Preface

In classical physics and engineering many problems are described by ordinary or partial differential equations. One usually considers an idealized situation, namely, the uncontrolled perturbations of a random system are excluded, although from the practical point of view taking into account not only the random evolution but also external random perturbations is of great interest.

To illustrate the motivation given above, consider the following situation. Let the evolution system be described by the differential equation

$$Lu + f(u) = 0,$$

Where L is some differential operator.

The solution to this equation (provided that it exists) is a deterministic function which completely characterizes the system. Suppose now that the system is subject to small external random perturbations. Under certain conditions it can be proved, using the central limit theorem, that the system is perturbed by a Gaussian white noise. Therefore, in a somewhat simplified situation, we can assume that the evolution system is described by the differential equation

$$Lu + f(u) = \xi,$$

where ξ is a random Gaussian perturbation.

The solution to this equation is no longer a deterministic function, and therefore the behavior of the system cannot be absolutely precise. However, it is possible to predict (in some sense optimally) the behavior of the system using more simple observations of the equation. For systems described by ordinary stochastic differential equations, this method allows to approach many practical problems in a new way, based on the new theory of automatic control of stochastic systems. But there are many problems, both theoretical and applied, solving of which requires new approaches. This often happens due to the fact that the investigated processes are described by stochastic partial differential equations. This problem is extremely extensive, so one cannot describe the whole range of applications associated with it.

To illustrate this let us focus on two problems that are important both from theoretical and applied points of view.

1. *The hydrodynamic instability problem.* In the modern approach for solving this problem, a fluid is considered as a nonlinear mechanical system with a very large number of degrees of freedom. Due to the nonlinearity of the dynamics various degrees of freedom mutually interact. In Eulerian description of the fluid motion such interactions are the inertial interactions between the inhomogeneities of the velocity field; the interaction constant is called the Reynolds number. Hence, for sufficiently large Reynolds numbers (for which the fluid motion may become unstable), the interactions between the degrees of freedom are very strong. Therefore, an unstable perturbation of a single degree of freedom rapidly leads to the perturbation of many other degrees of freedom, and the fluid motion becomes very complex and irregular, hardly amenable to the particular description. It is reasonable to describe this motion only statistically, using the methodology of the theory of random fields. And there, of course, arises the problem how to predict such a field based on the observations of another field with more simple structure.

   Hence, it becomes possible to obtain statistical estimates for the values of the process arising in many natural and technological phenomena such as the formation of sea waves, the resistance crisis when a fluid flows around curved profiles, and thermal convection.
2. *The problem of the optimal control of technological processes.* Consider the problem arising in the study of gases' absorption and desorption. Let a tube of length L be filled with an absorbing material (sorbent). We choose the tube axis as the coordinate axis, and assign the tube entrance, through which the gas–air mixture is supplied starting from the time moment $t = 0$, as the origin. If $u(x, t)$ is the gas concentration at time t in the tube layer x, then we have the following relation for the case of the low concentration of the supplied gas:

$$u_{kt} + \frac{\beta}{\nu} u_t + \beta\gamma u_x = 0,$$

   where β is the kinetic coefficient, ν is the gas velocity, and $1/\gamma$ is the Henry coefficient.

Many factors such as all sorts of irregularities in the distribution of the sorbent in the tube, the irregularity of the supply of gas mixture flow in time, its heterogeneity, and others make it necessary to consider equations with random coefficients and additional terms which reflect the diversity of random deviations from the process, which can be characterized using a multiparameter white noise. Here, again one comes across the problem of finding estimates for the parameters of the process. Among such problems there are of course the problems of existence and uniqueness of the solution to stochastic partial differential equations, which are of great theoretical and practical interest.

The book is focused on the study of the stochastic differential equations of hyperbolic type. Historically, the attention was focused first on the study of the stochastic Darboux equations, where the two-parameter Wiener field is taken as a noise. Therefore, the first question is how to construct a stochastic integral with respect to this field, and what are its basic properties. Similar to the one-parameter case, the stochastic integral was built with respect to a two-parameter martingale, which makes it possible to study the stochastic differential equations of hyperbolic type, where the two-parameter martingale was considered as noise. Such integrals and martingales are considered in Cairoli and Walsh [7], Etemadi and Kallianpur [19], Wong and Zakai [72], Gyon and Prum [36], and many others. One of the most fundamental results in the theory of stochastic differential equations is the Girsanov theorem, which provides the possibility to solve many recognition, estimation, filtration, forecasting, and optimal control problems. Such problems are considered completely enough in Gikhman and Skorokhod [26–29], Liptzer and Shiryaev [54], Novikov [63], and many other monographs. Similar problems arise for stochastic partial differential equations as well, and the two-parameter analog of Girsanov theorem [13, 50] gives us a key to find the solution for the above-mentioned estimation and control problems. On the other hand, related problems arise in the theory of evolution equations with Wiener Hilbert-valued process.

The problems described above are the basic content of this book. The authors tried to make the exposition of the material self-contained, in particular, to provide all necessary definitions and results used later on in the proofs.

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