

Modeling Hazardous, Free-Surface Geophysical Flows with Depth-Averaged Hyperbolic Systems and Adaptive Numerical Methods

D.L. George

Abstract The mathematical modeling and numerical simulation of gravity-driven, free-surface geophysical flows—such as tsunamis, water floods, and debris flows—are described. These shallow flows are often modeled with two-dimensional depth-averaged equations that possess similar mathematical structure: they are usually nonconservative hyperbolic systems with source terms. Some numerical challenges presented by these systems, particularly with regard to features common to depth-averaged models for flow over topography, are highlighted. The open-source software package GEOCLAW incorporates numerical algorithms designed to tackle many of the common difficulties presented by depth-averaged models, particularly for large multiscale problems featuring inundation influenced by topography. Some simulation results for tsunamis, floods, and debris flows highlight the capabilities of GEOCLAW.

1 Introduction

A wide variety of regularly occurring natural hazards can be classified as shallow, gravity-driven, free-surface geophysical flows. This class of flows includes tsunami propagation and inundation; coastal storm surges; riverine and overland flooding due to rainfall or dam and levee failures; and finally a subclass of flows involving variable granular-fluid mixtures such as mudflows and debris flows. The impact of these phenomena continues to grow due to expanding population and development coupled with anthropogenic alteration of landscapes and waterways, which may be further exacerbated by climate change because of changing rainfall patterns and vegetation. Therefore, there is an increasing interest in modeling and simulating

D.L. George (✉)
U.S. Geological Survey, Vancouver, WA, USA
e-mail: dgeorge@usgs.gov

such flows for the purposes of hazard assessment, land development and management, and infrastructure placement and design. More generally, simulations of these kinds of flows are becoming an integral part of the emerging field of sustainability science and engineering (see, e.g., [1]).

From a modeling and simulation perspective, these types of flows present similar challenges. Typically they are modeled with depth-averaged equations for conservation of mass and momentum for single or multiple constituent phases or layers. The resulting coupled partial differential equations (PDEs) are usually nonlinear hyperbolic systems. Aside from the usual well-known numerical challenges posed by hyperbolic systems (e.g., shocks and nonunique discontinuous weak solutions), depth-averaged equations for free-surface flows present a number of unique challenges. For instance, the flow usually advances over, or is routed by, complex topography, which introduces source terms in the momentum equations that can be discontinuous or singular. Additionally, the advancing inundation front represents a moving domain boundary that must be tracked or captured accurately and stably. A more subtle difficulty concerns numerical steady-state preservation: the dynamics of these free-surface flows are often small perturbations to an underlying balanced steady state where the flux gradients and source terms are large and counterbalanced. Finally, these flows have a large spatial extent and contain features with small length scales. Therefore, they are extremely multiscale in nature. For example, a tsunami wave might cross an entire ocean, yet coastal inundation can depend on meter-scale topographical features thousands of kilometers away from the source. A flood or debris flow might inundate an irregular region that is bounded by thousands of square kilometers, yet accurately modeling the flow can require sub-meter resolution.

For many applications involving water flow, such as tsunamis, river flow, and overland flooding, the use of simple depth-averaged models such as the shallow water equations has been well established for decades (see, e.g., [2,3]). For problems in which there may be more vertical variation in the flow-velocity profile, for example storm surges [4], some researchers have adopted multilayer depth-averaged models (see, e.g., [5]). These multilayer equations are also nonlinear hyperbolic systems, but they pose additional mathematical challenges, such as loss of hyperbolicity when flow velocities in different layers diverge beyond some threshold [5]. Nevertheless, it is possible to develop stable numerical schemes that avoid instabilities due to loss of hyperbolicity (see, e.g., [6,7]).

Depth-averaged models for free-surface flows involving mixtures of particles and fluid are much less well established, and simply developing the mathematical governing equations remains a significant challenge. Most models are based on depth-averaging two-phase flow with a theoretical approximation for the complicated stress that arises due to intergranular and granular-fluid interactions. The resulting equations are often nonlinear hyperbolic systems similar in mathematical structure to the shallow water equations, albeit with multiple phases and more complicated source terms (see, e.g., [8,9]). Determining the suitability of, and improving, such models is an active area of research.

The aim of this paper is to introduce and survey some of the common challenges inherent in modeling free-surface flows that move over topography. The common mathematical features of several depth-averaged equations are summarized. Additionally, numerical techniques developed to overcome these challenges are described. The full detail of the numerical techniques are left to the references—the aim here is to provide an introduction and survey of the methods implemented in GEOCLAW that are specifically relevant to these applications. The paper is organized as follows: in Sect. 2 mathematical models are introduced; in Sect. 3 some numerical challenges, methods, and software are described; and finally, in Sect. 4 some specific applications and a sample of simulation results are presented.

2 Mathematical Models

In this section hyperbolic systems are first described in general form before some depth-averaged models that belong to this class of equations are surveyed.

2.1 *Hyperbolic Conservation Laws and Nonconservative Balance Laws*

Hyperbolic systems of PDEs are commonly derived as integral conservation laws for mass, momentum, energy, or other conserved quantities. For a vector $q(x, t) \in \mathcal{R} \subseteq \mathbb{R}^m$ of m conserved quantities, a hyperbolic conservation law (in one spatial dimension) is of the form

$$\partial_t q + \partial_x f(q) = 0, \quad (2.1)$$

where $f : \mathbb{R}^m \rightarrow \mathbb{R}^m$ is a flux function and $\mathcal{R}$ represents physically admissible solution states. In order to satisfy the property of hyperbolicity, by definition the flux Jacobian $f'(q) \in \mathbb{R}^{m \times m}$ must be diagonalizable with real eigenvalues for all $q \in \mathcal{R}$. If $f'(q)$ satisfies these requirements only for some subset of states $q \in \mathcal{R}' \subsetneq \mathcal{R}$, the system is not strictly hyperbolic, and loss of hyperbolicity when $q \notin \mathcal{R}'$ results in an unstable or ill-posed system (see, e.g., [10]). Aside from this, even strictly hyperbolic systems present some well-known difficulties. For instance, nonlinear systems such as Eq. (2.1) lead to wave steepening and discontinuities, yet, due to the integral form from which Eq. (2.1) is derived, discontinuous weak solutions can be determined through Rankine–Hugoniot jump conditions based on boundary fluxes. However, this opens the possibility of nonunique weak solutions, which necessitates additional physically based admissibility conditions that are extraneous to Eq. (2.1). These are often called entropy conditions, analogous to the entropy requirements for the Euler equations of gas dynamics.

Most traditional finite difference schemes for PDEs do not converge to generalized weak solutions of Eq. (2.1), and in fact they are numerically unstable in the presence of discontinuities or sharp gradients. This has prompted the development of specialized shock-capturing schemes that can converge stably to admissible weak solutions, provided that some admissibility criteria are known and used in the design of the numerical scheme. Many of these methods are Godunov-type methods (i.e., methods making use of Riemann solvers for the numerical update). The high-resolution wave-propagation methods employed in GEOCLAW, described briefly in Sect. 3, are one example of Godunov-type methods.

For many applications, the solution $q(x, t)$ is not conserved but obeys a nonconservative balance law,

$$\partial_t q + \mathcal{A}(q) \partial_x q = \psi(q, x), \quad (2.2)$$

where $\psi : \mathbb{R}^m \times \mathbb{R} \rightarrow \mathbb{R}^m$ is a source term and $\mathcal{A} : \mathbb{R}^m \rightarrow \mathbb{R}^{m \times m}$ is some matrix-valued function. Similar to a conservation law, Eq. (2.2) is hyperbolic if $\mathcal{A}(q)$ is diagonalizable with real eigenvalues. The quasilinear form (2.2) is a more general form of a hyperbolic system: it includes Eq. (2.1) in the special case in which $\mathcal{A}(q) = f'(q)$ and $\psi(q) \equiv 0$. More generally, the term $\mathcal{A}(q) \partial_x q$ could be a nonconservative product (i.e., if $\mathcal{A}(q)$ is not a full derivative of a flux). Systems with nonconservative products present additional mathematical challenges. Because there is no flux function to provide Rankine–Hugoniot jump conditions for discontinuities, determining weak solutions near a discontinuity can be difficult (see, e.g., [11–14]), but finding such solutions can be advantageous in the design of numerical schemes (see, e.g., [13, 15]).

In two dimensions (2D), a hyperbolic system takes the general form

$$\partial_t q + \mathcal{A}(q) \partial_x q + \mathcal{B}(q) \partial_y q = \psi(q, x, y), \quad (2.3)$$

where similar to Eq. (2.2), $q(x, y) \in \mathbb{R}^m$, $\psi : \mathbb{R}^m \times \mathbb{R}^2 \rightarrow \mathbb{R}^m$, and $\mathcal{A}, \mathcal{B} : \mathbb{R}^m \rightarrow \mathbb{R}^{m \times m}$. The depth-averaged models described in Sect. 2.2 are of this general form.

