Chapter 2
Constant-Scale Natural Boundary Mapping Technique

2.1 Identifying Critical Boundaries, Unzipping and Zipping

The ‘divide tree’ or ‘tree of interruption’ is a network of boundaries that control a CSNB map. It grows from trunk lines of topographic extremes (ridges or valleys) identifiable on a shaded relief or topography map as strings of nearest neighbor summits (maxima) or pits (minima). For irregular objects, these can be thought of as maximum angular inflexions of planar orientation relative to the center of mass. Generally, the tree grows in either top down (from maxima) or bottom up (from minima), starting with the most recognizable and extreme cluster of maxima or minima. However, if the intent is to consider a certain class of features, such as plate boundaries, only maxima or minima associated with these boundaries should be used. The next step involves flattening: the transformation of the generated tree (and its surrounding area) from the spherical to the planar surface. See the example in Fig. 2.1.

Boundaries never change in length, only in orientation; boundaries define regions, and hence centerlines of regions. Centerlines are the pruned trunks of medial axes, which we discuss in Sects. 2.6 and 6.5. The medial axis is the set of center points of all circles equidistant from surrounding boundaries. It can be useful in locating centroids, direction vectors, and measures of distance in preparation for the next step. Centerlines change only in length, never in orientation. If these restrictions were relaxed, and the edge could be any imagined and stretchable line (such as a line of longitude), then conventional, formula-based projections would result.

How far the boundary tree grows depends on data resolution as well as preference for map segmentation or compactness to best illustrate processes. The splitting of boundaries, which creates segmentation, is known as *unzipping*, the reverse process, which creates compactness and a continuous surface as *zipping*. Highly segmented maps with largely unzipped boundaries illustrate morphological differences and patterns within an object, whereas compact maps, largely zipped, illustrate a global overall shape.

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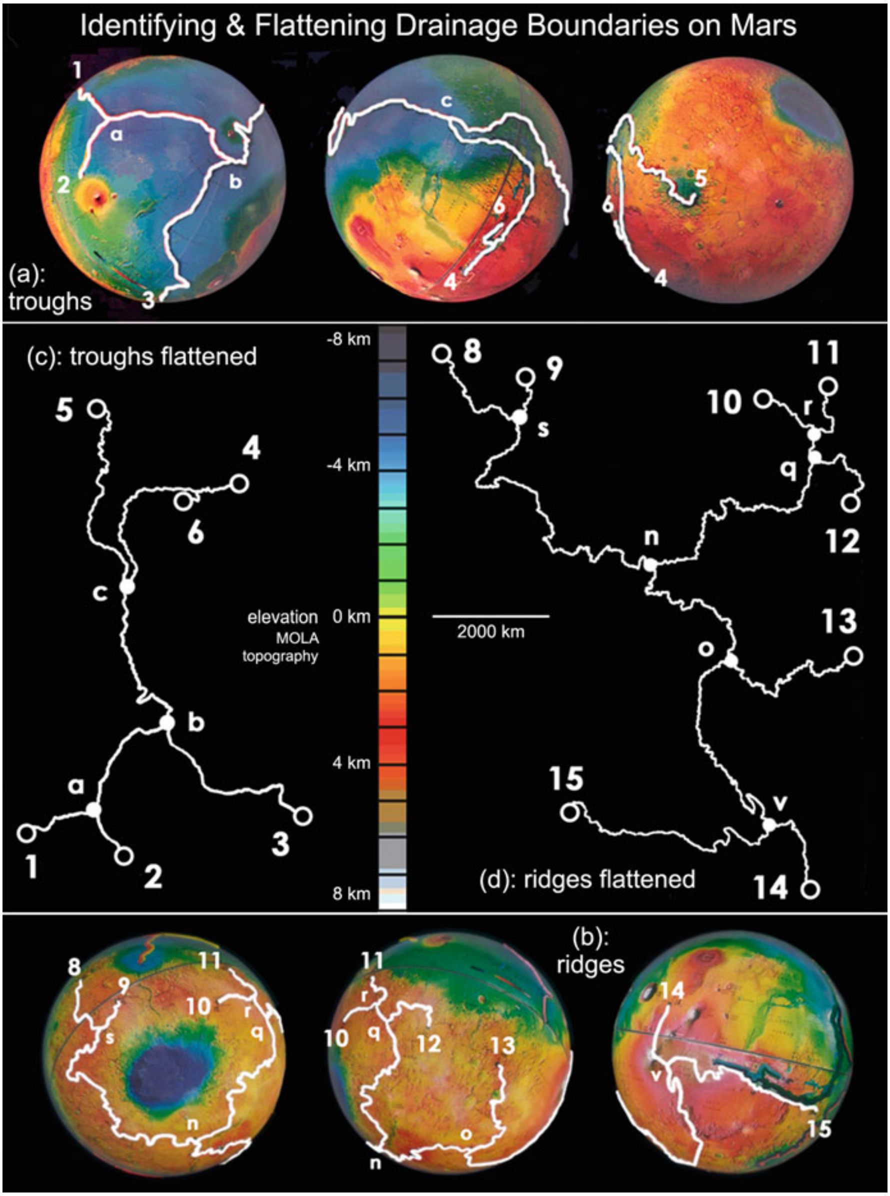

Fig. 2.1 CSNB mapmaking using (**a**) Martian trough and (**b**) ridge constant-scale boundary selections, then (**c** and **d**) flattening and hinging, as described in text (Clark 2004b) with help of René De Hon (Source data: Mars topography (MOLA) data (*red high* to *purple* low elevation). Courtesy of NASA)

2.2 Making Closed Shapes and Adjusting Proportions

We arrive at CSNB's second step—restoring a bounded surface.

A sphere‘s unbounded yet finite surface becomes a plane's unbounded, infinite surface as boundaries transform via radial unbending. Interruption of the surface happens by ‘unhooking’ one branch tip and unzipping the boundary along its length; additional tips also hinge. Rejoining the loose ends soon binds the once-infinite plane, reversing Fig. 2.1's relationship, as seen in Fig. 2.2.

