# Preface

This Lecture Notes volume is a fruit of two research-level summer schools jointly organized by the GTEM node at Lille University and the team of Galatasaray University (Istanbul). Both took place in Galatasaray University: "Geometry and Arithmetic of Moduli Spaces of Coverings" which was held between 09–20 June, 2008 and "Geometry and Arithmetic around Galois Theory" which was held between 08–19 June 2009. The second summer school was preceded by preparatory lectures that were delivered in TÜBİTAK Feza Gürsey Institute.

A group of seventy graduate students and young researchers from diverse countries attended the school.

The full schedules of talks for the two years appear on the next pages.

The schools were mainly funded by the FP6 Research and Training Network Galois Theory and Explicit Methods (GTEM) and the Scientific and Technological Research Council of Turkey (TÜBİTAK). Funding provided by the International Mathematical Union (IMU) and the International Center for Theoretical Physics (ICTP) have been used to support participants from some neighbouring countries of Turkey. We are also thankful to Galatasaray University and to University of Lille 1 for their support. Feza Gürsey Institute gave funding for the preparatory part of the summer school. The last named editor has been funded by TÜBİTAK grants 104T136 and 110T690 and a GSU Research Fund Grant during the summer school and the ensuing editorial process.

This volume focuses on geometric methods in Galois theory. The choice of the editors is to provide a complete and comprehensive account of modern points of view on Galois theory and related moduli problems, using stacks, gerbes and groupoids. It contains lecture notes on étale fundamental group and fundamental group scheme, and moduli stacks of curves and covers. Research articles complete the collection.

J. Bertin's paper, "Algebraic stacks with a view toward moduli stacks of covers", is an introduction to algebraic stacks, which focuses on Hurwitz schemes and their compactifications. It intends to make available to a large public the use of stacks gathering in a unified presentation most of the elements of the theory. Its goal is to study the moduli stacks of curves and of covers, which is the central theme of this collection of articles.

M. Romagny's article on "Models of curves" is a detailed account of the proof of Deligne-Mumford on semi-stable reduction of curves with an application to the study of Galois covers of algebraic curves. The author provides all the concepts and necessary ground making possible for the reader to understand the proof of the main theorem, supplying some complementary arguments, which are stated without proof in Deligne-Mumford's paper. The last part of the article is devoted to the problem of reduction of tamely ramified covers of smooth projective curves.

In her article on "Galois categories", A. Cadoret aims at giving an outline of the theory of the étale fundamental group that is accessible to graduate students. Her choice is to present the Grothendieck's theory of Galois categories in full generality, giving a detailed and self-contained proof of the main theorem not relying on Grothendiecks pro-representability result of covariant *lim*-compatible functors on artinian categories. The main example is that of the category of étale finite covers of a connected scheme, to which the rest of the article is devoted. All main theorems of the subject are proved in the paper, which contains also a complete description of the fundamental group of abelian varieties. Let us mention a very useful digest of descent theory given in appendix.

As a Galois category is equivalent to the category of continuous finite $\Pi$-sets for some profinite group $\Pi$, a Tannaka category is equivalent to the category of finite-dimensional representations of some affine pro-algebraic group. M. Emsalem's article on "Fundamental groupoid scheme" is an overview of the original construction by Nori of the fundamental group scheme as the Galois group of some Tannaka category $EF(X)$ (the category of essentially finite vector bundles) with a special stress on the correspondence between fiber functors and torsors. Basic definitions and duality theorem in Tannaka categories are stated, making the material accessible to non specialists. A paragraph is devoted to the characteristic 0 case and to a reformulation of Grothendieck's section conjecture in terms of fiber functors on $EF(X)$. Although this formulation is known from specialists, no complete reference was available.

Classically the structure theorem on the étale fundamental group of a curve is obtained by comparison with the topological fundamental group over $\mathbf{C}$.N. Borne's article on "Extension of Galois groups by solvable groups, and application to fundamental groups of curves" gives an account of the description of the pro-solvable $p'$-part of the étale fundamental group on an affine curve by purely algebraic means. The method inspired by Serre's work on Abhyankar's conjecture for the affine line relies on cohomological arguments, which are completely explained in the article, with a special stress on the Grothendieck-Ogg-Shafarevich formula.

