The Geometry of Diagrams and the Logic of Syllogisms

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Abstract Aristotle accounts for three figures on which syllogisms are formed. On the first figure it is possible to prove the completeness of all of the possible syllogisms. But on the second and on the third figures completion is not possible; therefore, premises based on the second or on the third figure are converted in such a way as to count as premises on the first figure. Aristotle's procedure leads to difficulties discussed and corrected in the course of this paper.

Keywords Short and long lines · Figure · Intervals · Terms · Syllogisms · Proof · Reduction

Mathematics Subject Classification Primary 03A10; Secondary 03B10

The aim of this paper is to argue the dependence of a demonstrative syllogism upon its figure. What a syllogism is can be briefly explained: a syllogism is formed from three linguistic parts: a major premise which affirms or denies a sentence composed of a subject and a predicate and a mark of affirmation or negation along lines suggested by the commentator John Philoponus; see his commentary on Aristotle's *Prior Analytics* [3, 66.27–67.14].

Aristotle's text differs from Alexander's and Philoponus's commentary in this way: having laid out the structure of a figure and its intervals Aristotle remarks on a basis which either is or is not adequate for the formation of a syllogism. But the commentators turn immediately to writing out a sketch of a combination of premises which either is or is not adequate for a conclusion. The commentators accordingly leave out the work which should be done by means of diagrams. Guenter Patzig [2], on the other hand, expresses high respect for the commentary of John Philoponus. W.D. Ross [4], in his commentary on the *Prior Analytics*, promotes the idea of diagrams drawn by Aristotle but not preserved.

Since it is argued that a demonstrative syllogism depends upon its figure, let us turn first to the shape of the figure by defining the three kinds of diagrams which guide the forming of the syllogisms as depending upon the figures presented by the diagrams. I call them Sophie diagrams, distinguishing them from Venn diagrams; the distinction between the two kinds will be made clear.

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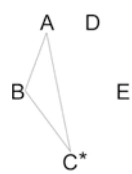
1 Affirmation, Negation, and the Resolution of Negation

I shall first try to make clear how a diagram is to be drawn. It is formed by marking points and by drawing lines. For example, a diagram of the first figure is drawn as follows: first, mark a point and let the point indicate a term; Aristotle assigns to such a term—for example—the letter A. We can then draw a line from the point A to the point B. The line indicates an interval terminated by two terms or limits. Every syllogism showing either plain belonging or belonging with necessity has two intervals indicated by the premises of the syllogism or, if the syllogism proves contingency, there are three intervals.

I have so far assumed that the premises which depend upon terms and intervals are affirmative. But affirmation itself is not registered on the diagram. For a diagram is formed from points and lines; premises of a projected syllogism, of course, are said to be either affirmative or negative. Now since affirmation and negation are acts of intelligence but are not either points or lines marked or drawn on a diagram which itself exists not as a spatial but rather as a temporal particular.

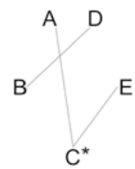
It is of course possible to resolve the negation embedded in a premise: denial is resolved not on a diagram but rather within the mind of the reasoner—resolved in favor either of correcting for deficiency or in favor of correcting for excess. Correction for deficiency is shown by drawing another line on the diagram (we will shortly have an example of this); correction of excess, on the other hand, is shown by adding another point opposite to the point A already registered (assuming that the major premise is negative). The force of correcting for excess is this: there are alternatives and so a choice is made, thinking "Let D belong to the whole of the limit B." But the choice is not joining terms but rather drawing a line from a point to a point. The faculty of reason will, for example, draw a line from the point D to the point B. This drawing helps complete a diagram.



The diagram above is drawn on the first figure which shows the concluding interval A-C composed of letters which stand for terms or limits one of which belongs to the upper end of an interval running to a middle limit; the interval is indicated on the diagram by a straight line running from an upper point, i.e. 'A', to a lower point, i.e. 'C'; the second interval runs from the middle point, i.e. 'B' to the extreme bottom point, i.e. 'C'. The concluding interval A-C is marked with an asterisk indicating that its long interval completes this instance of the first figure. The drawing of the long line is not meant to suggest that the long interval does not follow the two short intervals A-B and B-C.

