# Preface

The aim of this text is to introduce the reader to the core topics that constitute an introductory course in finance and financial engineering. Our particular emphasis is on illustrating principles and modeling through the Monte Carlo method. Monte Carlo is the uniquely appropriate tool for this purpose because the driving factors underlying the market are primarily random in nature. And just as the random dynamics works its way through the system and into financial observables, we may track the chain of influence every step of the way computationally.

The intended audience for the book is upper division undergraduates or beginning graduate students in mathematics, finance or economics. The reader is assumed to have knowledge of calculus through partial derivatives, Taylor series and LaGrange optimization, probability through an understanding of random variables, expectation, distribution and density functions including the normal distrubution, and basic matrix algebra through the solution of linear systems. A refresher for these topics is presented in the Appendices.

For additional background on probability with Monte Carlo methods, it may be useful to read through *Explorations in Monte Carlo Methods*, a textbook that I co-authored with Franklin Mendivil, published in Springer's *Undergraduate Texts in Mathematics* series ©2009.

In keeping with our presentation of the material in parallel with the Monte Carlo method, a majority of the exercises are primarily programming in nature. Hopefully this is where the real understanding takes place. A great enjoyment can derive from experimenting with parameters and seeing the results unfold, sometimes surprisingly, always in an interesting way.

Regarding programming, I prefer allowing students to use whatever language with which they are familiar; many use Matlab™, Maple®, R, C, and Java. However, some of the results can only be appreciated if presented graphically, for example as histograms or x-y plots. In the case of the first three, graphics is built-in; otherwise there is ample public domain software for rendering numerical output.

The programming background needed is quite modest, basically, branching, loops, and subroutines. In many cases, other than the boiler plate, programs span fewer than a dozen lines. Furthermore, programming code, given in a mathematical format, is presented in line with the text as encountered. I treat these in exactly the same way as displayed equations. Like equations, they are condensed and must be read with care.

## Organization of the Book

Two fundamental systems for analyzing market prices are presented in Chapter 1, the geometric Brownian motion (GBM) model and the binomial lattice model. GBM is preceded by a first principles introduction to Wiener processes. A more in-depth treatment is given in the Appendices if needed. Thus a rationale is provided for the Monte Carlo method and price simulation by the geometric random walk. In a kind of turn-about, we use the numerical method of the geometric random walk to derive the theoretical distribution for maturity stock prices, the lognormal distribution.

Starting out in this way allows for the implementation of the Monte Carlo method immediately. We take advantage of that by investigating one of the tenets of modern finance, the efficient market hypothesis (EMH). More generally we show that Monte Carlo can be used to test the antithesis of EMH, namely technical analysis (TA).

But in order to do that, we must have access to a database of historical prices. One of the best free sources of historical price information can be found at http://finance.yahoo.com. However this text must be independent of that lest its access change in the future. Therefore a database of prices, the FIMCOM prices, is supplied at the following URL:

http://people.math.gatech.edu/~shenk.

The FIMCOM database is in exactly the same format as that of finance.yahoo. Either can be used to test programs and to answer TA queries. Additionally, I have placed utility programs for working with the FIMCOM database or the finance.yahoo prices, on the aforementioned website. Further, any eventual errata to this text will also be found there.

Chapter 2 is devoted to basic investment science and to the important mean-variance theory of portfolio management. The central tenet of this theory is that diversification ameliorates risk. But, equally important, it is a completely quantitative theory providing for an exact measurement of risk. Strikingly specific investment policy, widely implemented in practice, derives from the theory. With regard to risk, the GBM model for stock prices lends itself to a natural explanation of the value at risk (VAR) and its Monte Carlo calculation.

Chapter 3 introduces forward contracts and options as tools for the alleviation of risk. This leads to the important topic of option pricing and its solution by the fundamental principle of no-arbitrage and the risk-neutral probability. In the interest of pedagogy, our approach builds on the binomial lattice model. Using it we derive techniques for pricing both European and American options. Separately, we obtain the Black–Scholes formulas for European options by straightforward integration of the maturity distribution. As a Monte Carlo technique for American puts, we introduce the notion of the exercise boundary.

Chapter 4 exhibits the power of the Monte Carlo method for it is here that we introduce exotic options and use the method to price them. Often these options require knowledge of the path prices take to maturity and some even require future knowledge in order to price. For some of these options Monte Carlo is the only applicable method. Moreover, the techniques illustrated here are not restricted to the GBM model for prices. They work just as well, for example, with Lévy models, the main topic of Chapter 6.

Chapter 5 deals with financial engineering and some practical aspects of options, namely option trading. Many of the most popular option strategies are investigated, among them are: covered calls, spreads, butterflies, straddles and condors. Here we introduce a novel use of Monte Carlo as a tool for the prediction of expected outcomes of these strategies under differing market conditions. Here also the option "greeks" are defined and studied. Their practical use for insulating a portfolio against market fluctuations in price and volatility is demonstrated.

In Chapter 6 we delve into more advanced processes for market prices, exponential Lévy processes. These are recent developments that include models allowing for discontinuities in prices and models that support "heavytailed" phenomena. The latter refers to market collapses that occur too frequently as predicted by the Gaussian model upon which Black–Scholes is based. Although a rigorous treatment of this material is beyond the scope of this text, we nevertheless strive to convey the essentials of the relevant mathematics without getting too involved in technicalities. Instead we focus on the application of these processes to financial modeling and the implications for option pricing.

Chapter 7 addresses the problem of optimally allocating resources among risky ventures. The solution, as formulated in a 1950s paper out of Bell Laboratory, and its application to finance has been controversial. Still the method is sound, interesting and valid in its claims.

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