It is not immediately clear that overlaps may always be eliminated, but pruning branches and altering hinge-arcs has always worked. In early CSNB maps (1992–1998), hinge-arcs were managed according to either of two simple criterions:

1. Equal or nearly so, thus equalizing distortion near hinges. Early maps, e.g., Figs. 1.3a and 1.8, were segmented, and conformality intuitively obvious. This strategy kept area and shape distortion to a minimum.
2. To maximize enclosed area. This tactic evolved as compact maps were attempted, e.g., Fig. 1.7a. (Also seen in Fig. 2.5 discussed in Sect. 2.4; note shape distortion near Hinge 6.)

Tracking only hinge-arcs inevitably created skewing in the reconstituted shape. Hinges must be set at a collective optimum, which exists, and may be found by correlating lengths across the flat shape with corresponding lengths on the object's spherical or irregular surface. If we, as Dürer, were mapping polyhedrons, we would make map lengths *equal* to corresponding object lengths (shown in Sect. 2.7). For natural bodies, we make map lengths *proportional* to corresponding object lengths.

Figure 2.3 shows this adjustment for the Mars trough-bound map. White lines are cross-map lengths, which are adjusted by equalizing their several ratios as a group, as if they were strategic lengths of springs. Hinges rotate in response; balanced spring strain ensures correct mid-map proportions. Where a suitable hinge is unavailable, as between Points 1 and 5, secondary bends, called *elbows*, are introduced between adjacent hinges. (Fig. 1.7 shows construction of a conformal elbow.) Elbows compromise a perimeter's angular metes and bounds, and thus degrade proportions, but the effect is slight, and local. Similarly, proportions will be inevitably distorted near hinges, but this effect is also localized.

A key distinction from, say, a circular or elliptical global map is the use of as few elbows as possible. This minimizes conformal distortions. The trade-off is in CSNB's extravagant shapes, which, it turns out, are precisely the shapes required to accurately capture antipodal proportions. This becomes clear as we cultivate our map's interiors.

2.3 Adjusting Internal Scale

In conventional cartography, scale is a function of latitude and longitude. In CSNB mapping, the interior points form a curving surface surrounded by a flat, constant perimeter. The interior surface points are projected to the plane by the two ancient

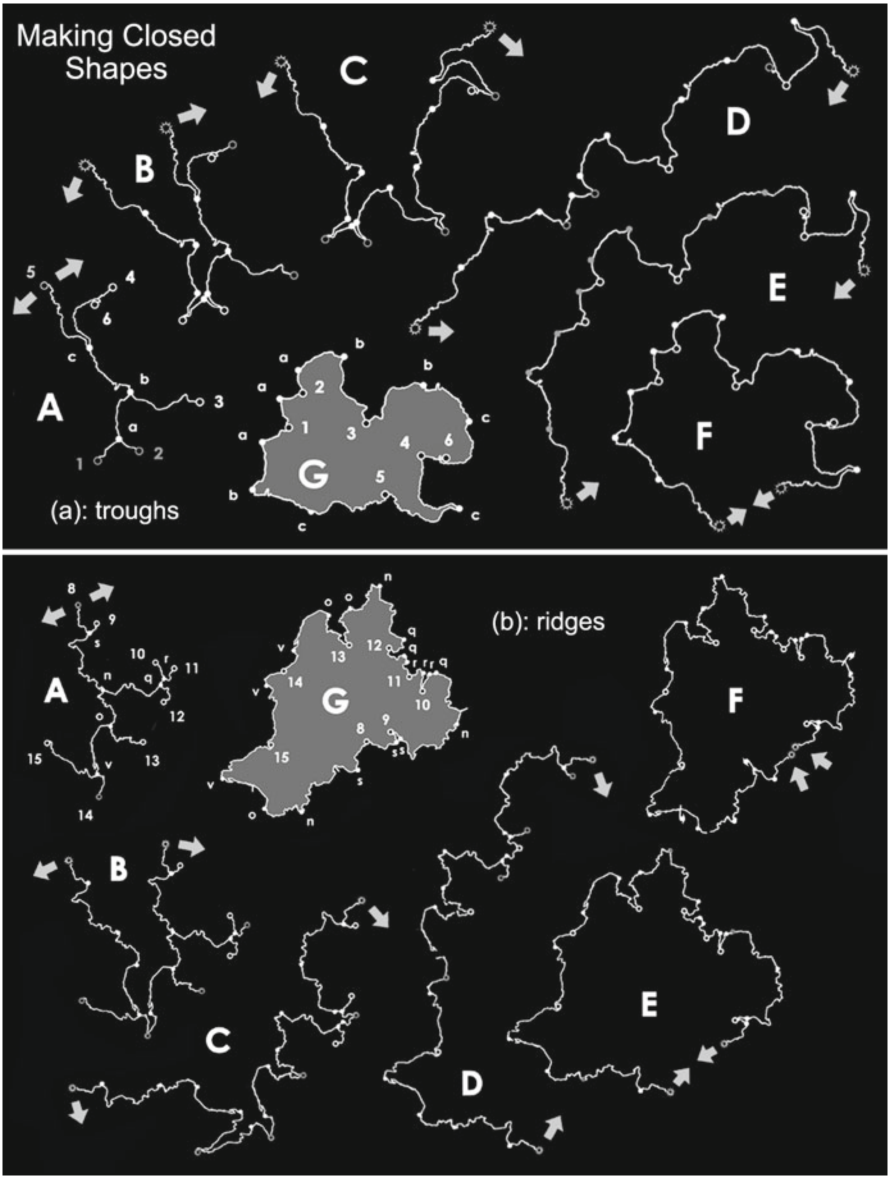

Fig. 2.2 Making a closed shape (using map in Fig. 2.1) via unzipping and hinging, as described in text Clark (2004b) with help of René De Hon