2.2 Depth-Averaged Equations

For many geophysical surface flows, it is common to apply the shallow approximation, that is, the assumption that the flow depth $h(x, y, t)$ is small relative to the characteristic horizontal length scale H (see Fig. 1). This assumption justifies integrating three-dimensional (3D) governing equations through the depth and applying boundary conditions at the basal and free surfaces, resulting in a 2D system that is more tractable computationally. This procedure also incorporates the free-surface boundary conditions directly in the governing PDEs, so that depth becomes a solution variable. In depth-averaged models the vertical variation in the flow field (of a particular layer) is either ignored or assumed to have a given profile. The models described in this section are for a fluid or continuum mixture with a free

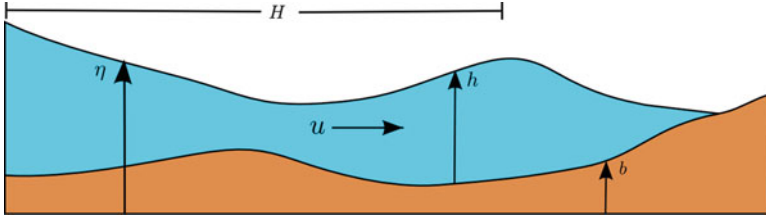


Fig. 1 Vertical cross section of a single-layer depth-averaged flow. The fluid (light shade) with depth h flows over fixed bottom topography b (dark shade). The shallowness parameter $\varepsilon = h/H$ is assumed to be small

surface at $z = \eta(x, y, t)$, flowing over fixed basal topography at $z = b(x, y)$. The total flow depth is then $h(x, y, t) = \eta(x, y, t) - b(x, y)$. The horizontal flow velocities in the x and y directions will be denoted by $u(x, y, t)$ and $v(x, y, t)$ respectively. Multilayer and multiphase flows will be denoted with subscripts for different layers or phases.

2.2.1 The Shallow Water Equations

The simplest depth-averaged equations are the well-known shallow water equations,

$$\partial_t h + \partial_x(hu) + \partial_y(hv) = 0, \quad (2.4a)$$

$$\partial_t(hu) + \partial_x(hu^2 + \frac{1}{2}gh^2) + \partial_y(huv) = -gh\partial_x b + S_x, \quad (2.4b)$$

$$\partial_t(hv) + \partial_x(huv) + \partial_y(hv^2 + \frac{1}{2}gh^2) = -gh\partial_y b + S_y, \quad (2.4c)$$

where g is the gravitational acceleration constant, and $hu(x, y, t)$ and $hv(x, y, t)$ are the depth-averaged momenta in the x and y directions, respectively. The terms $S_x(q, x, y)$ and $S_y(q, x, y)$ are basal friction terms obtained from an empirical friction law, such as the Chezy or Manning formula (see, e.g., [16, 17]).

The shallow water equations are derived (see, e.g., [2]) under the assumption that the fluid pressure is vertically hydrostatic, which can be justified if the shallowness parameter $\varepsilon = h/H$ is very small. In this case deviations from hydrostatic pressure can be formally shown to be $\mathcal{O}(\varepsilon^2)$ (see, e.g., [2]). These equations are commonly used, however, for problems in which this assumption is violated in some regions. Nevertheless, this system of equations has been repeatedly shown to perform very well for a wide variety of applications, at least for predicting general inundation patterns (see, e.g., [17, 18]). For problems in which the flow exhibits large vertical accelerations, and hence nonhydrostatic pressure that could significantly influence the dynamics, some authors have included nonhydrostatic pressure corrections to the equations (see, e.g., [19–23]). Inclusion of such correction terms introduces third-order dispersive terms, altering the mathematical nature of the system so that it is no longer of the form (2.3) (see, e.g., [19, 22]). This could potentially be dealt with numerically by splitting the hyperbolic and dispersive terms into separate integration steps.

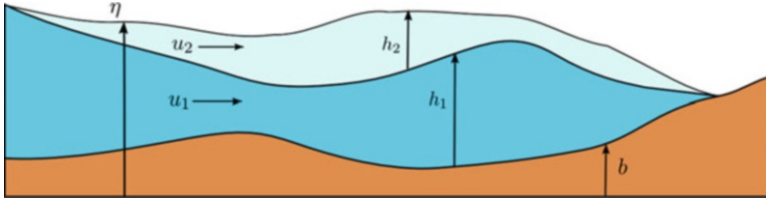


Fig. 2 Vertical cross section of a two-layer depth-averaged flow. The fluid (*lightest shade*) with depth h_2 flows over another fluid layer with depth h_1 and fixed bottom topography b (*dark shade*)

2.2.2 Multilayer Models

For problems in which there may be a large degree of vertical variation in the horizontal velocity profiles or fluid densities, researchers have used multilayer shallow water equations (Fig. 2). The solution $q(x, y, t)$ for an N -layer system takes the form

$$q = (\rho_1 h_1, \dots, \rho_N h_N, hu_1, \dots, hu_N, hv_1, \dots, hv_N)^T, \quad (2.5)$$

where ρ_n is the constant density, $h_n(x, y, t)$ is the depth, and $hu_n(x, y, t)$ and $hv_n(x, y, t)$ are the depth-averaged momenta of the n th layer. The exact form of the governing system depends on layer-coupling assumptions, but they are usually of the form (2.3) (see, e.g., [5–7, 24]). Multilayer models are often used for ocean dynamics problems in which a relatively shallow, more dynamic layer overlies denser and deeper layers in the water column. For instance, multilayer models have been used for storm surge generation and propagation [5]. One difficulty encountered with multilayer models is the loss of hyperbolicity due to the appearance of complex eigenvalues when layer velocities diverge significantly, i.e., when $\|u_n - u_{n-1}\| > M_n$ at some location for $n = 2, \dots, N$. The values M_n depend on the density ratio and depths of the adjacent layers. However, this problem can be overcome numerically with various specialized methods that prevent the loss of hyperbolicity (see, e.g., [6, 7, 25]). Interestingly, the problem can be avoided altogether if mass exchange is allowed between the layers [24].

2.2.3 Granular-Fluid Flows

Gravity-driven mass flows such as debris flows are commonly composed of saturated mixtures of solid particles, interstitial fluid, and suspended fine sediment (see, e.g., [26, 27] for an overview). The most notable difficulty in developing models for these flows is representing the complicated stress with some physical fidelity, while maintaining mathematical and computational tractability. The use of depth-averaged models for these mass flows was pioneered by Savage and Hutter [28, 29], who developed a model similar to the shallow water equations, yet accounted for solid stress using a solid-pressure model and Coulomb friction. In their model the fluid phase is ignored altogether, making it strictly appropriate for dry granular avalanches.

In order to model fluid-saturated flows, some researchers have treated granular flows as a single-phase continuum with a non-Newtonian rheology, such as a Bingham or viscoplastic medium (see, e.g., [30]). Iverson and Denlinger [31, 32] in contrast utilized a Coulomb-based Savage–Hutter-type model for granular flow, but included terms to explicitly account for the solid–fluid stresses in a mixture treated as a single phase. More recently, researchers (see, e.g., [8, 33–35]) have developed multiphase continuum mixture models, usually with one phase for the solid particles and another for interstitial fluid with suspended fine particles. These multiphase models allow for a more accurate accounting of the stress arising from solid–solid and solid–fluid interactions, but are inherently more complex. Typically the solid stress terms are based on Coulomb theory, and solid–fluid interactions are modeled with a drag term (arising from phase velocity differences) and a buoyancy term (arising from interstitial fluid pressure).

With a typical depth-averaged two-phase model, the solution vector might take the form

$$q = (\alpha_s h, \alpha_f h, hu_s, hv_s, hu_f, hv_f)^T, \quad (2.6)$$

where subscripts s, f indicate the solid or fluid phase respectively, and $\alpha_{s, f}$ are the volume fractions of the two phases. Various changes of variables with functions of the volume fractions and depth are of course possible as well, e.g., [33]. Because of the nature of the stress laws used, the governing PDEs are usually hyperbolic systems of the form (2.3) (e.g., [8, 33–35]). These models present the same mathematical difficulty as most multilayer shallow water models, in that complex eigenvalues can appear when there are large differences in phase velocities. Additionally, the eigenstructure cannot generally be determined analytically (see, e.g., [8, 34]).

