboundary of a Discrete 1-Form	202
7.5.5	Coboundary of a Discrete 2-Form	203
7.5.6	General Definition of Coboundary Process	204
7.5.7	Discrete Version of Stokes' Theorem	204
7.5.8	The Coboundary of the Coboundary Vanishes	207
7.5.9	Chains or Discrete Forms in Physics?	208
7.6	Coboundary Process in Two Dimensions	209
7.6.1	Coboundary Process on Primal Complex	209
7.6.2	Coboundary Process on Dual Complex	211
7.7	The Wonderful Role of the Coboundary Process in Physics	212
7.7.1	Algebraic Analogue of Derivative	213
7.7.2	Algebraic Analogue of Gradient	213
7.7.3	Algebraic Analogue of the 'Curl'	214
7.7.4	Algebraic Analogue of Divergence	215
7.8	Examples of Coboundary Process in Physics	216
7.8.1	Gauss's Law of Electrostatics	217
7.8.2	Equilibrium Law of Continuum Mechanics	218
8	Birth of Classification Diagrams	221
8.1	Classification Diagram of Physical Variables	221
8.2	Statics of Strings	224
8.3	A Time Diagram	226
8.4	How to Combine Space and Time	227
8.4.1	Transversal Vibrations of Strings	227
8.5	The Structure of the Diagrams	229
8.6	What the Diagram Shows	230
8.6.1	Global Variables Versus Field Variables	230
8.6.2	Reversible and Irreversible Links	231

8.6.3	Topological Relations	231
8.6.4	Scalar and Vectorial Theories	232
8.6.5	Exterior Differential Forms	233
8.6.6	Material Parameters	233
8.6.7	Configuration Variables are Material Dependent	233
8.6.8	Tensorial Nature of Field Functions	235
8.6.9	Composing the Equations	237

Part II Analysis of Physical Theories

9	Particle Dynamics	241
9.1	Fundamental Problem	241
9.2	Source Variables	242
9.2.1	Force	242
9.2.2	Impulse	242
9.2.3	Momentum	243
9.2.4	Force from Momentum or Vice Versa?	244
9.3	Configuration Variables	247
9.3.1	Radius Vector	247
9.3.2	Displacement	248
9.3.3	Velocity	248
9.3.4	Acceleration	249
9.4	Constitutive Laws	250
9.4.1	Momentum–Velocity Relation	250
9.4.2	Force–Radius Vector Relation	252
9.4.3	Force–Velocity Relation	252
9.4.4	Classification of Forces	252
9.5	Energy Variables	253
9.5.1	Power, Work, Kinetic Energy	253
9.5.2	Potential Energy	255
9.5.3	Potential Energy of a Particle in a Force Field	257
9.5.4	Potential Energy of a System	259
9.5.5	Kinetic Energy and Kinetic Co-energy	262
9.5.6	Lagrangian and Hamiltonian	264
10	Electromagnetism	273
10.1	Fundamental Problem	273
10.2	From Field Variables to Global Variables	273
10.3	Source Variables: Space and Time Classification	276
10.3.1	Electric Charge Content	276
10.3.2	Electric Flux	277
10.3.3	Birth of Electric Displacement	279
10.3.4	Critical Remarks	281
10.3.5	Electric Charge Flow and Electric Current	281

10.3.6	Birth of Electric Current Density Vector	282
10.3.7	Electric Vector Potential	283
10.3.8	Magnetic Field Strength	284
10.3.9	Magnetomotive Force	286
10.3.10	Scalar Magnetic Potential	287
10.4	Configuration Variables: Space and Time Classification	289
10.4.1	Electric Field Strength	289
10.4.2	Electromotive Force and Its Impulse	290
10.4.3	Electric Potential	291
10.4.4	Magnetic Flux	291
10.4.5	Magnetic Flux Density	293
10.4.6	Summary of Physical Variables of Electromagnetism	294
10.5	Field Laws	295
10.5.1	Gauss's Law	296
10.5.2	Gauss's Law for Magnetism	297
10.5.3	Faraday's Electromagnetic Induction Law	299
10.5.4	Ampère–Maxwell Law	300
10.6	Space-Time Representation of Maxwell's Equations	301
10.6.1	Visualizing Space-Time	303
10.6.2	Geometric View of Maxwell's Equations	304
10.7	Algebraic Formulation	306
10.8	Classification Diagrams of Electromagnetism	307
11	Mechanics of Deformable Solids	325
11.1	Introduction	325
11.2	Fundamental Problem	325
11.3	Source Variables	326
11.3.1	Impulse of Volume Forces	326
11.3.2	Impulse of Surface Forces	327
11.3.3	Momentum	327
11.3.4	Momentum Content, Flow and Production	328
11.3.5	Stress Vector, Stress Tensor, Pressure	328
11.4	Configuration Variables	330
11.4.1	Initial Position Vector	331
11.4.2	Relative Position Vector	331
11.4.3	Position Vector	331
11.4.4	Displacement	331
11.4.5	Relative Displacement	333
11.4.6	Displacement Gradient	334
11.4.7	Strain Tensor	334
11.4.8	Velocity	335
11.5	Field Laws	335
11.6	Rod Traction	335