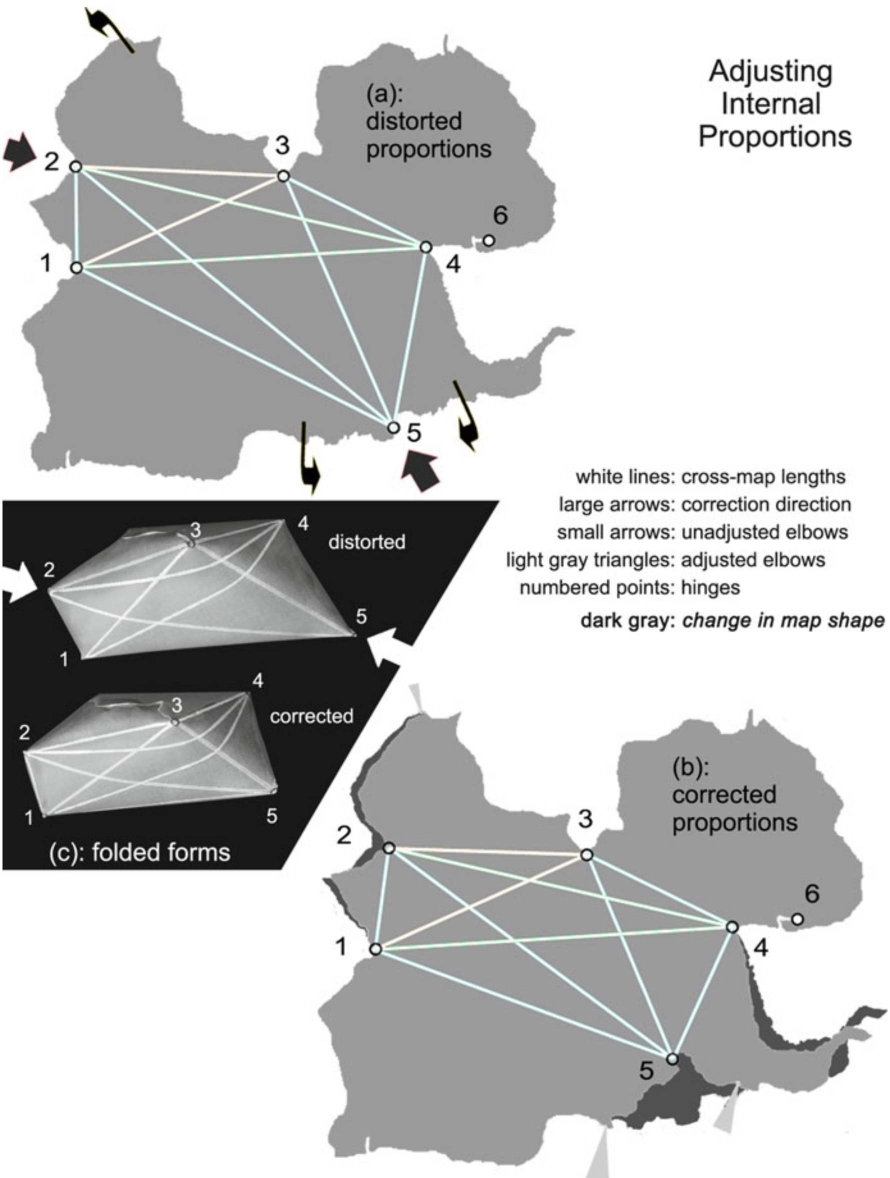

Fig. 2.3 Adjusting internal proportions (using map in Fig. 2.1) as described in text. Clark (2004b) with help of René De Hon

methods of geometry attributed first to the mapmaker Ptolemy (Sidoli and Berggren 2007) orthographic and stereographic projection (Fig. 2.4a). In an orthographic projection, the scale varies as smoothly as on a conventional map. Continents and oceans are both reduced evenly in size from their global dimensions. In a stereographic projection, scale may be specified as a function of distance from a given point, feature, or region, as in Fig. 2.4b, which concentrates areal distortion along mid-ocean ridges in order to show land at constant scale with the edge, i.e., continental size is preserved and ocean crust contracts in visual mimicry of a tectonic look backwards in time.

2.4 Drawing the Grid and Creating a Map

To provide orientation on maps and on surfaces of globes, space is usually subdivided into a grid of latitude and longitude, called graticles. Conventional map interiors are a precise and unambiguous coordinate point system within an arbitrary perimeter and graticles derive from a projection formula. CSNB interiors are controlled inexactly, as groups of points within a precise, unambiguous perimeter, and graticles are derived by adjacency, e.g., "neighborhood" axioms of Hausdorff (1914), with two added stipulations, the first obligatory, the second (usually) desirable:

1. Size of the smallest (most central) neighborhood is set arbitrarily, and
2. Sizes of other neighborhoods smoothly vary with distance from the edge.

Once this step is taken, the CSNB map has been created (Fig. 2.5).

We could use, and have used, a trial-and-error method for adjusting internal proportions. Graticles could be sketched in, compared to their on-globe counterparts, distortion appraised, and hinges adjusted to reduce skew. However, the approach outlined in Sect. 2.2 appears to allow more rapid minimization of distortion.

The development of the boundary network can begin on a conventional projection map. However, boundary lengths will need to be adjusted for their actual length to maintain constant scale.

Drawing natural boundaries on a flat, conventional map provides an interesting contrast to a CSNB map, particularly if the conventional map is reconfigured so that the natural boundaries form its edges (Fig. 2.6).

A succession of bordering lines may locate points in a map's field by organizing the map into a series of concentric neighborhoods. These "Hausdorff waterlines", which will be discussed in Sect. 2.6, move inward from the map edge a uniform distance y_1 y_2 y_3 …, and subdivide the field into neighborhoods of points relative to the border. Points in the inmost neighborhood(s) are, obviously, those points most distant from the edge, both on the map and on the globe.