Although multiphase models include buoyancy terms arising from pore-fluid pressure, accurately representing that pressure remains a significant challenge. The dynamics of granular flows are heavily influenced by the expansion and contraction of the pore space during solid grain motion, which creates a feedback loop between fluid pressure and pressure-mediated solid mobility (see, e.g., [27, 36]). In most models this effect is neglected and the fluid pressure evolves independently from the solid volume fraction (see, e.g., [8, 32]). A recently developed model [9] accounts for this feedback by using the concept of granular dilatancy: originally developed in the field of quasi-static soil mechanics (see, e.g., [37]). Although the details of the model are beyond the scope of this paper, in essence the pore-fluid pressure is coupled to the rate of expansion or contraction of the solid volume fraction, which evolves due to shearing, allowing feedback and coevolution of these variables (see [9, 36] for details). The variables modeled are

$$q = (h, hu, hv, \alpha_s, p)^T, \quad (2.7)$$

where p is the pore-fluid pressure. Although independent phase velocity fields are not tracked, the model is two-phase in the sense that the volume fractions and fluid pressure are retained and coevolve. The model has the added benefit of being strictly

hyperbolic with a simple analytically known eigenstructure. Because pressure is not a conserved quantity, the system has nonconservative products that must be handled numerically (described briefly in Sect. 3).

2.2.4 Hybrid Models

Some geophysical applications contain multiple interacting flow phenomena, each of which can be described by a distinct depth-averaged model. Hybrid models formed by coupling together individual depth-averaged equations are sometimes used for these applications. For instance, sediment transport problems in which water flows over loose sediment are modeled with the shallow water equations coupled with a hyperbolic advection equation for the entrained sediment, resulting in an extended system of the form (2.3) (see, e.g., [38, 39]). If the bed load transport is modeled in addition to suspended sediment, a two-layer set of equations can be used, with the lower layer representing the flowing granular material below the water and suspended load. Typically very simple models are used for the lower layer, because its dynamics are mostly due to the motion of the overlying water, rather than being independently gravity-driven (see, e.g., [38]). For problems involving submarine landslides and turbidity currents, the granular flows are gravity-driven and they have more influence on the motion of the overlying water column than does bed load. For these problems it can be advantageous to use multilayer models with the lower layer being governed by a granular-fluid model, such as those introduced above, and the water being modeled with the shallow water equations (see, e.g., [40]). Additionally, there is a growing interest in submarine-landslide-generated tsunamis (e.g., [41]), which could potentially be described by similar multilayer models.

3 Numerical Techniques and Challenges

In this section the numerical algorithms implemented in GEOCLAW [42] are briefly described in general terms, followed by some examples for depth-averaged flows. An exhaustive and detailed description of the algorithms is far beyond the scope and focus of this paper. Rather, the central aim of this section is to highlight some of the challenges and techniques specific to simulating free-surface flows over topography.

3.1 Finite-Volume Wave-Propagation Methods

GEOCLAW is based on two-dimensional finite-volume methods. The grids are logically rectangular, but various mappings, such as latitude–longitude mappings, are supported for modeling global-scale problems on the sphere (see Fig. 3).

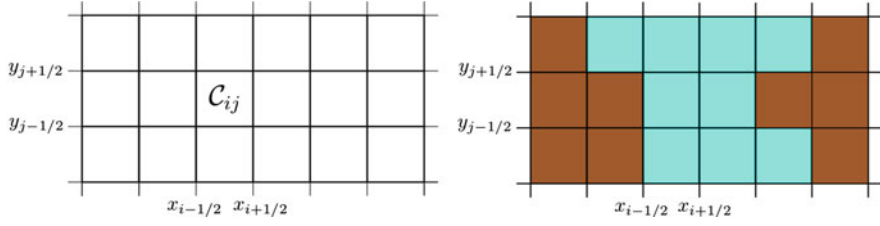


Fig. 3 *Left*: map view of a logically rectangular finite-volume grid in 2D. The x and y coordinates might represent Cartesian or latitude–longitude coordinates. *Right*: finite-volume grid for depth-averaged flows. The edge of the flow is captured as the boundary between the wet cells (indicated in a light shade) and the dry cells (indicated in a darker shade)

The numerical solution at a given time t^n is defined as piecewise constant,

$$Q_{ij}^n \approx \frac{1}{\lambda(C_{ij})} \int_{C_{ij}} q(x, y, t^n) dx dy, \quad (3.8)$$

where $\lambda(C_{ij})$ is the area of the grid cell $C_{ij} = [x_{i-1/2}, x_{i+1/2}] \times [y_{j-1/2}, y_{j+1/2}]$. In the following sections, capital letters and subscripts are used to denote the components, or functions of the components, of the numerical solution Q_{ij}^n (e.g., $U_{ij}^n = H U_{ij}^n / H_{ij}^n$). Each grid cell also has a topographic value,

$$B_{ij} = \frac{1}{\lambda(C_{ij})} \int_{C_{ij}} b(x, y) dx dy, \quad (3.9)$$

where $b(x, y)$ is determined from external data such as a digital elevation model (DEM). The numerical update $Q_{ij}^n \rightarrow Q_{ij}^{n+1}$ requires the solution of one-dimensional Riemann problems at each of the four grid-cell interfaces. For example, in solving a system defined by Eq. (2.3), at the interface between cells $C_{i-1,j}$ and C_{ij} , the system

$$\partial_t q + \mathcal{A}(q) \partial_x q = \psi(q, x, y), \quad \text{for } t^n \leq t \leq t^{n+1}, \quad (3.10)$$

is analytically solved with initial conditions,

$$q(x, y, t^n) = \begin{cases} Q_{i-1,j}^n, & \text{if } x \leq x_{i-1/2}, \\ Q_{ij}^n, & \text{if } x > x_{i-1/2}. \end{cases} \quad (3.11)$$

An analogous Riemann problem is solved for the interface between cells $C_{i+1,j}$ and C_{ij} . For the interfaces between cells C_{ij} and $C_{i,j\pm 1}$, the system

$$\partial_t q + \mathcal{B}(q) \partial_y q = \psi(q, x, y), \quad \text{for } t^n \leq t \leq t^{n+1}, \quad (3.12)$$

is solved with analogous initial conditions. Typically an approximate Riemann solver, developed for a given set of PDEs, provides an approximation to the

exact Riemann solution given arbitrary initial conditions in the form of Eq. (3.11). Approximate Riemann solvers must satisfy various requirements in order to ensure a consistent, stable, and numerically conservative global solution (see, e.g., [10,43]).

In GEOCLAW, the local wave structure in approximate Riemann solutions is used directly in order to build the numerical update via the wave-propagation algorithms of LeVeque [44], which are high-resolution, shock-capturing, total-variation-diminishing (TVD) schemes. A detailed description of the algorithms can be found in [10,44]. Unlike traditional Godunov-type methods, in which the numerical update is based on cell-interface fluxes determined from the Riemann solutions, the wave-propagation algorithms are applicable to systems with nonconservative products where there is no flux function.

Note that for an advancing or retreating flow, the location of the moving flow front is determined by the stair-step boundary between wet and dry cells at any given time step (see Fig. 3). While some inundation algorithms explicitly track the boundary by solving a separate set of kinematic equations for the edge trajectory (see, e.g., [45]), the algorithms in GEOCLAW capture the boundary by simply solving the Riemann problem between wet and dry cells, allowing dry cells to fill or wet cells to drain. This requires specialized Riemann solvers that carefully treat the source term resulting from variable topography, introduced in the next section.

3.2 Riemann Problems for Flow over Topography

Gradients in topography appear in the source terms of all of the models described in Sect. 2.2. Numerically including these terms in a satisfactory manner is surprisingly more difficult than might be expected, and the topic has generated a considerable amount of research (see, e.g., [15,46,47]). In GEOCLAW the topography is dealt with directly in the Riemann solver. To illuminate the challenges, consider a Riemann problem for the shallow water equations (2.4), as depicted in Fig. 4. The figure shows a step in topography, surface elevation, and fluid depth. It is physically intuitive that the waves in the Riemann solution arise due to deviations from the stationary steady state: $\eta_{i-1,j}^n = \eta_{ij}^n$, $U_{i-1,j}^n = U_{ij}^n = 0$, where η represents the free surface of the numerical solution, $\eta_{ij}^n = H_{ij}^n + B_{ij}^n$. Mathematically, the steady state exists due to a balance between nontrivial convective terms on the left-hand side of Eq. (2.4) (arising from depth gradients) and the nontrivial source term on the right (arising from topography gradients). Written compactly in the general form (3.10), the steady state satisfies

$$\mathcal{A}(q)\partial_x q = \psi(q, x, y). \quad (3.13)$$

(Note that for the Riemann problem between $\mathcal{C}_{i-1,j}$ and $\mathcal{C}_{ij}$, variation in y can be ignored.) Traditionally, Riemann solvers (e.g., [48, 49]) are designed for the homogeneous system