The fundamental group scheme of a scheme $X$ is an inverse limit of torsors under finite group schemes. In the context of Galois theory of étale fundamental group, a finite étale morphism $Y \to X$ has a Galois closure. The question addressed by M. Garuti in his article on "Galois Closure for finite morphism" is to characterize, in the case of positive characteristic, which finite morphisms are dominated by a torsor under a finite group scheme, thus what finite morphisms benefit from a "Galois Closure" in the context of Nori's fundamental group scheme. The article, which gives a complete satisfactory answer, recalls all the necessary material to get to the main theorem.

Cohomology which was a main tool in Borne's paper, is the core of J.-C. Douai article "Hasse principle and cohomology of groups". But here occurs non abelian cohomology: precisely, $H^1$ and $H^2$ of semi-simple groups defined over $K = k(X)$, where $k$ is a pseudo-algebraically closed field and $X$ a proper smooth curve over $k$. The main result is the fact that the non-abelian $H^2$ of a semi-simple simply connected group whose center has an order prime to the characteristic of $k$ consists in neutral classes.

With the article "Periods of mixed Tate motives, examples, $\ell$-adic side" by Z. Wojtkowiak, it is the motivic side of the area that comes into play. One hopes that the $\mathbf{Q}$-algebra of periods of mixed Tate motives over Spec($\mathbf{Z}$) is generated by values of iterated integrals on $\mathbf{P}^1(\mathbf{C}) \setminus \{0, 1, \infty\}$ of sequences of one-forms $\frac{dz}{z}$ and $\frac{dz}{z-1}$ (some numbers also called multiple zeta values). Assuming the motivic formalism, some variant of this is proved, and is then further studied in the $\ell$-adic Galois setting. Numerous examples are given that provide some ground for future research in this direction.

The article "On totally ramified extensions of discrete valued fields" of L. Bary-Soroker and E. Paran is devoted to a more arithmetical aspect. In the context of Artin-Schreier field extensions, they revisit and simplify a criterion for a discrete valuation of a Galois extension $E/F$ of fields of characteristic $p > 0$ to totally ramify. Interesting examples illustrate this criterion.

R.P. Holzapfel and M. Penkava's paper "An Octahedral Galois-Reflection Tower of Picard Modular Congruence Subgroups" studies a subgroup $\Gamma(2)$ of the Picard modular group $\Gamma$. The quotient of the complex 2-ball under this group becomes the projective plane after compactification. $\Gamma(2)$ has an infinite chain of subgroups that leads to an infinite Galois-tower of ball-quotient surfaces, making it possible to work with algebraic equations for Shimura curves, which is of importance in coding theory.

This volume has benefited very much from the precious and anonymous work of the referees. We are very grateful to them.

Finally we wish to thank all the members of the scientific committees and of the organization committees for their collaboration in the organization of the two events: Kürsat Aker (Feza Gürsey Institute), José Bertin (Institut Fourier), Özgür Kişisel (METU), Pierre Lochak (Paris 6), Hurşit Önsiper (METU), Meral Tosun (Galatasaray University), Sinan Ünver (Koç University), Zdzisław Wojtkowiak (Nice) and Stephan Wewers (Hannover). And we would like to extend our thanks to Celal Cem Sarıoğlu, Ayberk Zeytin, Neşe Yaman who also contributed at various levels to the organization during the long preparation process before and during the summer school.

