Preface

The thematic program on "Discrete Geometry and Applications" took place at the Fields Institute for Research in Mathematical Sciences in Toronto between July 1 and December 31, 2011. The core part of the book is based on my three lectures, delivered at the the Fields Institute under the titles "Contact numbers for congruent sphere packings" (September 23, 2011), "Rigid ball-polyhedra" (October 11, 2011), and "On a strong version of the Kepler conjecture" (November 17, 2011).

One can briefly describe discrete geometry as the study of discrete arrangements of geometric objects in Euclidean, as well as in non-Euclidean spaces. J. Kepler was the first to raise discrete geometry problems on packings of balls in the early 1610s, but the systematic research began in the late 1940s with the work of L. Fejes Tóth. The Hungarian school he founded focused mainly on packing and covering problems, while a number of great mathematicians helped to lay a broad foundation for the emerging field of discrete geometry, including H. S. M. Coxeter, J. H. Conway, B. N. Delaunay, B. Grünbaum, V. Klee, and C. A. Rogers. Two active areas that have been outstanding from the birth of discrete geometry are dense sphere packings and tilings. Both occupy a substantial part of my book as well. There is a chapter on unit sphere packings and there are a number of sections that apply the method of Delaunay and Voronoi tilings in the solutions of a variety of problems. Sphere packings and tilings have a very strong connection to number theory, coding, groups, and mathematical programming. In particular, the latter greatly helped to achieve recent breakthrough results. Extending the tradition of studying packings of spheres, another core topic of my book is the investigation of the monotonicity of volume under contractions of arbitrary arrangements of spheres. The research on this fundamental topic started with the conjecture of E. T. Poulsen and M. Kneser in the late 1950s. The third major topic of my book can be found under the sections on ball-polyhedra introducing an extension of the theory of convex polyhedral sets to the family of intersections of congruent balls. This part of my book is connected in many ways to the above mentioned major topics and it is also connected to some other important research areas as well including the one on coverings by planks (with close ties to geometric analysis). This topic is the forth major one in my book

discussed under coverings by cylinders. The research work on the latter topic started with a conjecture of A. Tarski in the early 1930s[1].

My book is aimed at advanced undergraduate and early graduate students, as well as interested researchers. In addition to leading the reader to the frontiers of geometric research on sphere arrangements, it gives a short introduction to the relevant modern parts of discrete geometry. I have structured the book in such a way that the four major research topics (unit sphere packings, contractions of sphere arrangements, ball-polyhedra, and coverings by cylinders) are surveyed in individual chapters (Chaps. 1, 3, 5, and 7) each followed by a chapter with a collection of selected proofs (Chaps. 2, 4, 6, and 8). The survey chapters are readable independently from each other. The selected proofs combine elementary and convex geometry with analytic and in some cases, probabilistic or topological ideas. They are the results of the author's joint work with a number of discrete geometers. In addition, an independently understandable collection of unsolved problems is compiled in Chap. 9.

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