$$\partial_t q + \mathcal{A}(q)\partial_x q = 0, \quad (3.14)$$

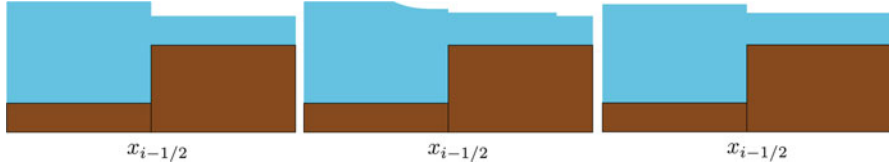


Fig. 4 Vertical cross-section of a one-dimensional Riemann problem for a typical depth-averaged free-surface flow, showing the fluid (*light*) and topography (*dark*) in cells $C_{i-1,j}$ and C_{ij} . *Left*: initial solution at t^n . *Middle*: the Riemann solution at $t > t^n$ consists of waves (smooth or discontinuous) that propagate away from the initial discontinuity at $x_{i-1/2}$, and represent deviations to the steady state. *Right*: the re-averaged solution at t^{n+1}

neglecting the source term. The source term is then accounted for in a separate integration step (from t^n to t^{n+1}) for the system of ordinary differential equations

$$\partial_t q = \psi(q, x, y). \quad (3.15)$$

These fractional step or split methods fail to preserve balanced steady states (i.e., nontrivial steady states) at practical grid resolutions, particularly when the balanced terms are large in magnitude. This should not be surprising, because the numerical updates based on Eqs. (3.14) and (3.15) must precisely cancel, yet might be large individually. Consequently, these methods also fail to correctly resolve small perturbations to such steady states, which characterize the dynamics of many shallow, free-surface flows [50]. Deep-ocean tsunami modeling provides a clear example: tsunamis represent a tiny perturbation to an underlying balanced steady state for still water over the variable seafloor.

3.2.1 Well-Balanced Methods

The challenges involving balanced steady states have motivated the development of well-balanced schemes, which are designed to incorporate the effect of the source term directly into the Riemann solution. Although simple in concept, this can be challenging to accomplish accurately. Again consider the Riemann problem for the shallow water equations depicted in Fig. 4. The source term due to the topographic gradient $\partial_x b$ is concentrated at the cell interface and is weakly defined as a delta function. However, note that h also appears in the source term of Eq. (2.4) and is discontinuous at the cell interface, creating a problem of overlapping distributions (see, e.g., [11, 12, 51]). The correct magnitude for the source term is therefore unclear, yet one wishes to exactly preserve (to machine precision) steady-state discontinuous weak solutions.

The correct solution to this problem can be illuminated by transforming the shallow water equations (neglecting friction) into a homogeneous system, which

can be accomplished by treating the topography $b(x, y)$ as another component of the solution vector, governed by the trivial equation $\partial_t b = 0$ (see, e.g., [15, 52]). Equation (2.4) can then be written as a homogeneous nonconservative system,

$$\partial_t \tilde{q} + \tilde{A}(\tilde{q}) \partial_x \tilde{q} + \tilde{B}(\tilde{q}) \partial_y \tilde{q} = 0, \quad (3.16)$$

where $\tilde{q} := (h, hu, hv, b)^T$ and $\tilde{A}, \tilde{B} : \mathbb{R}^4 \rightarrow \mathbb{R}^{4 \times 4}$. With the equations in this form, the jump discontinuity that remains at the cell interface for $t \geq t^n$ is a stationary wave in the Riemann solution variables $\tilde{q}$, which must be determined by jump conditions. To see how these jump conditions can be derived, consider a smooth steady-state solution in 1D $\tilde{q}(x, \cdot, \cdot)$, on some interval $x_0 \leq x \leq x_1$, with varying topography. From Eq. (3.16), the solution must satisfy

$$\tilde{A}(\tilde{q}) \partial_x \tilde{q} \equiv 0. \quad (3.17)$$

Consider this solution to be a curve in state space, parameterized by $x : \tilde{Q}(x) := \tilde{q}(x, \cdot, \cdot)$. From Eq. (3.17) it can be seen that at any point $x_p \in [x_0, x_1]$, $\partial_x \tilde{q}(x_p, \cdot, \cdot) = \tilde{Q}'(x_p)$ is an eigenvector $r_0(\tilde{q}(x_p))$ of $\tilde{A}(\tilde{q}(x_p))$ associated with a zero eigenvalue, $\lambda_0(\tilde{q}(x_p)) = 0$. The solution in phase space is therefore an integral curve of the vector field $r_0(\tilde{q})$:

$$\tilde{Q}'(x) = \alpha(x) r_0(\tilde{q}(x)), \quad x_0 \leq x \leq x_1, \quad (3.18)$$

where $\alpha(x)$ is a scalar proportionality constant. The solution varies smoothly along the curve in state space as x varies. The relevant discontinuous steady-state solution with a single jump in topography is therefore a jump discontinuity between two states lying on the same integral curve of $r_0(\tilde{q})$, which connects $\tilde{q}(x_0)$ and $\tilde{q}(x_1)$. This jump discontinuity provides the correct jump conditions for the two states adjacent to $x_{i-1/2}$, such as shown in the middle panel of Fig. 4 (see, e.g., [15, 53]). Well-balanced methods are designed to approximate this stationary wave as accurately as possible, though usually without determining the exact Riemann solution (see, e.g., [13–15, 50, 54–57]). Consequently, most well-balanced methods preserve some steady-state solutions (e.g., motionless water) but not all (e.g., steady flow). The well-balanced Riemann solver used in GEOCLAW preserves all one-dimensional steady states and the motionless steady state in 2D. However, exactly preserving general flowing steady states in 2D remains a challenging problem [15].

3.2.2 Wet–Dry States

Another challenge confronted in modeling free-surface flows that inundate dry land is accurately resolving the Riemann problem between a wet and dry cell. This is sometimes referred to as the dry-state problem or the filling and draining problem. Conventional approximate Riemann solvers are prone to producing negative depths for these and other very shallow Riemann problems. Simply resetting the negative

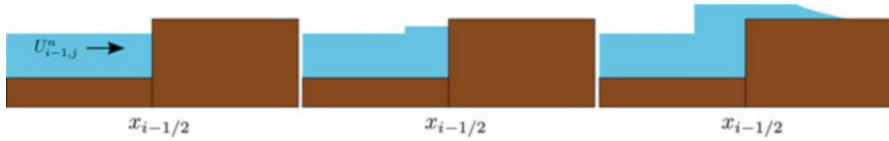


Fig. 5 The wet–dry Riemann problem with a step in topography. *Left*: the initial left state has fluid with positive velocity. *Middle*: the Riemann solution when the velocity is insufficient to inundate the dry cell. The solution consists of reflected waves. *Right*: the Riemann solution with inundation

depth to zero violates conservation and redistributes mass inaccurately in the inundation zone, where accuracy is typically the most desirable [17, 58]. A number of researchers have therefore developed so-called positivity-preserving or depth-positive-semidefinite approximate Riemann solvers that avoid this problem (see, e.g., [15, 59, 60]). These solvers accurately capture the draining of a wet cell.

Even when maintaining depth-positivity, solving the Riemann problem between wet and dry states when there is a step in topography remains a challenging problem, and it is well known that a standard discretization of $b(x, y)$ for these problems will produce spurious results for the advancing inundation edge [17, 61]. This becomes clear when one considers the inundation Riemann problems shown in Fig. 5. Consider the initial data in the wet cell $C_{i-1,j}$, which has a surface elevation $\eta_{i-1,j}^n < B_{ij}$ and a positive velocity $U_{i-1,j}^n$. It is clear that there are two distinct possibilities in the Riemann solution: either the fluid will be completely reflected by the step in topography, or it will inundate the dry cell to the right. The correct solution depends on the initial fluid velocity. However, the Riemann solution also depends on the source term, which is undefined at t^n for $x \geq x_{i-1/2}$. This ambiguity is physically intuitive, since in the case in which waves are entirely reflected, a value of B_{ij} higher than the fluid level has no influence on the solution. In GEOCLAW, before the wet–dry Riemann problem is actually solved, a preliminary test problem is conducted with a wall boundary condition (i.e., solution values symmetric to $Q_{i-1,j}^n$ are used in C_{ij}). The solution of this test problem determines whether $U_{i-1,j}^n$ is of sufficient magnitude to overtop the step in topography. If it is not, the reflected waves are retained for the update $Q_{i-1,j}^n \rightarrow Q_{i-1,j}^{n+1}$, and C_{ij} remains dry. If instead the waves reflected from the wall are higher than B_{ij} , then the original wet–dry Riemann problem is solved with a standard discretization of the source term. Although this ad hoc test determines whether inundation occurs, solving the inundation problem exactly when there is a step in topography is difficult (see, e.g., [62]). Because of the computational expense associated with determining the exact solution, the solution is approximated in GEOCLAW [15].