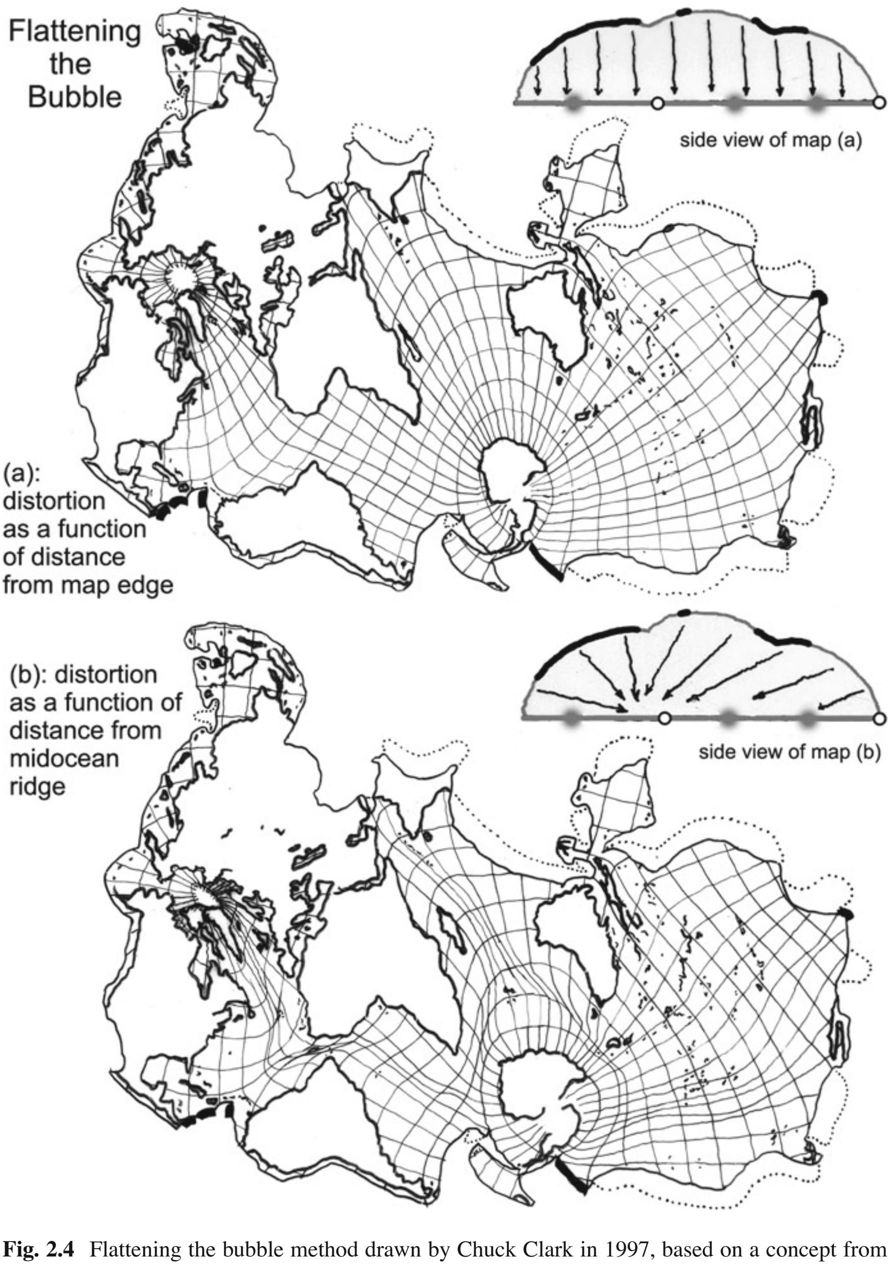

Fig. 2.4 Flattening the bubble method drawn by Chuck Clark in 1997, based on a concept from Ptolemy, for dealing with distortion, as described in the text

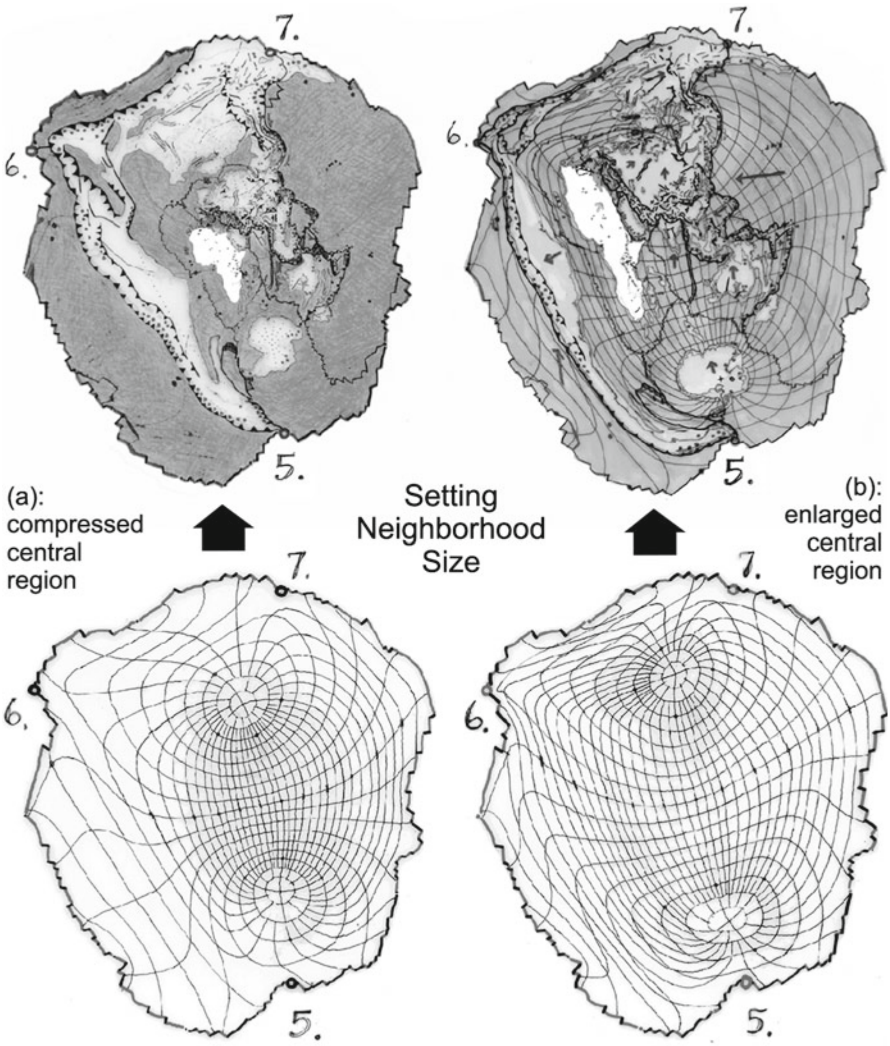

Fig. 2.5 The graticle-grid (*bottom*) used to create CSNB maps (*top*) (Clark 2003). All maps have constant-scale perimeter, but *left* and *right columns* illustrate different algorithms for varying distortion (compressed versus enlarged central region as seen on bottom) as a function of distance from perimeter. Note differences in neighborhood size and location (e.g., Africa) (Source data: Courtesy of NASA/GSFC, Global Tectonic Activity Map, Paul Lowman)

