

Contents

Preface	xi
Preface to the 2nd edition	xv
Notational conventions	xvii
1 Preliminary general material	1
1.1 Functional analysis	1
1.1.1 Normed spaces, Banach spaces, locally convex spaces	1
1.1.2 Functions and mappings on Banach spaces, dual spaces . .	3
1.1.3 Convex sets	6
1.1.4 Compactness	7
1.1.5 Fixed-point theorems	8
1.2 Function spaces	8
1.2.1 Continuous and smooth functions	9
1.2.2 Lebesgue integrable functions	10
1.2.3 Sobolev spaces	14
1.3 Nemytskiĭ mappings	19
1.4 Green formula and some inequalities	20
1.5 Bochner spaces	22
1.6 Some ordinary differential equations	25
I STEADY-STATE PROBLEMS	29
2 Pseudomonotone or weakly continuous mappings	31
2.1 Abstract theory, basic definitions, Galerkin method	31
2.2 Some facts about pseudomonotone mappings	35
2.3 Equations with monotone mappings	37
2.4 Quasilinear elliptic equations	42
2.4.1 Boundary-value problems for 2nd-order equations	43
2.4.2 Weak formulation	44
2.4.3 Pseudomonotonicity, coercivity, existence of solutions . . .	48

2.4.4	Higher-order equations	56
2.5	Weakly continuous mappings, semilinear equations	61
2.6	Examples and exercises	64
2.6.1	General tools	64
2.6.2	Semilinear heat equation of type $-\operatorname{div}(\mathbb{A}(x, u)\nabla u) = g$. . .	68
2.6.3	Quasilinear equations of type $-\operatorname{div}(\nabla u ^{p-2}\nabla u) + c(u, \nabla u) = g$	75
2.7	Excursion to regularity for semilinear equations	85
2.8	Bibliographical remarks	92
3	Accretive mappings	95
3.1	Abstract theory	95
3.2	Applications to boundary-value problems	99
3.2.1	Duality mappings in Lebesgue and Sobolev spaces	99
3.2.2	Accretivity of monotone quasilinear mappings	101
3.2.3	Accretivity of heat equation	105
3.2.4	Accretivity of some other boundary-value problems	108
3.2.5	Excursion to equations with measures in right-hand sides . .	109
3.3	Exercises	112
3.4	Bibliographical remarks	114
4	Potential problems: smooth case	115
4.1	Abstract theory	115
4.2	Application to boundary-value problems	120
4.3	Examples and exercises	126
4.4	Bibliographical remarks	130
5	Nonsmooth problems; variational inequalities	133
5.1	Abstract inclusions with a potential	133
5.2	Application to elliptic variational inequalities	137
5.3	Some abstract non-potential inclusions	145
5.4	Excursion to quasivariational inequalities	154
5.5	Exercises	157
5.6	Some applications to free-boundary problems	163
5.6.1	Porous media flow: a potential variational inequality	163
5.6.2	Continuous casting: a non-potential variational inequality .	166
5.7	Bibliographical remarks	169
6	Systems of equations: particular examples	171
6.1	Minimization-type variational method: polyconvex functionals . . .	171
6.2	Buoyancy-driven viscous flow	178
6.3	Reaction-diffusion system	186
6.4	Thermistor	188
6.5	Semiconductors	192

II	EVOLUTION PROBLEMS	199
7	Special auxiliary tools	201
7.1	Sobolev-Bochner space $W^{1,p,q}(I; V_1, V_2)$	201
7.2	Gelfand triple, embedding $W^{1,p,p'}(I; V, V^*) \subset C(I; H)$	204
7.3	Aubin-Lions lemma	207
8	Evolution by pseudomonotone or weakly continuous mappings	213
8.1	Abstract initial-value problems	213
8.2	Rothe method	215
8.3	Further estimates	230
8.4	Galerkin method	240
8.5	Uniqueness and continuous dependence on data	247
8.6	Application to quasilinear parabolic equations	251
8.7	Application to semilinear parabolic equations	261
8.8	Examples and exercises	264
8.8.1	General tools	264
8.8.2	Parabolic equation of type $\frac{\partial}{\partial t}u - \operatorname{div}(\nabla u ^{p-2}\nabla u) + c(u) = g$	266
8.8.3	Semilinear heat equation $c(u)\frac{\partial}{\partial t}u - \operatorname{div}(\kappa(u)\nabla u) = g$	277
8.8.4	Navier-Stokes equation $\frac{\partial}{\partial t}u + (u \cdot \nabla)u - \Delta u + \nabla \pi = g, \operatorname{div} u = 0$	279
8.8.5	Some more exercises	282
8.9	Global monotonicity approach, periodic problems	288
8.10	Problems with a convex potential: direct method	294
8.11	Bibliographical remarks	300
9	Evolution governed by accretive mappings	303
9.1	Strong solutions	303
9.2	Integral solutions	308
9.3	Excursion to nonlinear semigroups	314
9.4	Applications to initial-boundary-value problems	319
9.5	Applications to some systems	326
9.6	Bibliographical remarks	332
10	Evolution governed by certain set-valued mappings	335
10.1	Abstract problems: strong solutions	335
10.2	Abstract problems: weak solutions	339
10.3	Examples of unilateral parabolic problems	343
10.4	Bibliographical remarks	349
11	Doubly-nonlinear problems	351
11.1	Inclusions of the type $\partial\Psi(\frac{d}{dt}u) + \partial\Phi(u) \ni f$	351
11.1.1	Potential Ψ valued in $\mathbb{R} \cup \{+\infty\}$	351
11.1.2	Potential Φ valued in $\mathbb{R} \cup \{+\infty\}$	358
11.1.3	Uniqueness and continuous dependence on data	366

11.2	Inclusions of the type $\frac{d}{dt}E(u) + \partial\Phi(u) \ni f$	367
11.2.1	The case $E := \partial\Psi$	368
11.2.2	The case E non-potential	372
11.2.3	Uniqueness	375
11.3	2nd-order equations	377
11.4	Exercises	385
11.5	Bibliographical remarks	390
12	Systems of equations: particular examples	393
12.1	Thermo-visco-elasticity	393
12.2	Buoyancy-driven viscous flow	405
12.3	Predator-prey system	408
12.4	Semiconductors	412
12.5	Phase-field model	416
12.6	Navier-Stokes-Nernst-Planck-Poisson-type system	420
12.7	Thermistor with eddy currents	426
12.8	Thermodynamics of magnetic materials	432
12.9	Thermo-visco-elasticity: fully nonlinear theory	438
	References	449
	Index	469



<http://www.springer.com/978-3-0348-0512-4>

Nonlinear Partial Differential Equations with
Applications

Roubíček, T.

2013, XX, 476 p., Hardcover

ISBN: 978-3-0348-0512-4

A product of Birkhäuser Basel