Preface

The concept of associating ordinary functions with operators has arisen in many areas of science and mathematics and it can be argued that the earliest instance was Leibniz's attempt to define a fractional derivative. Up to the beginning of the twentieth century, many isolated results were obtained and culminated with the remarkable contributions of Heaviside and the efforts to put his methods on a sound mathematical footing. These developments were mostly based on associating a function of one variable with one operator, the operator generally being the differentiation operator.

With the discovery of quantum mechanics in the years 1925-1930, there arose, in a natural way, the issue that one has to associate a function of two variables with a function of two operators that do not commute. This has led to a wonderfully rich mathematical development that has found applications in many fields, including pseudo-differential operators, time-frequency analysis, quantum optics, wave propagation, differential equations, image processing, radar, sonar, chemical physics, and acoustics, among others. The earliest proposal for associating an ordinary function of two variables with an operator was that of Born and Jordan (1925), and subsequently Weyl (1929) and others proposed other rules. There are an infinite number of ways to associate a function of two ordinary variables with a function of two operators because ordinary variables commute while operators generally do not. The rules became known as rules of association, correspondence rules, or ordering rules.

Independently of these developments Wigner, in 1932, and Kirkwood, in 1933 devised a classical-like joint distribution where one can calculate operator averages in the standard probability manner, that is, by phase-space integration. No connection was made between these distributions and correspondence rules until Moyal in 1949 saw things clearly. He *derived* the Wigner distribution using the Weyl correspondence. Subsequently it was realized that for every correspondence rule there is a corresponding phase-space distribution. Now the field of correspondence rules and phase-space distributions are intimately connected. Remarkably, around the same time as Moyal, a similar development occurred in the field of time-varying spectral analysis whose aim was to understand signals with changing frequencies, human speech being the prime historical example. It was realized by Gabor and Ville that one can define time and frequency operators and make a mathematical analogy with the quantum case.

I have aimed to present the basic ideas and results of correspondence rules in a straightforward elementary manner and have included many examples to illustrate the ideas developed. I have strived to make the mathematics accessible to a wide audience and I have avoided delving into advanced formulations such as group theoretical considerations. The level of rigor and terminology are those of the original contributors and standard in mathematical physics and engineering.

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