In the course of the ancient tradition of teaching the syllogistic the commentator Alexander of Aphrodisias [1] tends to conflate an interval and a premise, even though Aristotle himself makes it clear that they are distinct and separate. Had Aristotle's diagrams survived, doubtless the tendency to merge a syllogism and its figure would not have developed.

Aristotle's text differs from Alexander's and Philoponus's commentary in this way: having laid out the structure of a figure and its intervals Aristotle remarks on a basis which either is or is not adequate for the formation of a syllogism. But the commentators turn immediately to writing out a sketch of a combination of premises which either is or is not adequate for a conclusion. The commentators accordingly leave out the work which should be done by means of diagrams.



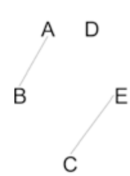
Aristotle does not himself make a distinction between the conclusion of a syllogism and the long interval drawn from the letter ‘A’ to the letter ‘C’. This interval is necessary for completing a second figure.

It is characteristic of every complete figure that there is a middle term whereby the long interval from the top to the bottom of the diagram can be shown. Showing the long interval on the first figure is the objective of a proof on the first figure. It is of course Aristotle’s view that the other two figures do not admit of a proof of their long intervals. For the long intervals are laid down by edict regarding the second two figures—i.e. on the second figure “Let the term A belong to the whole of the extreme limit C and let the term A belong to the whole of the middle term B.” (Discussion of the second two figures will add a crucial qualification to the initial description of the second two figures.)

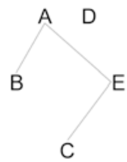
It was Aristotle’s view (as comes clear in early sections of the *Posterior Analytics* [4]) that it is not possible to prove an immediate interval, i.e. “The upper limit A belongs to the whole of the middle limit B.” In fleeing the difficulty of proving the existence of a short interval he confronts another: How is it possible to reshape a diagram in such a way that a long interval uniting two terms becomes a short interval joining two terms?

There is an important respect in which a syllogism differs from a diagram: a diagram is drawn and is not an immediate object of affirmation or negation. It is not a part of a diagram to have a part indicating negation. Negation, after all, is an act and not an object: affirming and denying, on the other hand, are contrary actions and not contrasting objects. For this reason no mark on a diagram indicates either affirming or denying; accordingly, an interval drawn between two points is neither an affirmation nor a negation. Therefore, in drawing a complete diagram it is necessary to resolve the negation embedded in the premises intended for forming parts of a syllogism.

Suppose we have the two premises on the first figure: “the limit A belongs to the whole of the limit B” and “the limit B does not belong to any part of the limit C.” Since a negative premise does not correspond to an interval running between two limits, it is useful to resolve the negation. The principle of the resolution of negation anticipates one of two possible ways in which negation can be resolved: in a first way we correct for the wrong term by drawing a contrary term on the diagram. Suppose we draw a line from the upper letter ‘A’ to the middle letter ‘B’; in resolving the negation of the second premise “the limit B belongs to none of the extreme bottom limit C” we should draw the following diagram on the first figure:



It is apparent that the diagram does not show a continuous long line from A to C. The diagram is therefore not suitable for showing a long interval necessary for a figure and sufficient for the formation of a syllogism. The point is not being made that we turn to showing by counterexamples that there is no conclusion; the point is rather that it is possible to correct the diagram for deficiency by adding an interval between A and E; for it is entirely possible that two species fall under the same genus A. The diagram can now be expanded, yielding:



There is accordingly a long interval A-E-C whereby the middle term E mediates the long interval. The syllogism formed upon the figure need not write a premise which describes the A-B interval.

The middle figure is particularly interesting in that it exemplifies the binary theory of the syllogistic regarding its longitudinal structure; the figure is of course also triadic with respect to a middle term whereby a long interval is mediated. A consideration of the middle figure also provides an opportunity to discuss a difficulty which Aristotle must face in his own dealing with the second figure—a difficulty to which I shall shortly turn.

