Preface

Before I speak, I have something important to say.
Groucho Marx

In the literature there exist a lot of traditional local search techniques that have been designed for problems where the objective function $F(y), y \in D \subset R^N$, has only one optimum and a strong a priori information is known about $F(y)$ (for instance, it is supposed that $F(y)$ is convex and differentiable). In such cases it is used to speak about *local optimization problems*. However, in practice the objects and systems to be optimized are frequently such that the respective objective function $F(y)$ does not satisfy these strong suppositions. In particular, $F(y)$ can be multiextremal with an unknown number of local extrema, non-differentiable, each function evaluation can be a very time-consuming operation (from minutes to hours for just one evaluation of $F(y)$ on the fastest existing computers), and nothing is known about the internal structure of $F(y)$ but its continuity. Very often when it is required to find the best among all the existing locally optimal solutions, in the literature problems of this kind are called *black-box global optimization problems* and exactly this kind of problems and methods for their solving are considered in this book.

The absence of a strong information about $F(y)$ (i.e., convexity, differentiability, etc.) does not allow one to use traditional local search techniques that require this kind of information and the necessity to develop algorithms of a new type arises. In addition, an obvious extra difficulty in using local search algorithms consists of the presence of several local solutions. When one needs to approximate the global solution (i.e., the best among the local ones), something more is required in comparison with local optimization procedures that lead to a local optimum without discussing the main issue of global optimization: whether the found solution is the global one we are interested in or not.

Thus, numerical algorithms for solving multidimensional global optimization problems are the main topic of this book and an important part of the lives of the authors who have dedicated several decades of their careers to global optimization. Results of their research in this direction have been presented as plenary lectures at dozens of international congresses. Together with their collaborators the authors have published more than a hundred of research papers and several monographs in English and Russian since 1970s of the twentieth century. Among these publications the following three volumes [117, 132, 139] can be specially mentioned:

1. Strongin, R.G.: Numerical Methods in Multi-Extremal Problems: Information-Statistical Algorithms. Nauka, Moscow (1978), in Russian
2. Strongin, R.G., Sergeyev, Ya.D.: Global Optimization and Non-Convex Constraints: Sequential and Parallel Algorithms. Kluwer Academic Publishers, DD (2000)
3. Sergeyev, Ya.D., Kvasov, D.E.: Diagonal Global Optimization Methods. FizMatLit, Moscow (2008), in Russian

Each of these volumes was in some sense special at the time of its appearance: the monograph of 1978 was one of the first books in the world entirely dedicated to global optimization; the second monograph for the first time has presented results of the authors in English in a comprehensive form giving a special emphasis to parallel computations—a peculiarity that was extremely innovative at that time; finally, the monograph of 2008 was one of the first books dedicated to global optimization and published in Russian since events occurred in the Soviet Union in 1990s filling so in the gap in publications in this direction that was 15–20 years long.

The decision to write the present Brief has been made due to the following two reasons. First, as it becomes clear from the format of this publication (Springer Brief) and the title of the book—*Introduction to Global Optimization Exploiting Space-Filling Curves*—the authors wished to give a brief introduction to the subject. In fact, the monograph [139] of 2000 has been also dedicated to space-filling curves and global optimization. However, it considers a variety of topics and is very detailed (it consists of 728 pages). Second, it is more than 10 years since the monograph [139] has been published in 2000 and the authors wished to present new results and developments made in this direction in the field.

The present book introduces quite an unusual combination of such a practical field as global optimization with one of the examples *per eccellenza* of pure mathematics—space-filling curves. The reason for such a combination is the following. The curves have been first introduced by Giuseppe Peano in 1890 who has proved that they fill in a hypercube $[a,b] \subset R^N$, i.e., they pass through every point of $[a,b]$, and this gave rise to the term *space-filling curves*. Then, in the second half of the twentieth century it has been independently shown in the Soviet Union and the USA (see [9, 132, 139]) that, by using space-filling curves, the multidimensional global minimization problem over the hypercube $[a,b]$ can be turned into a one-dimensional problem giving so a number of new exciting possibilities to attack hard multidimensional problems using such a reduction.

The book proposes a number of algorithms using space-filling curves for solving the core global optimization problem—minimization of a multidimensional, multiextremal, non-differentiable Lipschitz (with an unknown Lipschitz constant) function $F(y)$ over a hypercube $[a,b] \subset R^N$. A special attention is dedicated both to techniques allowing one to adaptively estimate the Lipschitz constant during the optimization process and to strategies leading to a substantial acceleration of the global search. It should be mentioned that there already exist a lot of generalizations of the ideas presented here in several directions: algorithms that use new efficient partition techniques and work with discontinuous functions and functions having Lipschitz first derivatives; algorithms for solving multicriteria problems and problems with multiextremal non-differentiable partially defined constrains; algorithms for finding the minimal root of equations (and sets of equations) having a multiextremal (and possibly non-differentiable) left-hand part over an interval; parallel non-redundant algorithms for Lipschitz global optimization problems and problems with Lipschitz first derivatives, etc. Due to the format of this volume (Springer Brief) these generalizations are not considered here. However, in order to guide the reader in possible future investigations, references to a number of them were collected and systematized (see p. 117).

In conclusion to this preface the authors would like to thank the institutions they work at: University of Calabria, Italy; N.I. Lobachevsky State University of Nizhni Novgorod, Russia; University of Cagliari, Italy; and the Institute of High Performance Computing and Networking of the National Research Council of Italy. During the recent years the research of the authors has been supported by Italian and Russian Ministries of University, Education and Science and by the Italian National Institute of High Mathematics "F. Severi." Actually research activities of the authors are partially supported by the Ministry of Education and Science of Russian Federation, project 14.B37.21.0878 as well as by the grant 11-01-00682-a of the Russian Foundation for Fundamental Research and by the international program "Italian-Russian University."

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