

Preface

A typical problem in offbeat integral geometry is as follows. Let \mathbb{R}^n be the n -dimensional real Euclidean space, $\mathbf{M}(n)$ the group of Euclidean motions of \mathbb{R}^n , and A a bounded subset of \mathbb{R}^n of positive Lebesgue measure. Consider the following problem: describe the class of locally integrable functions f such that

$$\int_{gA} f(x)dx = 0 \quad (*)$$

for each $g \in \mathbf{M}(n)$. This problem has various generalizations and modifications. For instance, in place of $(*)$ one can investigate solutions of a system of convolution equations with fixed distributions.

The first studies in this area were carried out in 1929, by the Rumanian mathematician D. Pompeiu, who investigated the question on the existence of non-trivial functions satisfying $(*)$ for some A . D. Pompeiu erroneously assumed that if A is a ball then equation $(*)$ has only the trivial solution. Later on, F. John showed that a function $f \in C^\infty$ with zero integrals over all balls of fixed radius r is uniquely defined by its values in the ball of radius r .

After that, F. John, J. Delsarte, L. Hörmander, L. Zalcman, C.A. Berenstein, and other authors discovered deep connections between these questions and many areas of contemporary mathematics and its applications.

In recent years, local versions of the above problem have become a point of attention, in which a function f is defined in a bounded domain \mathcal{O} and equality $(*)$ holds for $g \in \mathbf{M}(n) : gA \subset \mathcal{O}$. The transition from the global to the local case makes the problem considerably more complicated, which is related to the breakdown of the structure of a group action on the solution set of equation $(*)$. Among first results in this direction we point out Hörmander's approximation theorem for solutions of a convolution equation on convex domains and the local two-radii theorem by C.A. Berenstein and R. Gay. Until recently research in this area was carried out mostly using the technique of the Fourier transform and corresponding methods of complex analysis. A remarkable result by the first author at the end of the last century was the development of a universal method for the complete solution of many problems of this kind, which allowed one, in particular, to remove virtually all superfluous assumptions imposed by his predecessors. This method is based on the representation of solutions of a broad class of convolution equations by series in special functions. The results obtained by this method

were summarized in the monograph [V.V. Volchkov, *Integral Geometry and Convolution Equations*, Kluwer Academic Publisher, 2003] in which the author puts forward more than 50 new problems designed for further developments in this area. In particular, it seems a promising program to develop these techniques for various classes of homogeneous spaces with invariant measure. This task is now close to completion in the case of two-point homogeneous spaces. The present book describes recent advances in this direction of research. A key role here is played by an analytic tool developed in the recent monograph [V.V. Volchkov and Vit.V. Volchkov, *Harmonic Analysis of Mean Periodic Functions on Symmetric Spaces and the Heisenberg Group*. Springer-Verlag London Limited, 2009].

How is the book organized? Subdivision is into Parts, Chapters and Sections. Formulas are numbered consecutively within each chapter (ignoring subdivision of chapters into sections). In references to formulas within the same part the number of the part is not indicated. When a formula from another part is referred to the number of the part is added on the left. Analogous numbering is used for theorems, propositions, lemmas and so on. For example, Theorem I.4.3 is the third theorem of the fourth chapter of Part I, and Chapter I.2 is the second chapter of the first part.

Each chapter begins with a summary and ends with bibliographical notes. In these notes and in the text our books [2003] and [2009] are abbreviated to **[IG]** and **[HA]**. All the chapters contain some exercises and further results with explicit references. The reader will find also many open problems and an invitation to work in this exiting field. It is hoped that the diversity of the problems tackled in this book illustrates the possibilities offered by the theory considered, so that it might be a source of inspiration to mathematicians. In order to make this book as self-contained as possible we have gathered all prerequisites needed in the first part. The subject matter and notation of the first part are used throughout the book.

Some of the material in this book has been the subject of lectures delivered by the authors over a number of years. We have received helpful comments and suggestions from many colleagues; of these we mention R. Trigub, O. Ochakovskaya, V. Zastavnyi, D. Zaraisky, A. Grishin, V. Burskii and V. Ryazanov. We thank them all.

The first author owes very much to L. Zalcman who invited him to come and work at his Seminar in 1993, 1996, 2001, 2004, and the department of Mathematics and Computer Science of Bar-Ilan University (Israel) for its hospitality and library facilities during the stay. Thanks are also due to participants of Zalcman's Seminar for useful discussions related to the topics of the book.

We are very indebted to the National Fund for Scientific Research for supporting our work. It is a pleasure to thank P. Masharov and O. Ochakovskaya for their expert and conscientious \TeX setting of the manuscript. We are very grateful to our home institution, the Donetsk National University, for working conditions we enjoy.

Donetsk,
June 2012

Valery Volchkov
Vitaly Volchkov



<http://www.springer.com/978-3-0348-0571-1>

Offbeat Integral Geometry on Symmetric Spaces

Volchkov, V.V.; Volchkov, V.V.

2013, X, 592 p., Hardcover

ISBN: 978-3-0348-0571-1

A product of Birkhäuser Basel