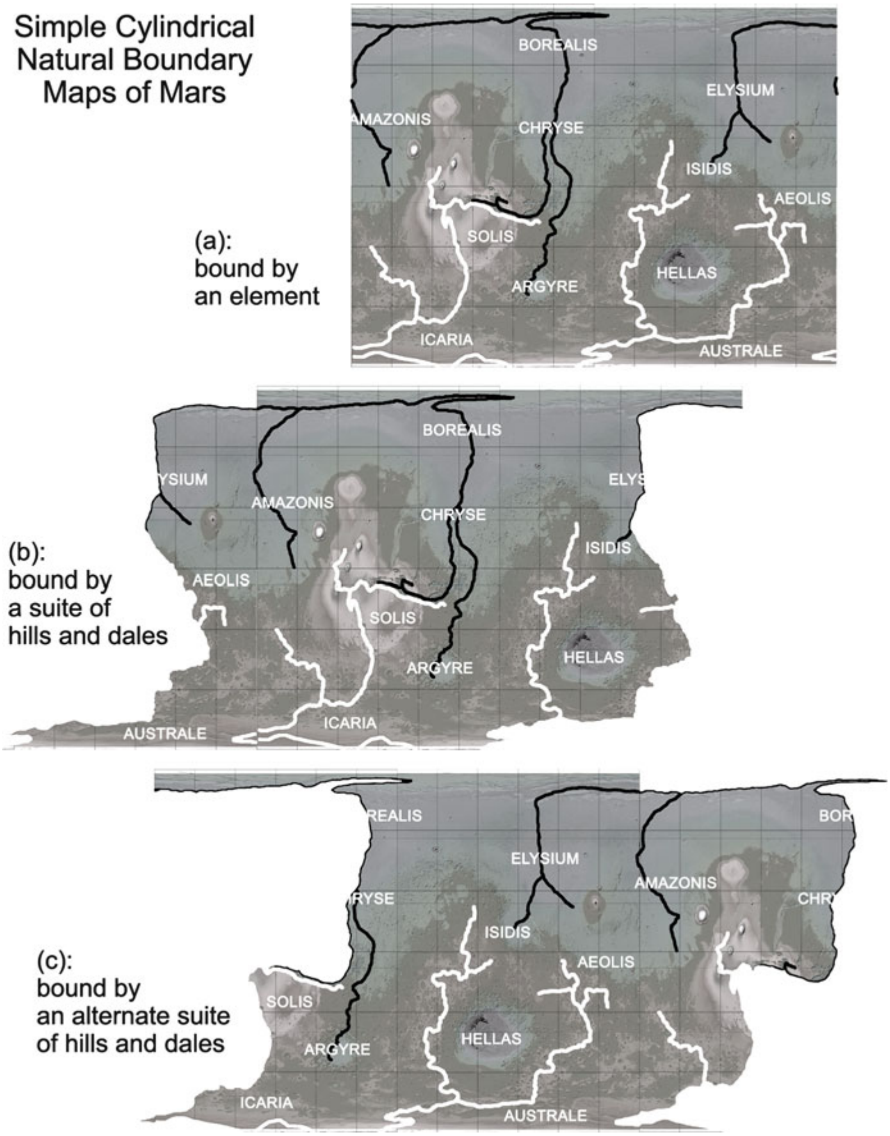

Fig. 2.6 Recomposing Mars conventional maps to represent natural boundaries. Spilhaus (1991) used this method for terrestrial maps. Clark (2004b) with help of René De Hon (Source data: MOLA courtesy of NASA)

2.5 Folding

We can test the accuracy of CSNB maps by folding into a 3D model, as illustrated in Figs. 2.3 and 2.7. Folding is a unique property of this approach and signifies the ease of 3D-2D-3D transition. CSNB maps should fold neatly, without gaps. If not, a mistake exists in the map's border. Forms should 'close' in a similar sense as an accurate survey does when metes and bounds are plotted.

We see a larger inference than mere drafting accuracy in this property. The lower form in Fig. 2.3c is a smaller volume because of its smaller surface. While it shouldn't surprise topologists that CSNB maps always fold, the volumetric change within a constant perimeter, Panofsky's prototopological property, demonstrates that constant edge scale is the sole necessity for foldability. This is dramatically illustrated in Fig. 2.7: a CSNB map made from a fine-branched tree of severely limited global extent is accompanied by its bizarre but nevertheless neatly closed folded form. Unlike conventional maps, CSNB maps have conformal antipodes, and unlike geodesic maps, which also fold, CSNB antipodes are naturally bounded. CSNB folded forms are not planar-faced, straight-line-edged solids, i.e., polyhedra; instead, they have one, variable and irregular face with multiple vertices. These solids may be novel geometrical entities. By inspection, they appear to be self-organizing amalgams of cones, cylinders, and twisted sheets. We might call such solids *unihedrons* or *unitopes*.

When CSNB maps are partially folded, exterior or interior appendages, such as submerged tectonic plates, can be modeled, as we saw in Fig. 1.9. And when CSNB maps are completely folded, they have weak and strong axes, which may mimic an object's actual conditions, such as Earth's trenches and spreading ridges. We envision adjustable mechanical globes with interlaced and manipulable boundaries, able to demonstrate the push-pull of plate tectonics. (An example is in Sect. 4.2.)

For regular objects, the only true 3D representations are triaxial ellipsoids, so such models, while instructive in showing relationships, will not be true representations. However for irregular objects such as asteroids, CSNB models made from highly segmented maps will be the truest representations, as discussed in Chap. 6. And printed flat sheets can be folded to make 3D photoreal models, a capability of obvious economic virtue, a dollop of public fascination (Jacobs 2009) and high educational value (Lakdawalla 2008).

