

Preface

In this second edition we have made minor revisions and added new material to Chapter 8 and partly also to Chapters 2, 3 and 6. We quote from the Preface to the first edition in order to clarify the original motivation behind publication of this book:

“There are many books and monographs devoted to the methods of nonlinear analysis and their applications. Typically, such a book is either dedicated to a particular topic and treats details which are difficult to understand for a student, or it deals with an application to complicated nonlinear partial differential equations in which a lot of technicalities are involved. In both cases it is very difficult for a student to get oriented in this kind of material and to pick up the ideas underlying the main tools for treating the problems in question. The purpose of this book is to describe the basic methods of nonlinear analysis and to illustrate them on simple examples. Our aim is to motivate each method considered, to explain it in a general form but in the simplest possible abstract framework, and finally, to show its application (typically to boundary value problems for elementary ordinary or partial differential equations). To keep the text free of technical details and make it accessible also to beginners we did not formulate some key assertions and illustrative examples in the most general form.

In order to make the text self-contained, we decided to comment on several notions and statements in footnotes. To place the material from the footnotes in the text could disturb a more advanced reader and make the exposition more complicated. In order to emphasize the role of the statements in our exposition we identify them as Theorem, Proposition, Lemma and Corollary. However, the reader should be aware of the fact that this by no means expresses the importance of the statement within the whole of mathematics. So, several times, we call important theorems Propositions, Lemmas or Corollaries.

Although the book should primarily serve as a textbook for students on the graduate level, it can be a suitable source for scientists and engineers who have need of modern methods of nonlinear analysis.

At this point we would like to include a few words about our good friend, colleague and mentor Svatopluk Fučík to whom we dedicate this book. His work in the field of nonlinear analysis is well recognized and although he died in 1979 at the

age of 34, he ranks among the most important and gifted Czech mathematicians of the 20th century.

We would like to thank Marie Benediktová and Jiří Benedikt for an excellent typesetting of this book in L^AT_EX 2_ε, excellent figures and illustrations as well as for their valuable comments which improved the quality of the text. Our thanks belong also to Eva Fašangová, Gabriela Holubová, Eva Kaspříková and Petr Stehlík for their careful reading of the manuscript and useful comments which have decreased the number of our mistakes and made this text more readable. Our special thanks belong to Jiří Jarník for correction of our English, Ralph Chill and Herbert Leinfelder for their improvements of the text and methodological advice.”

The exposition of the material in the second edition is at two levels, visually differentiated by different font sizes. The basic material is contained in the body of the first seven chapters. The more advanced material is contained in appendices to a number of sections and is presented in a smaller font size. The last chapter contains selected special material from the theory of partial differential equations. The basic material is independent of the more advanced material, is self-contained, and can be read by students new to the subject. It should prepare an undergraduate student in mathematics to read scientific papers in nonlinear analysis and to understand applications of the methods presented to more complex problems. Each chapter contains a number of exercises which should provoke reader’s creativity and help develop his or her own style of approaching problems. However, the exercises play an additional role. They carry some of the technical material that was omitted in simplifying some of the basic proofs. They are thus an organic part of the exposition for graduate students who already have experience with the methods of nonlinear analysis and are interested in generalizations.

In the second edition we organize the material differently than in the first edition. We dedicate an extra chapter to monotonicity methods, where we newly include the elements of maximum principles both for ordinary differential equations and partial differential equations. Most of the material about boundary value problems for partial differential equations is now subsumed into the body of Chapters 5–7 as an immediate application of the methods presented there. The last Chapter 8 contains technically more involved material which was not included in the first edition. It deals with semigroups, semilinear evolution partial differential equations, application of fixed point methods to solve some quasilinear partial differential equations as well as some partial differential equations on manifolds. In particular, to prepare the reader for subsections on evolution partial differential equations we extend some parts of Chapters 2 and 3.

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