October 6, 2012 Istanbul, Lille and Paris

The Editors

## 2008 Summer School Schedule

### “Geometry and Arithmetic of Moduli Spaces of Coverings”

### Lectures

| Lecturer | Minicourse |
|---|---|
| Bertin, José | Introduction to stacks |
| Cadoret, Anna | Galois categories |
| Dèbes, Pierre | Foundations of modular towers, inverse Galois theory and abelian varieties |
| Emsalem, Michel | On the fundamental groupoid scheme |
| Fried, Michael | Modular towers |
| Garuti, Marco | $p$-adic representations of the fundamental group scheme |
| Korkmaz, Mustafa | Mapping class groups |
| Litcanu, Razvan | Intersection theory on algebraic stacks |
| Lochak, Pierre | Profinite complexes of curves and another geometric view of the GT group |
| Romagny, Matthieu | Models of curves |
| Schneps, Leila | Grothendieck-Teichmuller theory |
| Türkelli, Seyfi | Connected components of Hurwitz schemes and Malle’s conjecture |
| Tossici, Dajano | Weak and strong extension of torsors |
| Ünver, Sinan | Multi-zeta values and the Grothendieck-Teichmuller group |
| Wewers, Stefan | Algebraic patching and covers of curves |

## 2009 Summer School Schedule

### “Geometry and Arithmetic around Galois Theory”

### Lectures

| Lecturer | Minicourse |
|---|---|
| Aker, Kürşat | Hurwitz Schemes (at FGI) |
| Borne, Niels | Extensions of Galois groups by solvable groups, and application to fundamental groups of curves |
| Cadoret, Anna | Descent theory for covers |
| Çakçak, Emrah | An Introduction to Algebraic Fundamental Groups (at FGI) |
| Dèbes, Pierre | Geometric Galois Theory: an Introduction (at FGI) |
| Dettweiler, Michael | Middle convolution and the Inverse Galois Problem |
| Feyzioğlu, Ahmet | Infinite Galois Theory (at FGI) |

| | |
|---|---|
| Fehm, Arno | Ample Fields IV |
| Geyer, Wulf-Dieter | Ample Fields III |
| Haran, Dan | Ample Fields II |
| İkeda, İlhan | Higher-dimensional Langlands correspondence |
| Jarden, Moshe | Ample Fields I |
| Özden, Şafak | Fields of Norms (at FGI) |
| Ramero, Lorenzo | Lectures on logarithmic algebraic geometry |
| Türkelli, Sefyi | Malle's conjecture and number of points on a Hurwitz space |
| Wojtkowiak, Zdzisław | Galois actions on fundamental groups and on torsors of paths |

## Research Talks

| **Speaker** | **Talk Title** |
|---|---|
| Antei, Marco | On the fundamental group scheme of a family of curves |
| Bary-Soroker, Lior | Frobenius automorphism and irreducible specializations |
| Cadoret, Anna | A uniform open image theorem for $\ell$-adic representations |
| Cau, Orlando | Irreducible components of Hurwitz spaces |
| Collas, Benjamin | Action on torsion-elements of mapping class groups by cohomological methods |
| Dettweiler, Michael | On the automorphy of hypergeometric local systems |
| Douai, Jean-Claude | Principe de Hasse et cohomologie des groupes |
| Hatami, Omid | A short talk on Class field theory |
| Holzapfel, Rolf-Peter | Galois reflection towers |
| Kim, Minyong | Diophantine geometry and fundamental groups |
| Mendes, Sergio | Class field theory and the principal series of SL(2) |
| Neftin, Danny | On arithmetic field equivalences and crossed product division |
| Pal, Ambrus | The real section conjecture and Smith's fixed point theorem |
| Paran, Elad | Power series over generalized Krull domains |
| Petersen, Sebastian<br>Poineau, Jérôme | Inverse Galois problem for convergent arithmetic power series |
| Schmidt, Johannes | Rigid $G_2$ Representations and Motives of Type $G_2$ |
| Türkelli, Seyfi | Homological stability of Hurwitz schemes |
| Yafaev, Andrei | Andre-Oort and Manin-Mumford conjectures: a unified approach |

Springer

http://www.springer.com/978-3-0348-0486-8

Arithmetic and Geometry Around Galois Theory
Dèbes, P.; Emsalem, M.; Romagny, M.; Uludağ, A.M.
(Eds.)
2013, XII, 404 p., Hardcover
ISBN: 978-3-0348-0486-8
A product of Birkhäuser Basel