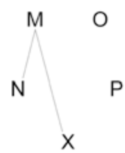
At the center of my study of diagrams and in what ways they provide plans for the formation of syllogisms is its theory of the resolution of negation, briefly discussed above. This theory makes it possible to give a coherent account of diagrams, on the one hand, and, on the other, to show how Aristotle's need of a reduction to the first figure can be eliminated; in place of such a reduction is a procedure for proving three classes of syllogisms: one class depends upon the first, another on the second, and the third class is established on the third figure. The demonstration of the equality of the three figures in relevant ways also provides a rationale for holding that there are only three distinct and separate figures.

The incoherence in Aristotle's execution of his syllogistic program is due to several demands which cannot be coherently satisfied together. To make the conflict clear let us begin with the view of proof which the philosopher evidently endorses: all proof is executed by means of the posit of a middle term. Let us accordingly imagine three terms so arranged that the upper extreme term A belongs to the whole of the middle term B and that the middle term belongs to the whole of the lower extreme term C. If the interval between the limits A and B and the interval between the limits B and C are immediate, Aristotle supposes that the existence of such immediate intervals (each limited by a pair of terms) cannot be proved. The long interval between A and C can be proved by means of a middle term which unites the two intervals A-B and B-C. For this reason Aristotle holds that syllogisms proved on the first figure are complete; but the premises based upon the second and the third figures cannot be the source of a complete syllogism a pair of premises of which has been formed upon the basis of either the second or upon the basis of the third figure. Like wood, prepared for becoming the parts of a chair formed by one mill, but which must be taken to a different mill for becoming a real and complete chair, the premises which originate upon the second or upon the third figure must be transformed into sentences which reflect the first figure thereby becoming the premises of a syllogism which itself depends immediately upon the first figure.

But what do you suppose happens to the sentence underway from the second back to the first figure? When it was first formed on the second figure, let us say, it had a job, namely to describe either a short or a long interval between two terms: perhaps it described a short interval from the limit A running to the limit B; perhaps, on the other hand, it described a long interval from A to C. And yet it is just this responsibility—of bearing a specified use sufficient for representation—that is dropped. For by means of its being converted it falsifies its own prior statement of the interval which was to be represented. The shame of the loss and the deception of Aristotle's pretensions are ameliorated by supposing that there never was a figure or a diagram presenting terms, their intervals, and their modalities—never was a figure, I was about to write, the representation of which was entrusted into the hands of the departing premise or sentence.

The job of a figure or a diagram is to present the terms and their intervals as they are—as we would expect from a floor-plan or from a map of a neighborhood. (A premise, on the other hand, is not a drawing of terms and intervals.) The stability of a diagram is particularly important for its presentation of the modalities of necessity and contingency; for if A is necessary for B, it does not follow that B is necessary for A. If C depends upon B, it does not follow that B depends upon C.

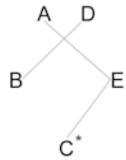
Consider the following two diagrams, defining the second figure ([4], I.5; 26b34–38), and reduction to the first figure ([4], I.5; 26b36–39):



Suppose that a diagram of the second figure shows both a long line connecting the letters 'M' and 'X', as the diagram just above shows, and also shows a short line connecting 'M' and 'N'. But the information provided by the diagram is destroyed by converting the long line 'M-X' into a short line 'M-X' shown above on the second diagram. In trying to show that there is a short, concluding line from 'M' to 'X', conversion turns the initial long line 'M-X', into a short line 'M-X'; this move destroys the long line which was meant to represent a long interval between M and X. So Aristotle's solution to the problem of forming syllogisms on the second and the third figures is in effect to undercut the point of the second and of the third figures and to undercut the uses of sentences which originally have uses whereby a grid of terms and relations can be indicated. We should accordingly expect that Aristotle's mishandling of the second and the third figures would have the effect of making the syllogistic a largely linguistic matter which would be interpreted pretty much as Alexander and John Philoponus did interpret it. Further, such an interpretation blurs a picture and a diagram of the general structure of the syllogistic: first comes the formation of demonstrative syllogisms ([4], I.4ff) upon the basis of a figure diagrammed to show a grid of terms and relations and then, having completed the project of the forming of syllogisms, Aristotle turns ([4], I.32ff) to the dissolution of a syllogism back down to its proper figure.

