Preface

Ricci flow deforms the Riemannian metric proportionally to the curvature, such that the curvature evolves according to a heat diffusion process and eventually becomes constant everywhere. Ricci flow is a powerful tool in geometric analysis for studying low dimensional topology. It has been successfully applied for the proofs of Poincaré's conjecture and Thurston's geometrization conjecture. Recently, Ricci flow has started making impacts on practical fields and tackling fundamental engineering problems. This book focuses on the theories and algorithms of discrete surface Ricci flow, and its applications on surface registration and shape analysis.

General Ricci flow is defined on arbitrary dimensional Riemannian manifolds. Surface (two-manifold) Ricci flow has unique characteristics, which are crucial for developing discrete theories and designing computational algorithms. *First*, surface Ricci flow never blows up, namely, the Gauss curvature during the flow is always bounded. This phenomenon ensures the numerical stability of discrete surface Ricci flow. In contrast, three-manifold Ricci flow will produce singularities; thus topological surgery is unavoidable. *Second*, surface Ricci flow is conformal, namely, the deformation of the Riemannian metric preserves angles. This fact greatly simplifies both theoretical arguments and algorithmic designs. General Ricci flow is governed by tensor differential equations, whereas surface Ricci flow is described by scalar differential equations. *Third*, surface Ricci flow has intuitive geometric interpretations, which directly lead to the design of data structures. A conformal deformation transforms infinitesimal circles to infinitesimal circles. This elucidates the geometric nature of the flow. *Finally*, Ricci flow is variational, namely, Ricci flow is the negative gradient flow of Ricci energy. Accordingly, discrete surface Ricci flow can be formulated as a convex optimization problem, which has a unique global optimum and can be carried out using the efficient Newton's method.

For the purpose of surface registration and shape analysis, discrete surface Ricci flow has the following unique merits: (a) by Ricci flow, all shapes in real life can be unified to one of the following three canonical shapes: the sphere, the plane, or the hyperbolic disk; (b) therefore, most 3D geometric problems can be converted to 2D image problems, which greatly simplifies the computation; (c) furthermore,

this conversion is conformal and preserves the original geometric information; (d) finally, by deforming Riemannian metric, Ricci flow can be used to compute general diffeomorphisms between surfaces.

Ricci flow has demonstrated its great potential by solving various problems in many fields, which can be hardly handled by alternative methods so far. The following are some examples: (1) nonrigid surface registration and tracking in computer vision, (2) global surface parameterization in computer graphics, (3) conformal brain mapping and virtual colonoscopy in medical imaging, (4) the shortest word problem in computational topology, (5) delivery guaranteed greedy routing and load balancing in wireless sensor network, and so on. We believe that more and more researchers will realize and appreciate the intrinsic power and beauty of Ricci flow, and more and more fields in engineering and medicine will be impacted by Ricci flow.

This book is mainly for graduate students and researchers in the fields of computer science, applied mathematics, engineering, and medical imaging. The book provides both theoretical foundations and computational methods for surface Ricci flow. The introduction to the smooth geometry theories is self-contained. The discrete theories and computational algorithms are written using elementary mathematical tools, and all the details are well exposed, such that students with engineering background can easily follow and digest them. In order to help students and researchers reproduce the algorithms in the book for their own research projects, sample codes and data sets are available on the authors' web sites: http://www.cs.stonybrook.edu/~gu and http://www.cs.fiu.edu/~wzeng. These computational tools are also valuable for professionals in the fields related to surface registration and shape analysis, such as digital media and digital entertainment industry, geometric modeling and computer aided design industry, medical imaging industry, and biometrics industry.

In this book, the first chapter (Chap. 1) gives an overview of the whole contents; the rest of the book is organized into two parts. The first part (Chaps. 2 and 3) gives brief introduction to the theoretical foundations necessary to understand Ricci flow, mainly algebraic topology, surface differential geometry, and Riemann surface theory; the second part (Chaps. 4 and 5) provides complete proofs, algorithmic details for discrete surface Ricci flow, and applications in practice. Students emphasizing engineering applications may start reading the second part directly. An overview of each chapter in the book is as follows:

- Chapter 1 introduces the fundamental concepts of shape space and mapping space, including different transformation groups (such as diffeomorphisms, isometries, conformal transformations, and rigid motions) and group actions on shape spaces. In order to perform surface registration and shape analysis in the shape space and the mapping space, Ricci flow is introduced, which leads to the celebrated uniformization theorem.
- Chapter 2 briefly reviews the fundamental concepts and theorems in algebraic topology, surface differential geometry, and surface Ricci flow.

- Chapter 3 briefly introduces the Riemann surface theory, including quasi-conformal mapping, Teichmüller space, and surface harmonic maps. Finally, the Teichmüller theory of harmonic maps is covered.
- Chapter 4 systematically introduces the discrete surface Ricci flow theory. The whole theory is explained thoroughly using variational principle on discrete surfaces based on derivative cosine law. Complete proofs for most theorems and lemmas are given in detail.
- Chapter 5 focuses on the computational algorithms and direct application examples. The algorithms have been fully tested to handle various problems in real world for many years and are mature for broad practical applications.

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