2.6 Waterlining

The nature and impact on selected intra-plate stress fields generated by tensional or compressional physical processes at boundary lines or points can be captured semi-quantitatively by using the CSNB waterlining technique (Fig. 2.8). To waterline is to draw a series of concentric lines paralleling an initiating line (Blum 1967); such lines represent stress contours (Zoback 1992; Christensen 1999; Siddiqi and Pizer 2005).

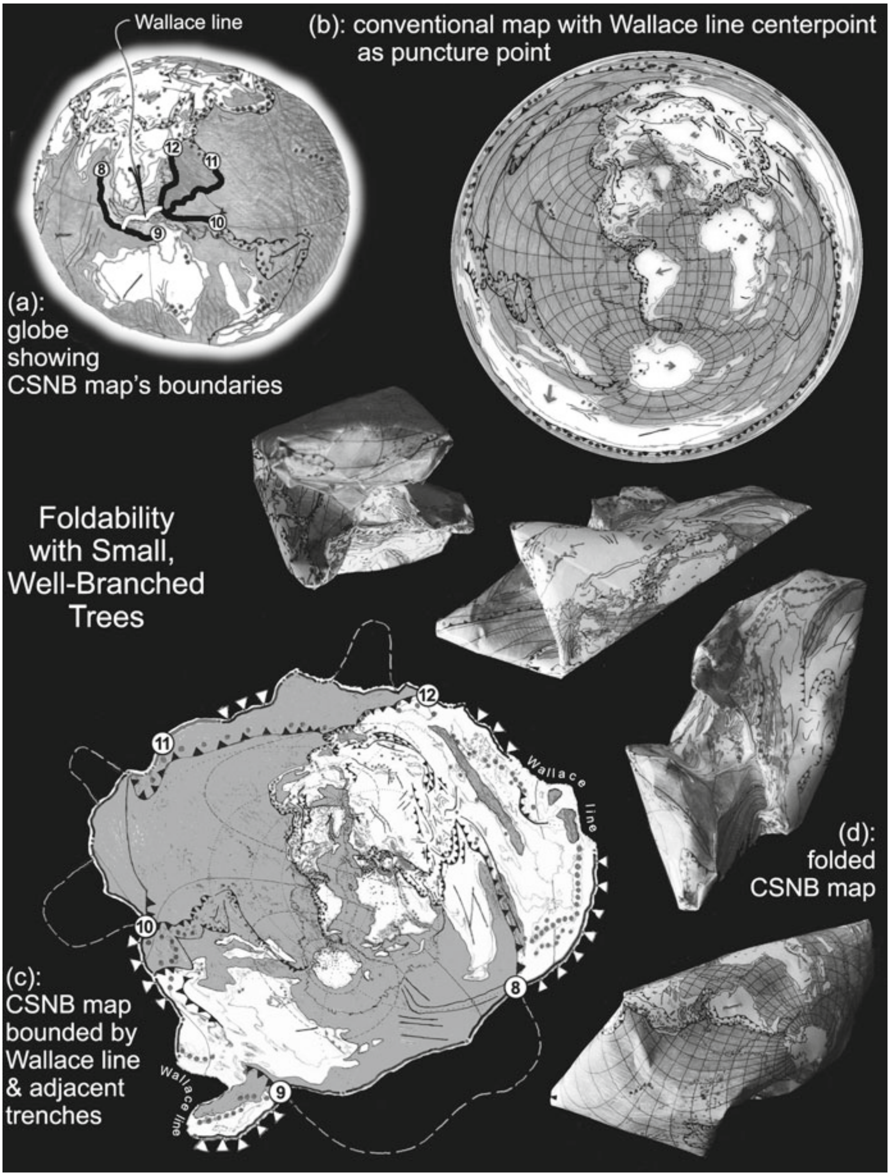

Fig. 2.7 Earth from the viewpoint of Wallace's line (Whitmore 1981), a boundary based on species distribution, and on nearby tectonic activity, as discussed in Sect. 4.2 (Clark 2004a). Even a localized, well-branched tree generates a map and folded form, though the result is bizarre (Source data: courtesy of NASA/GSFC, Global Tectonic Activity Map, Paul Lowman)