But let us return to my story of the black-market which has flourished in the obscurity of what happens when sentences formed as premises on either the second or the third figure are transported to the first figure now pregnant with a syllogism, illegitimate on the first figure but no longer at home on the second or the third figures.

Here is an example of the second figure and its diagram:



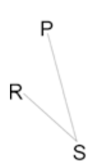
The two lines D-B and A-E-C indicate two intervals one long and one short. (This instance of the binary theory is the result of the resolution of the negation embedded in what Alexander would call a major, universal, and privative premise.) It is obvious that the short and concluding interval does not run between the terms B and C; for in that case the bottom extreme would fall under contrary genera—which is impossible. Therefore, there is a short and concluding interval between the terms E and C forming the concluding interval necessary for completing both the figure and the syllogism formed upon the basis of it.

The long line A-C represents the long interval A-C which in truth is mediated by the term E. The long interval being mediated by means of the middle limit E is not necessarily shown on the diagram drawn just above; nor is the fact mentioned in the syllogism formed on the basis of the second figure. If we were to draw the long interval first from A to E and then from E to C, we would have a different version of the proof of mediation (namely in the style of the second figure and not in that of the first figure). The question arises what difference there is between the proof which depends upon a proper figure and the proof which depends upon contraposition and return to the first figure for the basis of the syllogism. A syllogism on the second figure can be formed as follows:

1. All B are D (or, restoring negation): "No B is A"
2. All C are A
3. So, no C are B

It is evident upon the basis of the diagram that if some C were B, some part of B would fall under a contrary upper term, which is impossible. It is therefore not necessary either to convert or to contraposit the names of the terms in order to form the conclusion.

But let us consider Aristotle's text further—a text which makes it clear in what way the philosopher plays unfairly with the formation of the third figure and therefore with the syllogism to be formed upon its basis. We again require two diagrams: the first is the initial sketch of the third figure; the second shows the result of conversion in order to forge a copy of the first figure. According to Aristotle's suggestions for drawing the two figures, here again is one possibility:



Defining the third figure ([4], I.6; 28a10–22)

Reduction to the first figure ([4], I.6; 28a10–17)

In defining the third figure Aristotle identifies the possibilities whereby the same limit S is the common term to which both the upper term and the middle term belong, as indicated on the third figure. Aristotle remarks at the beginning of I.6 that he calls middle that of which both the limits predicated are predicated. The extreme and major limit P is further from the middle limit S; the limit closer to P is minor. The middle limit is outside the extreme limits and is last by position. Therefore, a complete syllogism does not come to be nor a complete syllogism which depends upon this figure. But a syllogism will be possible both when the terms in relation to the middle limit are whole and not whole.

In order to correct for the deficiency of the position of the limit S we convert the minor interval and thereby make our return to the first figure holding now that P belongs to the whole of S and that S belongs to some R, as the second diagram just above shows. Accordingly, it is proved on the first figure that the limit P belongs to part of the lower limit R.

When a figure has been identified and an interval drawn on a diagram according to which the limit B does not belong to any part of the lower limit C, an argument can be made similar to that argument laid out above concerning the second figure. The denial refers to the middle limit B, to be sure, but does not indicate that middle term by means of which a long interval can be identified by means of which the long interval is mediated. We are therefore ignorant of how the force of the upper term is delivered to the lower term.

The two mistakes attributed to Aristotle's handling of the second figure have counterparts in his handling of the third figure: rather than converting the upper interval he now converts the lower interval R-S forming the short interval S-R. The two unlawful conversions somewhat differ: the first disturbs the interval from the top down; the second disturbs the interval from the middle down.

What is disturbing in particular is the distinct roles of the two forms of modality: the upper term may be necessary for the middle term but the upper term does not determine the middle; the middle is rather determined by the bottom term. So Aristotle's solution to the problem of forming syllogisms on the second and the third figures is in effect to undercut the point of the second and of the third figures and to undercut the uses of sentences which originally have uses whereby a grid of terms and relations can be indicated.

