

Preface

This monograph is intended to present the current state of the art of the so-called theory of *geodesic convexity* in finite, simple, connected graphs. It has been designed with the objective of being useful and stimulating for research workers in the field and for developing courses on convexity in graphs for graduate students as well as advanced undergraduates.

Convexity, in both continuous and discrete structures, has been studied in many contexts. These contexts have been generalized to the concept of a *convexity space*, which is a pair (V, C) , where V is a set and C is a collection of subsets of V , called *convex sets*, such that $\emptyset, V \in C$ and C is closed under arbitrary intersections and nested unions.

Although convex sets have been introduced in different settings, the most useful definitions are based on the notion of *betweenness*. For example, given a metric space (X, d) , a set A is *d-convex* if for every two points $x, y \in A$, the segment $[x, y]$ connecting x and y lies completely in A , where the segment $[x, y]$ stands for the set of points which are *between* x and y , i.e., $[x, y] = \{z \in X : d(x, z) + d(z, y) = d(x, y)\}$.

Given a finite, simple, connected graph $G = (V, E)$, it can be treated as a metric space by means of their shortest paths (also known as *geodesics*), as follows. For any pair of vertices $u, v \in V$, the distance between u and v is the length of any geodesic joining x and y . The convexity induced by this metric is called the *geodesic convexity* of G .

The main purpose of this lecture notes is to survey the main ideas, concepts, and results related to *geodesic convexity* in graphs. A glance at the table of contents shows that the book is organized in seven chapters; it contains a complete bibliography on the subject, a short glossary of definitions directly related to this topic, an index of terms, and finally, an index of symbols. Furthermore, unless otherwise expressly stated, all terms, invariants, and results pertain to the *geodesic convexity*.

In order to make the book as self-contained as possible, the monograph begins with an introductory chapter organized in four short sections: *Graph Theory*, *Metric Graph Theory*, *Convexity Spaces*, and *Graph Convexities*. Each of them ends up

with a list of reference citations including books, surveys, and the most significant papers related to the topic.

The rest of the book is organized in six chapters. The first of them, under the title *Invariants*, contains the most basic and significant terms, concepts, results, and techniques, and hence it constitutes, roughly speaking, the core of the monograph. Except in a few cases, mainly due to space reasons, each nonobvious result is followed by the corresponding detailed proof, written in concise and schematic style, always starting from the original proof of the cited paper.

After carefully analyzing each of the more than 200 papers that have approached, directly or indirectly, the study of the geodesic convexity in graphs, most of them being published in the last two decades, and excluding those results included in Chap. 2, the rest of them have been grouped in five short chapters: *Graph Operations*, *Boundary Sets*, *Steiner Trees*, *Oriented Graphs*, and *Computational Complexity*. Each of them has been designed in the same way as Chap. 2, that is to say, including, in almost every case, result plus proof.

This monograph contains about 10 tables, 40 figures, 100 proofs, 25 sketches of proofs, 20 conjectures (or open problems), and 182 cited references.

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