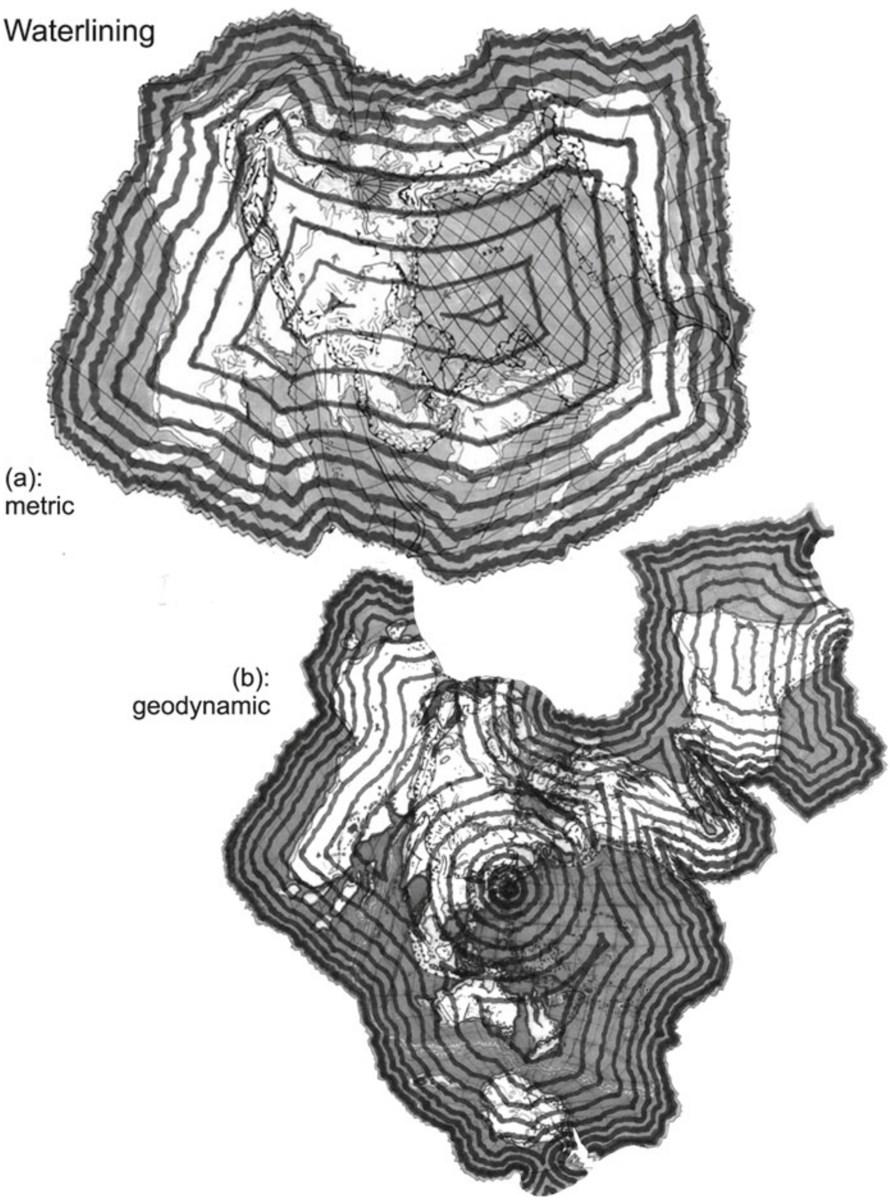

Fig. 2.8 Illustration of (**a**) metrical and (**b**) geodynamical waterlining as discussed in text (Source data: courtesy of NASA/GSFC, Global Tectonic Activity Map, Paul Lowman. Drawn by Chuck Clark)

The boundary may be thought of as a stimulus, an initiating wave; hinges may be thought of as first order, mid-plate, uniaxial stress vectors best fit to the data (Lowman 1999). First order stresses express as inward wavefronts, and second order stresses as functions carried on the first order waves. Hinge rotation and location could be adjusted to model observed stresses.

Similarly, a series of concentric lines representing stress movement may be drawn around a single, point-like event occurring anywhere on the map. The resulting interference patterns may reveal previously undiscovered associations, and show correlations with terrane types or features, as suggested in Fig. 2.8. One set of waterlines works inward from the map's edge (a segment of the mid-ocean ridge system), and expresses the effects of sea floor spreading (metrical) (Fig. 2.8a). A second set of waterlines radiates outward to express the effects of the 2011 Tōhoku megathrust earthquake (geodynamic) (Fig. 2.8b).

The medial axis can be useful in locating centroids, vectors, and distances. Waterlines can be used to locate the medial axis, as in Fig. 1.8, where ocean-basin labels follow a medial axis established by waterlines radiating from shore. The medial axis may include or be merged with fractures, faults and trenches, and wavefronts or critical boundaries redrawn accordingly. Inversely, the medial axis, a topological skeleton akin to a CSNB tree, could be portrayed in the greatest detail, and its stresses, including Coulomb stresses associated with earthquakes, emphasized by becoming critical boundaries, a CSNB map edge.

2.7 Demonstration

A pipe cleaner demonstration (Fig. 2.9) is helpful in visualizing the CSNB method. In moving pipe cleaners from globe to table, the only geometry in play is *development* and not, as in conventional maps, *projection*. A sphere's finite, unbounded surface becomes a plane's infinite, bounded surface, as the pipe cleaners leave the grapefruit and flatten on the table. At A, the object's surface has not yet been interrupted. This happens at B by unhooking one pipe cleaner and unzipping the boundary along its length (CDE). Reconnecting the pipe cleaner soon binds the plane's infinite field. At this stage, while nothing is required *in* the field of the map, a shape model exists, as seen in the folded forms (as we saw in the foreground of Fig. 1.11).

Figure 2.10 illustrates CSNB mapping of a regular polyhedron, in this case a brick. A tree marks its edges. The tree is fully inter-digitated: if a branch were lopped, the result would be deficient; if it grew another branch, a circuit (prohibited in this simplified exposition) would occur. As we saw in Fig. 2.1, the sequence ABCD shows the flattened tree unzipping, branches swinging to enclose a shape. Sequence D´EFG shows the shape's internal proportions adjusting via diagonal distancing (dashed lines). Shape D is, of course, the 'developed geometry' begun by Dürer. Shapes DEF fold to forms DEF, which are prototopological equivalents to the brick; form G is the brick's geometrical replica.