A solution of the difficulty is possible in two steps: the first is an adequate diagram whereby the binary theory which represents two separate terms at the top of the diagram is given adequate representation; the binary theory also requires a noting of two middle terms; the lower level requires only one indication of a term usually taken by the commentators to be a class of particulars (something which Aristotle himself suggests in the *Prior Analytics* [4, I.27]).

Let us turn now to the logic of syllogisms which employ the modalities of necessity and contingency. The syllogisms which posit necessity have aroused less puzzlement than those which posit contingency. But even so, a certain disagreement broke out presumably during Aristotle's life concerning the logic of necessary syllogisms and, in particular, the conclusion which follows from premises one of which indicates pure belonging and the other necessity.

Diagrams can help eliminate irrelevant confusion. Consider the following example.

As the diagram stands, we can interpret the figure which it presents in the following way: the upper limit A is necessary for the whole of the limit B; the limit B, in turn, simply belongs to the whole of the extreme limit C. The part of the diagram drawn as ‘n-’ indicates not the necessity of the term A for the whole of the extreme lower term C; it rather indicates the modality of the concluding and long interval A-C, namely as necessary for the completion of the figure. By contrast the mark ‘n’ attached to the short line A-B (which presents the short upper interval A-B) indicates the modality of the upper term A for the whole of the middle term B. It follows that the diagram does not present the upper term as being necessary for the extreme term C. But commentators who assume that the middle term B is contained within the upper term A and that the lower term C is contained within the middle term B, are bound to confuse the two uses of the modalities.

A point made about the diagramming of the second figure can be made again: the long line indicates the long interval A-C but without showing the course of the long interval and thereby making clear what the mediating term is (nor would this information be shown by a syllogism which depends upon the figure); we are also not told how the middle term is able to receive an interval from a necessary upper term and mediate the long interval in such a way that the interval B-C is of plain belonging. Nevertheless, the interval is necessary for the coherence of the figure and for the syllogism formed upon the figure.

A syllogism formed on the third figure can be written as follows:

1. All C are B.
2. All C are A.
3. So, all B are A.

It is again evident upon the basis of the diagram that if some B are not A, then some part of C would be both A and not A. It is equally obvious that the long interval is mediated by the middle limit B. This feature of the figure is not made evident by a syllogism based upon it.

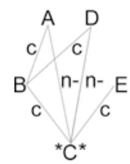
In Sect. I.14 of the *Prior Analytics* [4] Aristotle continues his study of modal syllogisms and their figures by taking up contingency. Just as some terms are necessary for others, so some terms are contingent for others. Unlike the posit of necessity the posit of contingency requires a correlated posit of two terms from which two intervals connect with a single term for which there is contingency. For example, both sitting and walking are contingent for humans and waking and sleeping are contingent for animals. A diagram which presents such terms and the modality of contingency requires two upper terms and a single lower term upon which the two intervals converge. It is not possible to draw a diagram of this sort of modality without a coordinated pair of intervals converging upon a lower term.

It was a mistake of Aristotle’s to try to expand the syllogistic into both the modality of necessity and the modality of contingency while maintaining a single column of terms. It was in fact a strange mistake for the philosopher to make since he was aware of and worked within a binary theory in several respects. Aristotle follows a modal path which is particularly rocky when some intervals are necessary, some contingent and others mixed both with the modalities and also with plain belonging. Aristotle has succeeded in puzzling his commentators; he leaves some modern commentators both perplexed and dismayed. Application of a binary theory is the least of what help lies at hand along with the resolution of negation. It is then possible to sketch diagrams which make consistent sense of the philosopher's project.