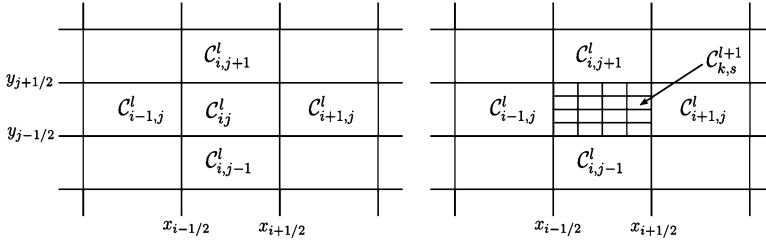


Fig. 6 Block-structured adaptive mesh refinement. A single level- l grid cell C_{ij}^l is refined to 16 level- $(l+1)$ cells, with refinement ratios $r_x^l = r_y^l = 4$

3.3 Adaptive Mesh Refinement

In order to accommodate the multiple spatial and temporal scales in geophysical problems, GEOCLAW utilizes block-structured, patch-based adaptive mesh refinement (AMR), designed for logically rectangular finite volume grids [63–65]. These algorithms allow multiple grid levels, $l = 1, \dots, L$, with different resolutions. Spatial refinement ratios between grid levels are user-defined integers for each direction: r_x^l, r_y^l , for $l = 1, \dots, L-1$. For instance, Fig. 6 depicts the refinement of a single level- l grid cell C_{ij}^l to a level- $(l+1)$ grid with $r_x^l = r_y^l = 4$. Typically, many adjacent grid cells of a given level- l grid would be refined to generate a single logically rectangular level- $(l+1)$ subgrid. The individual subgrids of a given level are arranged to form a patchwork that evolves with features in the solution. By using many levels of refinement, the diversity of spatial scales in a given problem can be accommodated.

The AMR algorithms used in GEOCLAW are based on methods developed for shock-capturing methods applicable to general hyperbolic conservation laws by Berger, Colella, and Oliger [63, 64]. These were later generalized to accommodate wave-propagation algorithms for conservative and nonconservative problems [65]. The algorithms are designed to interpolate and coarsen (*antinterpolate*) the solution between coarse and fine grids upon refinement and de-refinement. The interpolation/antinterpolation schemes must be designed to maintain properties of the underlying numerical methods, such as numerical conservation and TVD maintenance in the case of shock-capturing schemes. Special techniques must be used at subgrid boundaries in order to maintain conservation and prevent reflections. Details of these algorithms can be found in [63–65].

Free-surface environmental flows present unique challenges for AMR algorithms, due particularly to the time-varying domain, vanishing depths near flow edges, and need for well-balanced preservation of steady states. Standard interpolation/antinterpolation schemes (e.g., [63–65]) fail to preserve steady states, destroying the underlying well-balanced properties of the numerical scheme, as well as introducing energy in the form of hydraulic head. Additionally, because of nonlinear topography, it is possible to generate finite momentum in cells

with vanishing fluid depths near the flow front, introducing unbounded velocities and kinetic energy. To overcome these challenges, an extensive set of interpolation/antepolation schemes tailored to free-surface flows over topography were developed and implemented in GEOCLAW [17, 66]. In summary, the schemes satisfy several properties simultaneously. Steady states are maintained numerically, at and away from the flow edge. Energy (hydraulic head) is not introduced upon interpolation, and velocity interpolation is TVD (i.e., new extrema in fluid velocities are not introduced). For free-surface flows, AMR interpolation/antepolation can violate conservation of mass. For example, with a body of water near the shoreline, it is impossible to maintain conservation of mass and steady states simultaneously [66], due to the changing representation of the shoreline on grids of different resolutions. However, the AMR schemes in GEOCLAW conserve momentum when motionless mass is introduced near a shoreline, and if mass with nonzero momentum is removed upon coarsening, the momentum associated with that mass is removed as well [66]. In practice, lack of conservation near shorelines is not particularly problematic, since refinement ideally occurs just prior to the arrival of waves carrying momentum. The unique interpolation/antepolation schemes in GEOCLAW confer stability of the AMR algorithms for free-surface flows over topography [66].

4 Applications and Some Simulation Results

4.1 Tsunami Modeling: The 2011 Tohoku-Okii Tsunami

Tsunamis are typically modeled with the shallow water equations (2.4), since the depth of the ocean is very small compared to typical wavelengths. Tsunamis are typically generated by the vertical displacement of the seafloor,

$$b(x, y) \rightarrow b(x, y) + \delta b(x, y, t),$$

which disturbs the idealized motionless steady-state ocean,

$$0 \equiv \eta(x, y, \cdot) \rightarrow \eta(x, y, t) + \delta b(x, y, t),$$

thereby generating waves. Seafloor motion can occur due to earthquake-generated fault motion or large submarine landslides. The former generation mechanism is commonly known to produce large teletsunamis, which propagate across entire ocean basins. The initial spatial scales of a teletsunami are related to those of the initial seafloor disturbance $\delta b(x, y, t)$, which has horizontal length scales typically on the order of 100 km with amplitudes on the order of 1 m or less [17]. The wave speeds of a tsunami are equivalent to the eigenvalues of $f'(q)$ for the shallow water equations (2.4), which are $u, u \pm \sqrt{gh}$. The dependence of the wave speed on water depth means that the leading edge of a wave slows in shallow coastal

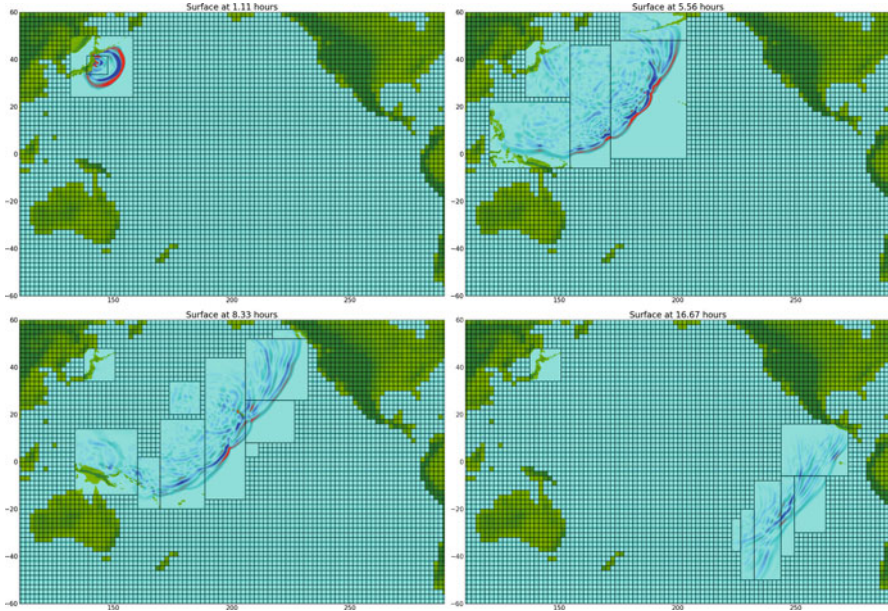


Fig. 7 A GEOCLAW simulation of the 11 March 2011 Tohoku-Oki tsunami. The steady-state ocean is maintained on extremely coarse level-1 grids, while level-2 grids (refined by 16^2) resolve the waves as they cross the ocean. Grid lines are omitted from level-2 grids for clarity. AMR provides a significant efficiency enhancement over static grids: this simulation required 27 min of computational time to model 24 h of real time (on a 64-bit Intel i7 quadcore desktop) (color figure online)

waters, which compresses the tsunami considerably and results in extreme energy focusing, giving rise to the tsunami's destructive potential. Upon inundation, the highly variable topographic features near the shore (seawalls, for example) can require meter-scale resolution for accurate modeling [17]. These disparate spatial scales imply that tsunamis are extremely multiscale in nature. Furthermore, because the waves propagate throughout the domain but are localized at any given time, the required grid resolution is highly spatially and temporally dependent, making AMR valuable if not essential for efficient modeling.

A typical refinement scenario for a GEOCLAW simulation is depicted in Figs. 7 and 8. In these simulations of the Tohoku-Oki tsunami of 2011, a single coarse level-1 grid with a resolution of 160 km was used for the simulation domain. The role of the coarse grid was to maintain the motionless steady state, while propagating waves were resolved on level-2 grids, refined by $r_x^1 = r_y^1 = 16$, resulting in a 10-km resolution. The simulation depicted in Fig. 7 used only these two levels.