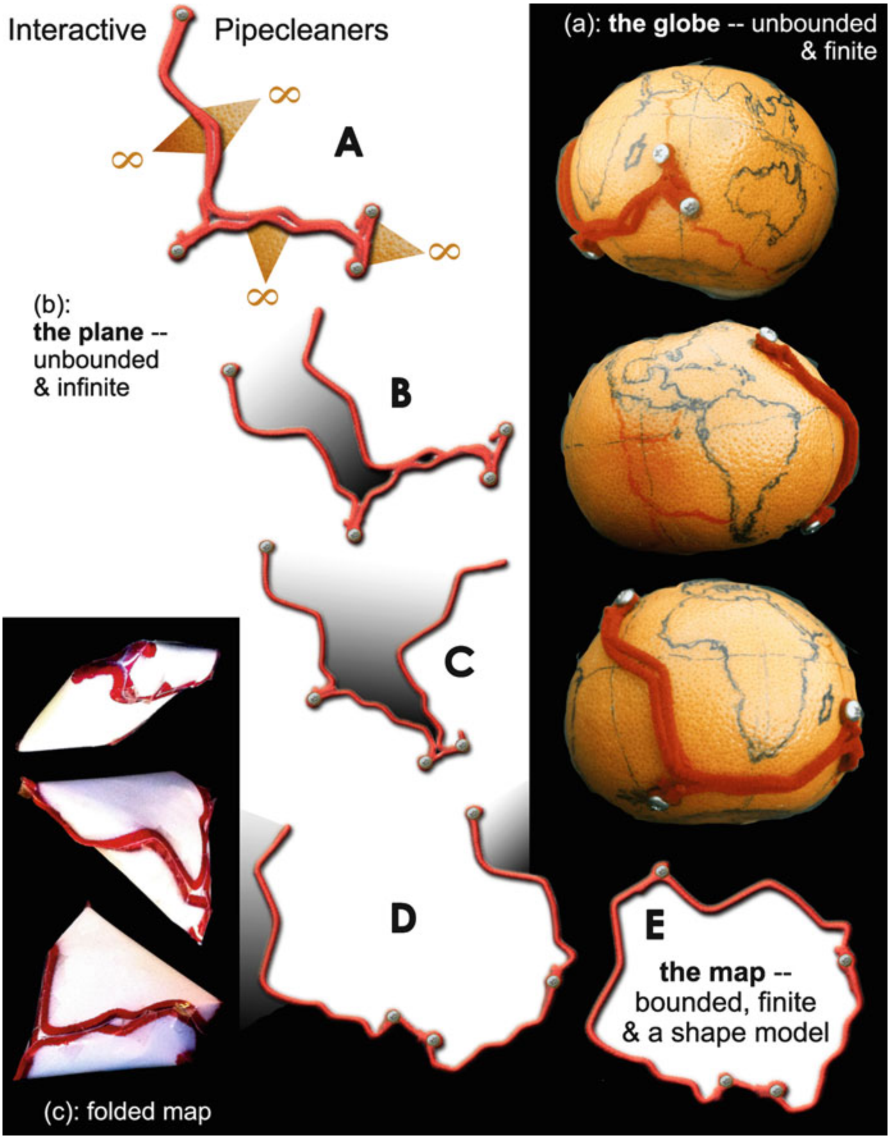

Fig. 2.9 (**b**) Demonstration of developing, unzipping, hinging, and reconnecting boundaries (Clark 2003), (**a**) their relationship to original object, and (**c**) 3D shape model. Note that (**c**) is prototopologically equivalent to (**a**), given the selected boundaries. From demonstration at Advances in Extraterrestrial Mapping workshop (2003)

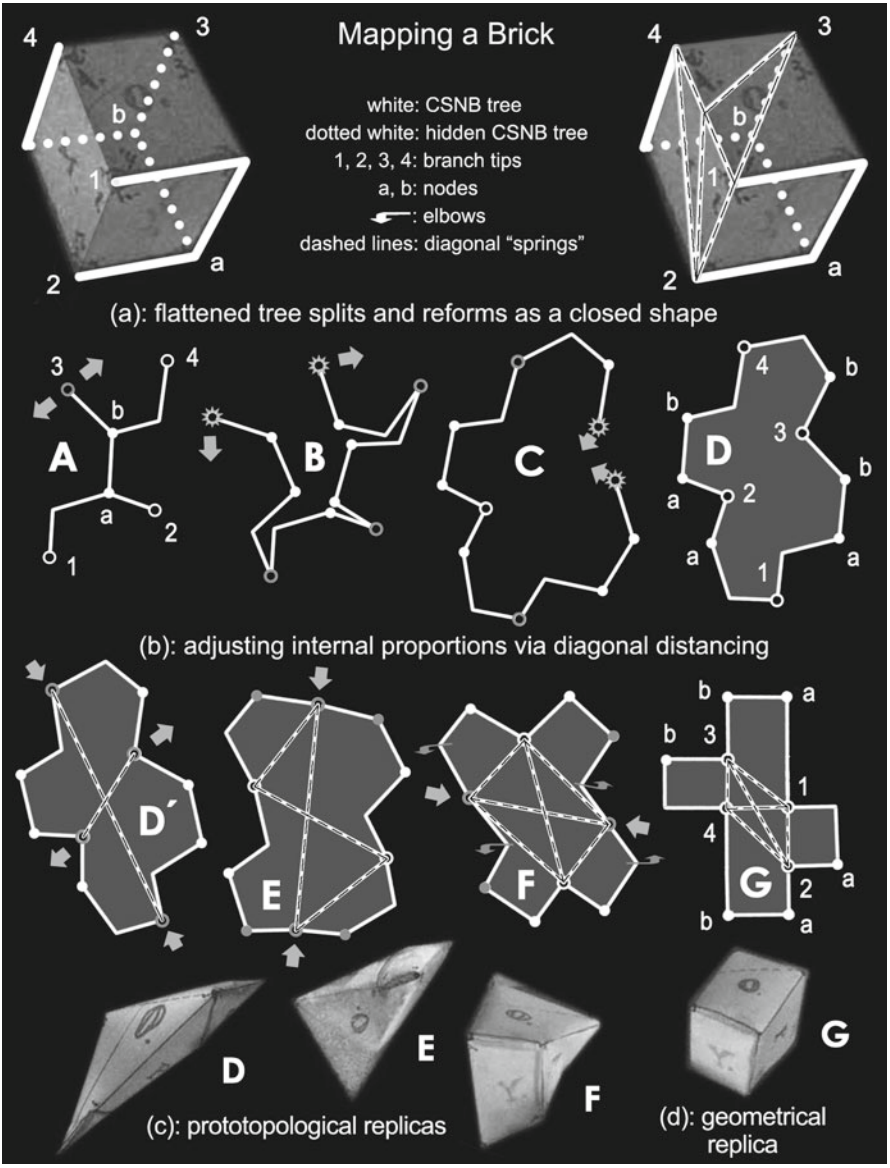

Fig. 2.10 Step-by-step CSNB mapping of a brick, as discussed in text (Clark 2007) (Suggested by P.E. Clark and Gunther Kletetschka)

2.8 Summary of Implications for Global Mapping

CSNB maps have these distinctive features:

1. The map edge is a deformable wavefront, with resolution dependent on boundary complexity.
2. Models, folded along edges, are condensations of the object, and, for irregular solids, such as asteroids, can be true representations.
3. Resolution is highest at the most recognizable, pivotal, and defined feature, forming the boundary, where local proportions are preserved.
4. The CSNB approach, unlike conventional map projections, preserves antipodal geometry (maps are conformal for antipodal areas).
5. Local interior proportions are also preserved. Equalizing ratios of hinge connector cross-map-lengths to lengths between corresponding object points minimizes shape distortion.
6. As resolution increases and/or more natural boundaries are discovered, additional edges subdivide and refine the original shape model.

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