Let's begin with contingency defined in such a way as to accommodate the following diagrams. The text in question begins with Sect. I.14 of the first part of the *Prior Analytics* [4]. Aristotle writes:

> So when the limit A is contingent for the whole of the limit B and the limit B for the whole of the limit C, there will be a complete syllogism [showing] that the limit A is contingent for belonging to the whole of the limit C. This is plain from the definition; for we spoke in this way of the fact that being contingent for belonging to the whole is so. [4, I.14; 32b38–33a1]

The following diagram presents the figure described above:



Four premises need to be written in order to form a syllogism which depends upon the whole of the figure represented by the diagram above. The first premise represents the short contingent interval A-B; the counterpart premise represents the second short contingent interval D-B; the lower premise represents the short contingent interval B-C, and the second lower premise represents the short contingent interval E-C. There are two concluding long intervals both necessary for the completion both of the figure and also for the completion of the syllogism formed upon the basis of the figure. The course of the first long interval is evident, namely mediated by the middle term B. But how is the second long interval mediated? The binary theory applied to this instance of a figure suggests mediation by the middle term E.

Before turning to counterexamples employed by Aristotle to prove that there are no further syllogisms in addition to those already proved there is a question as to whether a distinction between the ontic and the modal should be part of the logic of syllogisms. If we begin with actuality—as Aristotle seems to do in laying down paradigms of figures—we may assume that Aristotle does not suppose that a posit of actuality implies the presence of necessity; on the other hand, he does suppose that a posit of necessity implies a posit of actuality; he makes an analogous assumption regarding contingency and actuality. And yet it is clear that a posit of actuality does not cohere with a posit of contingency. It is possible to hold that both graying and keeping the color of one's hair are contingent for belonging to Fred. The simultaneous actuality of both contingencies is not possible. Therefore, it is possible for there to be the contingency of A for B and also possible for there to be the contingency of D for B.

A parallel question arises regarding necessity, namely whether it is possible to hold that the limit A is necessary for B even though it is not implied that A actually belongs to B. Plausible examples support the suggestion. My neighbor's new house depends upon a foundation necessary for it even though a house has not yet been built.

There remain two matters to be discussed in bringing this paper to a conclusion. The first is about counterexamples and the second and last is about reduction to the impossible.

Regarding the first John Philoponus complains that Aristotle sometimes doesn't do a good job of laying out terms and their relations in showing that a syllogism is not possible. The question appropriate to this paper is whether Aristotle would have used a diagram in giving his argument or would rather have formed premises which are to make it clear that no syllogism can be formed by means of such premises. The first examples appear in I.4 of [4] after Aristotle has shown how terms and intervals in the first figure make syllogisms possible; at I.4; 26a2ff he lays out terms and intervals to show when there is not a figure sufficient for the formation of a syllogism. He writes:

> But if the first interval follows upon the middle term but if the middle limit belongs to none of the extreme limit, there will not be a syllogism of the extreme terms; for nothing necessary results by virtue of these limits and intervals; for it is possible for the first limit to belong both to the whole and to none; but when nothing necessary arises by means of these, there will not be a syllogism. [4, 26a2–8]

It is evident that at the same time as Aristotle lays out his argument, he is not forming a syllogism; he must rather be sketching a diagram in order to show why there cannot be a syllogism. To make his point obvious he then likely mentions terms on a diagram sketched out with indications of limits and intervals. How then does such a diagram look when exemplary terms are indicated on the diagram? It is likely that he would draw two intervals; the first would be drawn from A to B; perhaps we can guess how the second would look upon the basis of his example.

He writes, "terms of belonging to the whole: animal-human-horse; of belonging to none: animal-human-stone." Since Aristotle stands by the principle that syllogisms are formed upon the basis of a figure consisting of three terms and two intervals, he has inadequate means for producing a diagram whereby it is clear that no figure is formed which is sufficient for a syllogism.

Although it is clear to Philoponus (see his commentary [3, 76.1–76.24]) that Aristotle is dealing with terms and not with premises, the commentator does not diagnose Aristotle's failure to lay out a figure by means of a diagram clear enough to persuade us that there is a diagram either deficient or excessive for diagramming a figure—a figure which would itself be adequate for the formation of a syllogism.

The last topic to be taken up in this paper is Aristotle's account of reduction to the impossible. Book II of the *Prior Analytics* takes up topics which seem to imply that there is no perfecting correspondence between a syllogism and its underlying structure of terms and their relations. Aristotle's account of a syllogism with false premises and a true conclusion implies the absence of perfecting correspondence; such a syllogism is therefore not a demonstrative syllogism but should rather be called hypothetical.