Inundation modeling requires multiple (> 2) levels of grids, with high refinement ratios. Typically a region or coastline of interest is selected prior to the simulation. Figure 8 shows a GEOCLAW simulation for potential inundation in Hilo, Hawaii, caused by the Tohoku-Oki tsunami. Level-3 to level-5 grids were used to resolve

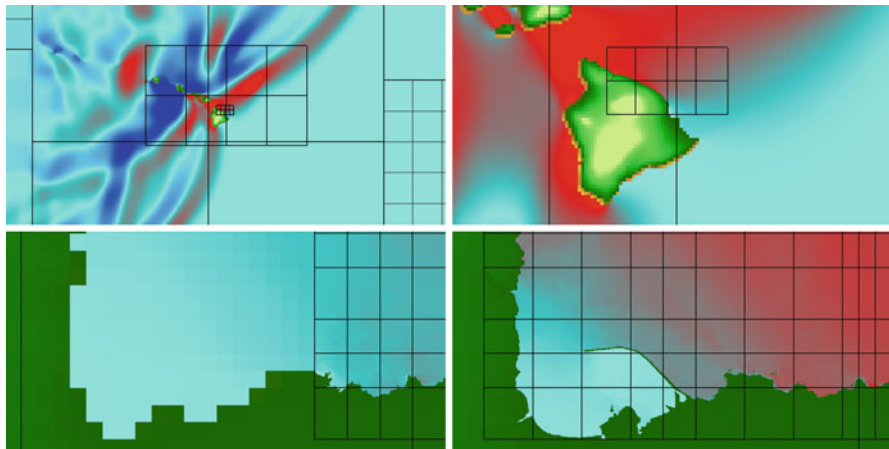


Fig. 8 A GeoClaw simulation showing inundation modeling, requiring extremely high refinement. *Upper panels:* as tsunami waves approach the northern coast of Hawaii, level 2–4 grids appear. *Lower panels:* inundation modeling of Hilo, Hawaii, on level-5 grids with ≈ 5 m resolution. The level-5 grids appear just as the waves approach the Hilo harbor. Grid lines are omitted from level 3–5 grids for clarity. AMR allows transoceanic propagation and local inundation to be modeled in single global-scale simulations (color figure online)

the waves as they compressed in the shallow coastal waters, with refinement ratios $(r_{x,y}^2, r_{x,y}^3, r_{x,y}^4) = (4, 16, 32)$. With $r_{x,y}^1 = 16$, these ratios resulted in a total refinement factor of 2^{15} in each direction, yielding level-5 grids with approximately 5 m resolution in a global-scale computational domain.

Tsunami modeling illuminates the importance of well-balanced methods, described in Sect. 3.2. In the deep ocean, tsunami amplitudes on the order of 1 m, with a wavelength of ≈ 100 km imply that the surface elevation varies by centimeters or less between 10 km grid cells. In the same distance, the seafloor elevation can vary by thousands of meters. Because the steady state arises from the balance of topography gradients on the right-hand side of Eq. (2.4) and depth gradients on the left-hand side of Eq. (2.4), tsunamis are in fact a tiny perturbation to the motionless steady state. At practical grid resolutions, the use of methods that are not well balanced can produce spurious waves much larger than the actual tsunami. Transforming Eq. (2.4) to a nonconservative system for η , u , and v can diminish this problem. However, these nonconservative systems are not appropriate for shock-capturing in the near-shore domain [66].

4.2 Flood Modeling: The 1959 Malpasset Dam Failure

Traditionally, flood prediction has been done without solving governing PDEs, but increasingly, engineers are using codes that solve the shallow water equations (2.4), typically on terrain-fit meshes (see, e.g., [67, 68]). Designing meshes that

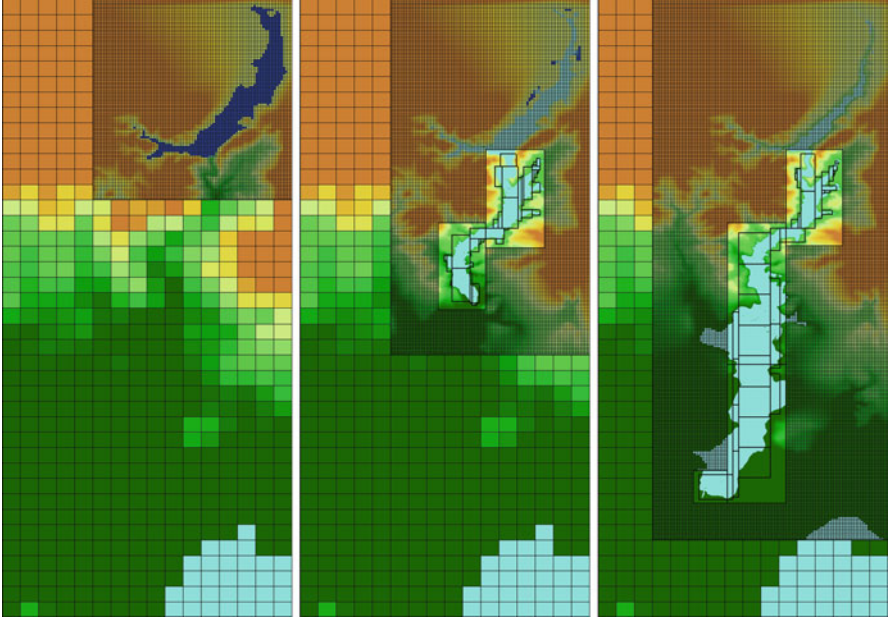


Fig. 9 A GEOCLAW simulation of the Malpasset dam-break flood, described in more detail in [18]. Grids evolve to track the flood as it progresses from the reservoir (*upper right*) toward the Mediterranean Sea (*lower right*). Grid lines are shown only for level-1 and level-2 grids. Individual level-3 and level-4 grids are outlined (color figure online)

conform to the topography of a region can be difficult and time-consuming, and the results obtained are still less than ideal because the optimal resolution evolves in time. Because floods can spread widely through variable terrain, AMR confers an enormous advantage by allowing grids to track a flood along its evolving route. Additionally, as described in [18], well-balanced methods and methods that correctly solve the wet–dry-interface Riemann problem help to accurately resolve the edge of the flood and prevent a spurious numerical sloshing effect that can occur at flow boundaries.

As described in [18], GEOCLAW was used to simulate the flood resulting from the Malpasset Dam failure that occurred on the Côte d’Azur in southern France. This 60-meter-high arch dam failed catastrophically on 2 December 1959, sending a rushing flood into the Reyran River Valley and killing 421 people, mostly in the nearby town of Fréjus [69]. This disaster has served as a benchmark problem for model validation due to the extensive survey work that documented the extent of flooding and timing of the flood arrival.

Results from the GEOCLAW simulation are shown in Figs. 9 and 10. A more detailed description and comparison to field data is provided in [18]. Generally, simulations using the shallow water equations compare surprisingly well to the actual event (see, e.g., [18, 68, 70]). As in the tsunami simulations described above,

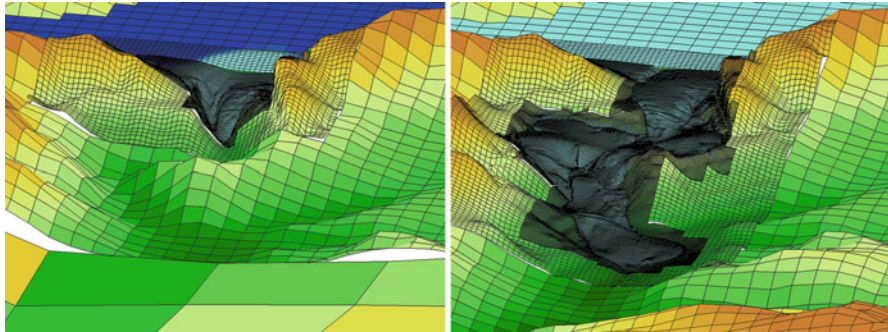


Fig. 10 Close-up oblique view of the Malpasset Dam moments after the initial failure. Level-3 and level-4 grids evolve with the advancing flood waves (color figure online)

a very coarse level-1 grid comprises the domain, which is roughly 5 km by 10 km. The reservoir is resolved on a level-2 grid. Level-3 and level-4 grids then resolve the flood wave as it winds down into the river valley toward the Mediterranean Sea. The refinement ratios were $(r_{x,y}^1, r_{x,y}^2, r_{x,y}^3) = (8, 4, 4)$, yielding level-4 grids with 3-meter resolution. Figure 10 shows a close-up oblique view near the dam seconds after the dam failure, showing the high-level grids tracking the winding flood path.

4.3 Granular-Fluid Flow Modeling

The depth-averaged hyperbolic model described in [9] was developed for granular-fluid flows, for simulating a variety of phenomena that can be broadly classified as debris flows (volcanic lahars, shallow water-saturated landslides, shallow mudflows, etc.); see, e.g., [27] for an overview of these phenomena. This granular-fluid model has been implemented in GEOCLAW for simulating natural debris flows in mountainous topography, to be described in future publications. GEOCLAW's AMR capability is utilized for these problems in a manner analogous to that described in Sect. 4.2, where grids evolve and follow the flow along its winding route.