11.7	Vibrations in One Dimension	339
11.8	Classification Diagrams of Deformable Solids	339
12	Mechanics of Fluids	355
12.1	Particles and Points	355
12.2	Some Peculiarities of the Fluid Field	356
12.3	Fundamental Problem	357
12.4	Fluids and Flows	358
12.4.1	Kinds of Fluids	358
12.4.2	Kind of Flows	359
12.5	Variables Used in the Steady Motion of a Perfect Fluid	360
12.6	Source Variables	360
12.6.1	Volume and Surface Forces and Their Impulses	361
12.6.2	Mass Content and Mass Density	361
12.6.3	Mass Flow, Mass Current and Mass Current Density	362
12.6.4	Stream Function	362
12.6.5	Stream Vector	363
12.6.6	Mass Production, Mass Source	364
12.6.7	Momentum Content and Momentum Density	364
12.6.8	Momentum Flow and Momentum Current	364
12.6.9	Momentum Production and Momentum Source	364
12.7	Configuration Variables	365
12.7.1	Line Integral of Velocity and Velocity Potential	365
12.7.2	Vortex Flux and Vorticity	367
12.7.3	Relative Velocity and Velocity Gradient	367
12.7.4	Strain Rate Tensor and Volume Dilatation Rate	368
12.7.5	Global Form of Mass Balance	369
12.7.6	Global Form of Momentum Balance	370
12.8	Constitutive Laws	372
12.9	Classification Diagrams of Fluid Dynamics	372
13	Other Physical Theories	385
13.1	Equilibrium Thermodynamics	385
13.2	Non-equilibrium Thermodynamics	386
13.2.1	Internal Energy	386
13.3	Thermal Conduction	387
13.3.1	Fundamental Problem	387
13.3.2	Source Variables: Space and Time Classification	387
13.3.3	Configuration Variables: Space and Time Classifications	388
13.4	Gravitational Field	389
13.4.1	Relativistic Gravitation	392
13.5	Quantum Mechanics	392

Part III Advanced Analysis

14	General Structure of the Diagrams	415
14.1	Introduction	415
15	The Mathematical Structure	429
15.1	Introduction	429
15.1.1	Discovery of Adjointness	431
15.1.2	Topological Equivalent of Algebraic Formulation	433
15.2	From Differential Operators to Algebraic Operators	436
15.2.1	Differential Operators: Some Specifications	437
15.2.2	Algebraic Equivalent of Differential Formulation	439
15.3	Physics Needs Couples of Vector Spaces	440
15.3.1	Bilinear Forms as Scalar Products	442
15.3.2	From Transposed Matrix to Adjoint Operator	444
15.3.3	Inhomogeneous Boundary Condition: Convex Set	445
15.3.4	Adjoint Operator	446
15.3.5	Further Extension of Notion of Adjoint Operator	447
15.3.6	Role of Boundary Conditions	448
15.3.7	Symmetric and Self-Adjoint Operators	449
15.3.8	Operators at the Same Level Are Mutually Adjoint	450
15.3.9	Formal Operator 'curl' is Self-Adjoint	452
15.4	The Three Kinds of Partial Differential Equations	455
A	Affine Vector Fields	457
A.1	Affine Fields	457
A.1.1	Affine Scalar Field	458
A.1.2	Affine Vector Field	459
B	Tensorial Notation	467
B.1	Summary of Tensorial Notation Used in This Book	467
B.1.1	Generalized Kronecker Delta	469
B.1.2	Permutation Symbol	470
B.1.3	Main Use of Levi-Civita Pseudotensor	472
B.1.4	Vector Components	472
B.2	Algebraic and Metric Duals	473
B.3	Bivectors	474
B.3.1	Exterior Product of Two Vectors	475
C	On Observable Quantities	477
D	History of the Diagram	481
D.1	Historical Remarks	481

E	List of Physical Variables	485
F	List of Symbols Used in This Book	493
	References	505
	Index	513

The Mathematical Structure of Classical and Relativistic
Physics

A General Classification Diagram

Tonti, E.

2013, XXXVI, 514 p. 164 illus. in color., Hardcover

ISBN: 978-1-4614-7421-0

A product of Birkhäuser Basel