There are several questions and difficulties which arise in taking up an examination of Aristotle's account of reduction to the impossible. The first question (1) is just what a syllogism *by the impossible* is; a second question (2) is what the role of reduction to the impossible is in completing the syllogistic project; a third question (3) is how it is possible to present the impossible within the general project of the syllogistic. The fourth question (4), finally, is whether there are two ways of resolving the impossibility shown on a reduction diagram.

Aristotle opens his discussion of a syllogism by or through the impossible in the following way:

> So then what converting is and how [it is effected], depending upon each figure, and what syllogism comes to be, is clear. [4, II.11; 61a17–18]

(Comment. A pair of premises depends upon, for example, the second figure; the application of conversion is required in order to transform a pair of premises on the second figure into the premises of a complete syllogism formed on the first figure. In earlier parts of this paper it has been shown how there can be syllogisms some of which fully depend upon the second and others upon the third figure. It is still unclear how we're to understand reduction to the impossible if the procedure implies that there are syllogisms which fully depend upon the second and the third figures. Aristotle writes:)

> So what then converting is and how on each figure and what syllogism comes to be, is clear. [4, II. 61a17–25]

(Comment. Aristotle's summary remark leaves the question unanswered just when and how a syllogism is formed on either the second or the third figure. When we left the first book, we had no reason provided as to how complete syllogisms have been formed on the second and the third figures. But concerning a syllogism by the impossible Aristotle writes:)

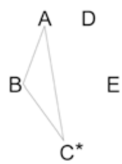
> Now the syllogism by the impossible is demonstrated when the contradictory of its conclusion has been set out and another premise is taken in addition; [such syllogisms come to be depending, respectively, upon] all the figures: for [the procedure] is like conversion, although it differs to the extent that [the conclusion] of the syllogism, which has come to be and of both of the premises which have been assumed, is reduced to the impossible not by virtue of the fact that the contrary has been agreed to before, but rather because it is apparent that it is true. And the terms are disposed in a similar way in both [figures]. For example, if the limit A belongs to the whole of the limit B, and the limit C is middle and if the limit A is set down either belonging not to the whole or to none of the limit B but to the whole of the limit C. [4, II.11; 61a17–25]

The first question stated above is answered in this way: a syllogism by the impossible is a syllogism which is demonstrated with the help of a second syllogism, a premise of which is the contrary or the contradictory of the conclusion of the first syllogism; a premise of the first syllogism is copied into the framework of the second syllogism. A conclusion emerges which is in apparent conflict with one of the premises of the original syllogism. Some debater in the discussion, who holds that premise dear, may find the conclusion of a reduction syllogism intolerable and therefore endorses the conclusion of the original syllogism.

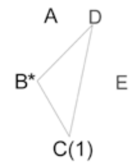
Aristotle contrasts two procedures for generating a second syllogism: just as conversion of a premise (part of a pair of premises on either the second or the third figure) results in a syllogism constructed upon the first figure, so reduction to the impossible results in a second syllogism which is based on the first figure (as Aristotle remarks in I.7) one premise of which is inconsistent with the conclusion of the second syllogism.

But the two procedures differ in this way: conversion begins with a syllogism already agreed upon; reduction to the impossible is different. For the conclusion of the first syllogism is not agreed upon; the second syllogism is constructed beginning with the denial of the conclusion of the first syllogism. For, in constructing the second syllogism, one assumes that the conclusion of the first syllogism has not been proved (or is not evident to those who are arguing with one another). Further, an additional premise for the second syllogism is copied from a premise in the first syllogism. The conclusion of the second syllogism conflicts with a premise helping to compose the first syllogism—a premise held to be true by those arguing, as just suggested.