In order to rigorously test this granular-fluid model, a 1D numerical code (DIGCLAW) has been developed for validation comparisons with experimental data collected at the U.S. Geological Survey's debris-flow flume in Oregon (Fig. 11). This ≈ 90 m concrete flume is equipped with instrumentation allowing the collection of data during controlled field-scale experiments (see, e.g., [71]). In addition to flow depth, the sensors measure the basal normal stress and pore-fluid pressure as the flow evolves, revealing fundamental feedback processes controlling flow mobility.

There are two principal types of experiments conducted at the debris-flow flume. In one type (e.g., [71]), loose sediment (composed of gravel, sand, and silt) is deposited in a hopper behind a closed steel headgate. The sediment is saturated and flows immediately when the controlled doors are released. In these gate-release



Fig. 11 The USGS debris-flow flume. Instrumentation allows the collection of flow depth, basal normal stress, and pore-fluid pressure during controlled experiments

experiments, the flow is initially far from equilibrium and the unstable mass fails instantaneously. Most numerical models are initialized in a similar way—as a dam-break far from equilibrium. The second type of experiment (e.g., [72]) consists of sediment deposited on the flume bed and slowly saturated in a controlled manner. The mixture can fail spontaneously if the pore-fluid pressure reaches a critical threshold and reduces the shear strength to an unstable level. These mobilization experiments are more representative of natural debris-flow initiation, in which a balanced equilibrium is slightly perturbed, leading to a propagating feedback between grain shearing and changes in pore-fluid pressure, possibly resulting in a dynamic flow [36].

A key motivation in the development of the granular-fluid model [9] is to enable capturing the transition from failure to flow mathematically and numerically. Although a full description of the mathematical model and numerical scheme is beyond the aim of this section, modeling the transition described above provides another example of the importance of well-balanced numerical methods. Much like a tsunami, debris flow motion is often characterized by a near-balanced steady state

$$\mathcal{A}(q)q_x \approx \psi(q, x). \quad (4.19)$$

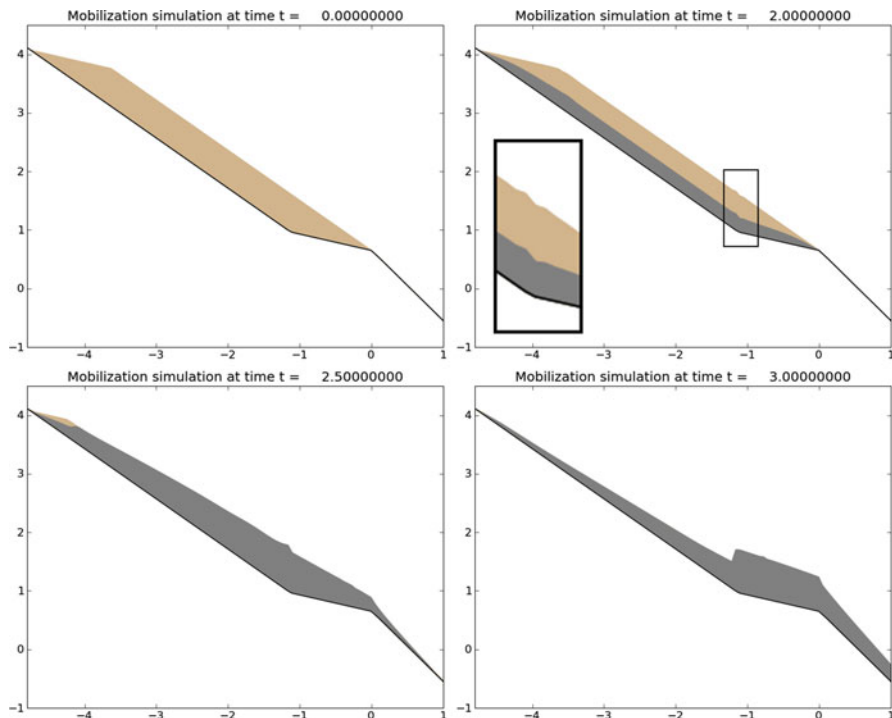


Fig. 12 Preliminary 1D simulations of debris-flow mobilization. *Upper left*: a vertical cross-section of dry debris is shown (*lightly shaded*). As the pore-fluid pressure rises, the shear strength is reduced. Pore pressure is indicated by the *dark shading*, scaled linearly between zero (*no shading*) and lithostatic (entire depth is *shaded*). When the pressure reaches a critical value, the material begins to shear at some location. This can be seen as a small disturbed region in the *upper right panel* near $x = -1.0$ (enlarged in the inset). This instability leads to a rapidly expanding feedback of shearing, grain consolidation, increasing pore pressure, and eventual motion of the entire mass (*lower panels*). In these numerical experiments, the initial fluid pressure rise was dictated numerically prior to motion, after which it evolved spontaneously

However, for a debris flow, large driving forces on the left-hand side of Eq. (4.19) are balanced by high shear resistance and friction on the right-hand side. Initial motion is a small perturbation to this balance that must be accurately captured, particularly in order to correctly resolve the potential transition to rapid mobility or alternatively restabilization and a return to a motionless steady state.

Preliminary use of DIGCLAW to simulate flume experiments indicates that it is capable of capturing these processes. Because the stress terms in the model equations evolve, a sediment mass can remain stable at the beginning of a simulation and potentially fail as the pore pressure and volume fractions coevolve. Alternatively, a flowing mass can stabilize and become motionless. In contrast, models that use a static friction law (constant Coulomb friction) or rate-dependent rheology (e.g., a viscoplastic fluid) must either produce flow initially or remain motionless permanently. Some preliminary numerical results using the 1D DIGCLAW code are shown in Fig. 12.

5 Final Remarks

Free-surface environmental flows in nature exhibit complicated three-dimensional physics and complex flow regimes. Because of the large scale of these types of flows and the inherent uncertainties associated with them (generation mechanisms, exact physical properties of mixtures, precise topography, etc.), predicting all aspects of these flows with three-dimensional models is typically not feasible. However, two-dimensional depth-averaged models can provide a tractable alternative with a high degree of fidelity, owing in part to the shallowness of these flows. (There is extensive literature on the validation and verification of depth-averaged models; see, e.g., [18, 66, 73–76].) Aside from the questions about model fidelity, numerically solving these systems accurately and efficiently presents unique challenges. Not only do mathematical models of these flows present the standard difficulties common to hyperbolic systems, they also require specialized techniques specific to free-surface flows over topography and multiscale resolution. Despite these challenges, it is possible to design algorithms that can effectively and efficiently simulate a range of free-surface flows.

Acknowledgements The work described in this paper has resulted from an active collaboration of researchers in mathematics, computer science, and the geosciences. In particular, Randall LeVeque at the University of Washington, Seattle; Marsha Berger at the Courant Institute, NYU; and Richard Iverson and Roger Denlinger at the U.S. Geological Survey, Vancouver, WA. GEOCLAW software development for future applications continues on an active basis, thanks to contributions from Kyle Mandli at the University of Texas, Austin; Dave Yuen at the University of Minnesota; and many others.

References

1. J.R. Mehelcic, J.C. Crittenden, M.J. Small, D.R. Shonnard, D.R. Hokanson, Q. Zhang, H. Chen, S.A. Sorby, V.U. James, J.W. Sutherland, J.L. Schnoor, *Environ. Sci. Technol.* **37**, 5314 (2003)
2. J.J. Stoker, *Water Waves: The Mathematical Theory with Applications* (Interscience Publishers, New York, 1957)
3. G. Whitham, *Linear and Nonlinear Waves* (John Wiley and Sons, New York, 1974)
4. D.T. Resio, J.J. Westerink, *Physics Today* pp. 33–38 (2008)
5. K.T. Mandli, Finite volume methods for the multilayer shallow water equations with applications to storm surges. Ph.D. thesis, University of Washington, Seattle, WA (2011)
6. R. Abgrall, S. Karni, *SIAM Journal on Scientific Computing* **31**(3), 1603 (2009)
7. F. Bouchut, V. Zeitlin, *Discrete and continuous dynamical systems-series B* pp. 739–758 (2010)
8. E.B. Pitman, L. Le, *Phil. Trans. R. Soc. A* **363**, 1573 (2005)
9. D.L. George, R.M. Iverson, in *The 5th intl. conf. on debris-flow hazards*, ed. by R. Genevois, D. Hamilton, A. Prestininzi (Italian Journal of Engineering, Geology and Environment, Padova, Italy, 2011), pp. 415–424
10. R.J. LeVeque, *Finite Volume Methods For Hyperbolic Problems*. Texts in Applied Mathematics (Cambridge University Press, Cambridge, 2002)
11. G. Dal Maso, P.G. LeFloch, F. Murat., *J. Math. Pures Appl.* **74**, 483 (1995)