In Sect. 12 Aristotle remarks that all the projections in the first figure are proved by means of the impossible; the whole affirmative projection based on the first figure is not proved by the impossible [4, II.12; 62a20–22]. If we draw a proper diagram, it is possible to suggest a reason why Aristotle thinks that reduction to the impossible is not possible regarding the first figure. Consider the following pair of diagrams:

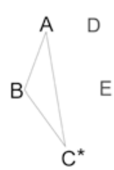


A diagram of a first figure wanting proof

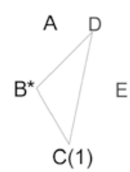


A diagram of a reduction to the impossible

It is evident from the diagrams above that the reduction diagram posits the denial of the long concluding interval charted on the first diagram: that denial is read from the long interval from D to part of the limit C. This first step is not problematical for Aristotle. We then copy the interval indicated by the minor premise into the reduction diagram. The interval and their terms yield a conclusion according to which the limit D belongs to some part of the middle term B. It is commonly supposed that if there is denial of the major interval A-B, and if we assume that negation is embedded not in the interval but in the premise "A-B", it follows that the statement of the denial yields: "So D belongs to some B". But this cannot be since no highest term belongs to part of the middle term when it belongs at all. On the other hand, if we were denying a major premise on account of universal quantification, we would take the resulting major premise to be "So D belongs to some B". But the middle term is neither quantified nor universalized. Therefore, the diagrams should be paired as follows:

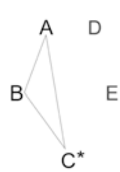


A diagram of a first figure wanting proof

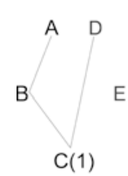


A diagram of the third figure

The second diagram is not part of the process of reduction to the impossible—a process in the service of proving the validity of the diagram on the left and therefore proving the validity of a syllogism formed upon the basis of the figure. Aristotle's mistake is perhaps evident: if we're to show the necessity of the long interval A-C for completing both the figure and also the syllogism formed upon the basis of the figure, we must posit the contrary of the long concluding interval A-C. When we do so, we form the following pairs of diagrams:



A diagram of a first figure wanting proof



A diagram showing the impossible

The point is not to continue syllogistic formation as if to complete a demonstration of impossibility. A syllogism itself is not defined for that task; the task rather falls to the drawing of diagrams which, at the same time, are not intended to present the impossible buried in its figure. What we need is what the second diagram just above gives us, stripped bare of any presumption to reveal the actual situation within a syllogistic figure. No part of the bottom term can fall under both of the two upper indications of the upper terms.

The final question before us is whether there are two ways of resolving the impossibility. With the resolution of the negative prefix 'im' I answer in the affirmative and in the following way. When impossibility becomes apparent upon the basis of a diagram, the faculty of intellect can return to itself with an apparent need to resolve the impossibility either for deficiency or for excess. Resolution of the impossible opens upon two paths: correcting for deficiency takes one path and posits possibility; correcting for excess takes the other and posits the necessity of the syllogism. The necessity is hypothetical necessity with its subject as the figure under discussion. But what is proved with hypothetical necessity is not the relative necessity of a final interval relative to the other intervals described by the diagram; nor is the relative necessity of the conclusion proved relative to its premises.

It is tempting to think that a choice between two courses of resolution depends upon the structure of the two sorts of diagrams: the first plotting a long interval as conclusion and the second two plotting short intervals as conclusions. If a reduction diagram shows impossibility regarding a sketch of a syllogism, resolution of the impossible opens upon two paths, as suggested just above: correcting for deficiency takes place on one path and writes 'possibility' below that projected syllogism; correcting for excess takes the other and posits the necessity of the syllogism. The modality is necessity, and its subject is the syllogism in question. If a reduction diagram rather shows possibility regarding a sketch of a syllogism, the diagram in question is itself possible but not sufficient for the construction of a syllogism.

At the beginning of the paper I argued that the formation of a demonstrative syllogism should not depend upon the conversion or the contraposition of premises of a syllogism to be. The paper has shown how syllogisms can be completed without invoking either conversion or contraposition. Reinforcement is provided by means of diagrams. I hope it has come clear how Sophie diagrams and Venn diagrams differ: by virtue of overlapping a Venn diagram denies the role of intervals which keep the representation of terms apart. A Sophie diagram, on the other hand, give space for all the parts.

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