12. L. Gosse, *Math. Comp.* **71**, 553 (2002)
13. M.J. Castro-Díaz, T. Chacón Rebollo, E.D. Fernández-Nieto, C. Parés., *SIAM J. Sci. Comput.* **29**, 1093 (2007)
14. R.J. LeVeque, *Journal of Scientific Computing* **48**, 209 (2010)
15. D.L. George, *J. Comput. Phys.* **227**(6), 3089 (2008). DOI 10.1016/j.jcp.2007.10.027
16. V.T. Chow, *Open Channel Hydraulics* (McGraw-Hill, 1959)
17. D.L. George, Finite volume methods and adaptive refinement for tsunami propagation and inundation. Ph.D. thesis, University of Washington (2006)
18. D.L. George, *Int. J. Numer. Meth. Fluids* **66**(8), 939 (2011). DOI 10.1002/fld.2298
19. A.E. Green, P. Naghdi, *Journal of Fluid Mech.* **78**, 237 (1976)
20. P.K. Stansby, J.G. Zhou, *Int. J. Numer. Meth. Fluids* **28**, 541 (1998)
21. Z. Li, B. Johns, *Int. J. Numer. Meth. Fluids* **35**, 299 (2001)
22. J. Sainte-Marie, M. Bristeau, *DCDS (B)* **10**(4), 733 (2008)
23. Y. Yamazaki, Z. Kowalik, K. Cheung, *Int. J. Numer. Meth. Fluids* **61**, 473 (2008)
24. E. Audusse, M. Bristeau, B. Perthame, J. Sainte-Marie, *Mathematical Modelling and Numerical Analysis* (2009)
25. M.J. Castro-Díaz, E.D. Fernández-Nieto, J.M. González-Vida, C. Parés., *Journal of Scientific Computing* **48**, 16 (2010)
26. T. Takahashi, *Annual review of fluid mechanics* **13**, 57 (1981)
27. R.M. Iverson, *Rev. Geophys.* **35**(3), 245 (1997)
28. S.B. Savage, K. Hutter, *J. Fluid Mech.* **199**, 177 (1989)
29. S.B. Savage, K. Hutter, *Acta Mech.* **86**, 201 (1991)
30. A.M. Johnson, *Physical Processes in Geology* (W. H. Freeman, 1970)
31. R.M. Iverson, R.P. Denlinger, *J. Geophys. Res.* **106**(B1), 537 (2001)
32. R.P. Denlinger, R.M. Iverson, *J. Geophys. Res.* **109**, F01014 (2004). DOI10.1029/2003JF000085
33. J. Kowalski, Two-phase modeling of debris flows. Ph.D. thesis, ETH Zurich (2008)
34. M. Pelanti, F. Bouchut, A. Mangeney-Castelnau, J.P. Vilotte., in *Hyperbolic Problems: Theory, Numerics, Applications.*, ed. by S. Benzoni-Gavage, D. Serre (Springer, 2008). Proc. 11th Intl. Conf. on Hyperbolic Problems, Lyon France, July 2006.
35. M. Pelanti, F. Bouchut, A. Mangeney, *Mathematical Modelling and Numerical Analysis* **42**(5), 851 (2008)
36. R.M. Iverson, *J. Geophys. Res.* **110**, F02015 (2005)
37. T.W. Lambe, R.V. Whitman, *Soil Mechanics* (John Wiley and Sons, 1969)
38. M.J. Castro-Díaz, E.D. Fernández-Nieto, A.M. Ferreira, *Comput. Fluids* **37**, 299 (2008)
39. J. Murillo, J. Burguete, P. Brufau, P. García-Navarro, *Int. J. Numer. Meth. Fluids* **49**, 267 (2005). DOI 10.1002/fld.992
40. T. Morales de Luna, M.J. Castro Díaz, C. Parés, E.D.F. Nieto, *Communications in Computational Physics* **6**, 848 (2009)
41. P. Lynett, P.L.F. Liu, *Proceedings of the Royal Society of London A* **458**, 2885 (2002)
42. M.J. Berger, D.L. George, R.J. LeVeque, K. Mandli, *Advances in Water Resources* p. in press (2011). DOI 10.1016/j.advwatres.2011.02.016
43. E.F. Toro, *Riemann Solvers and Numerical Methods for Fluid Dynamics* (Springer-Verlag, Berlin, 1997)
44. R.J. LeVeque, *J. Comput. Phys.* **131**, 327 (1997)
45. V.V. Titov, C.E. Synolakis, *Journal of Waterways, Ports, Coastal and Ocean Engineering* **124**(4), 157 (1998)
46. J.M. Gallardo, C. Pares, M. Castro, *J. Comput. Phys.* **227**, 574 (2007)
47. F. Marche, P. Bonneton, P. Fabrie, N. Seguin, *Int. J. Numer. Meth. Fluids* **53**, 867 (2007)
48. P.L. Roe, *J. Comput. Phys.* **43**, 357 (1981)
49. A. Harten, P.D. Lax, B. van Leer, *SIAM Review* **25**, 235 (1983)
50. R.J. LeVeque, *J. Comput. Phys.* **146**, 346 (1998)
51. J.F. Colombeau, *Multiplication of Distributions: A tool in mathematics, numerical engineering and theoretical physics* (Springer Verlag, Berlin, 1992)

52. T. Gallouët, J.M. Hérard, N. Seguin, *Comput. Fluids* **32**, 479 (2003)
53. M.J. Castro, P.G. LeFloch, M.L. Munoz, C. Parés., *J. Comput. Phys.* **227**, 8107 (2008)
54. J.M. Greenberg, A.Y. LeRoux, *SIAM J. Numer. Anal.* **33**, 1 (1996)
55. P. García-Navarro, M.E. Vázquez-Cédon, *Comput. Fluids* **29**, 17 (2000)
56. L. Gosse, *Math. Mod. Meth. Appl. Sci.* **11**, 339 (2001)
57. F. Bouchut, *Nonlinear Stability of Finite Volume Methods for Hyperbolic Conservation Laws and Well-Balanced Schemes for Sources* (Birkhäuser Verlag, 2004)
58. E.F. Toro, *Shock Capturing Methods for Free Surface Shallow Flows* (John Wiley and Sons, Chichester, United Kingdom, 2001)
59. E. Audusse, M.O. Bristeau, *J. Comput. Phys.* **206**, 311 (2005)
60. A. Kurganov, G. Petrova, *Commun. Math. Sci.* **5**(5), 133 (2007)
61. E. Audusse, F. Bouchut, M.O. Bristeau, R. Klein, B. Perthame, *SIAM J. Sci. Comput.* **25**, 2050 (2004)
62. R. Bernetti, V. Titarev, E. Toro, *J. Comput. Phys.* **227**, 3212 (2008)
63. M.J. Berger, J. Oliger, *J. Comput. Phys.* **53**, 484 (1984)
64. M.J. Berger, P. Colella, *J. Comput. Phys.* **82**, 64 (1989)
65. M.J. Berger, R.J. LeVeque, *SIAM J. Numer. Anal.* **35**, 346 (1998)
66. R.J. LeVeque, D.L. George, M.J. Berger, *Acta Numerica* **20**, 211 (2011). DOI10.1017/S0962492911000043
67. J.M. Hervouet, A. Petitjean, *J. Hydraul. Res.* **37**, 777 (1999).
68. A. Valiani, V. Caleffi, A. Zanni, *J. Hydraul. Eng.* **128**(5), 460 (2002)
69. USBOR, Arch dam failures: Malpasset and St. Francis. Tech. rep., Unites States Bureau of Reclamation (unknown)
70. D. Pianese, L. Barbiero, *J. Hydraul. Eng.* **128**(5), 941 (2004)
71. R. Iverson, M. Logan, R. LaHusen, M. Berti, *J. Geophys. Res.* **115**, 1 (2010)
72. R.M. Iverson, M.E. Reid, N.R. Iverson, R.G. LaHusen, M. Logan, J.E. Mann, D.L. Brien, *Science* **290**(5491), 513 (2000)
73. A. Mangeney, P. Heinrich, R. Roche, *Pure appl. Geophys.* **157**, 1081 (2000)
74. R.P. Denlinger, M. Iverson, *J. Geophys. Res.* **106**, 553 (2001)
75. M. Briggs, C. Synolakis, G. Harkins, *Proc. of Waves—Physical and Numerical Modeling* (1994)
76. V.V. Titov, C.E. Synolakis, *Tsunami 93 Proc. IUGG/IOC Intl Tsunami Symp.* pp. 627–636 (1993)

Computational Challenges in the Geosciences

Dawson, C.; Gerritsen, M. (Eds.)

2013, IX, 167 p. 57 illus., Hardcover

ISBN: 978-1-4614-7433-3