

Preface

This book is an essay in epistemology of classifications, not a mathematical textbook or monograph. Except in a few pages, we do not present results of our own. When it is the case, we must recognize they are often elementary ones. Indeed, our main purpose is not to provide an exposition of an actual mathematical theory of classifications, that is, a general theory which would be available to any kind of them: hierarchical or not hierarchical, ordinary or fuzzy, overlapping or not overlapping, finite or infinite, and so on, founding all possible divisions of the real world. For the moment, such a theory is but a dream. We are essentially listing questions.

Our aim is, in fact, to expose the “state of the art” of this moving field and the philosophy one may eventually adopt to go further. To say a little more, we shall speak of some advances made in the last century, discuss a few tricky problems that remain to be solved, and, above all, show the very ways open for those who do not wish to stay any longer on the wrong track.

Let us briefly explain the importance of the subject.

A Long History

Finite classifications, as an economical way of grouping objects to simplify our understanding of the world (one may start with this “intuitive” definition), have been known since Antiquity and may already be found in the writings of Plato or Aristotle (see [371, 376, 377, 383]).¹

¹Of course, the views of Plato and Aristotle about classifications are very different. All along the Dialogues, and especially in the latest ones (*Parmenides*, *Sophist*, *Politicus*, *Philaebus*...), Plato obviously used to class a lot of things (Ways of Life, Political Constitutions, Pleasures, Arts, Jobs, Kinds of Knowledge, etc.), generally in relation with the “distance” that separates them from their archetypal forms—which gives some order (or preorder) on them. But, as Aristotle argues, it is quite impossible to explain what exactly this participation or imitation is. The properties that the forms have (eternal, unchanging, transcendent, etc.) are not compatible with material objects and the metaphor of participation or imitation breaks down in a number of cases. For instance, what

Though scholastic thought put these classifications to good use, taxonomy as a fully fledged discipline actually began to develop with the birth of natural science in the Classical Age, and with the need to organize floras and faunas in connection with the growth of human population on earth, in the context of the beginning of agronomy (see [115, 117]). A century later, following the progress of technology, Chemistry itself began to be a science. Coming after the works of Lavoisier and Auguste Laurent, and a lot of, often unsuccessful, essays, Mendeleev's Periodic Table of the Elements were given all the credit for winning the victory. Since that time, all sciences have made use of classifications (especially physics, where important fields need to be organized by non-trivial mathematical structures: for instance, discrete groups in crystallography, or Lie groups in quantum mechanics and elementary particle physics).

Progress in Library Classifications

At the end of the XIXth century, the development of scientific research, which raised the question of information storage and retrieval, encouraged the constitution of voluminous library catalogs: e.g., Dewey's decimal classification, Otlet and La Fontaine's universal decimal classification (see [138]), Herbert Putnam's Library of Congress classification (see [426]). In the course of the XXth century, new modes of indexing and original classification schedules appeared in this domain with S.R. Ranganathan (see [411, 412]) and his faceted classification, while, in the middle of the 1950s, a Classification Research Group was constituted and many international conferences on scientific information organized (London 1946, Chicago 1950, Dorking 1957, Washington 1958, Cleveland 1959, Elsinore 1965...).

Since that time, some rare attempts to develop a formalized approach to classifications were made by authors like R.A. Fairthorne, C.N. Mooers, B.C. Vickery (see [489, 490]) or J. Farradane (see [161, 162]), the very start of a *mathematics of classifications* having already existed since the end of the 1940s, as mentioned in a brief paper of the *Journal of Symbolic Logic* (see [89]) where Alonzo Church's review studied the work of R.A. Fairthorne, including discussions of A.B. Agard Evans, T.H. O'Beirne and E.M.R. Ditmas.² In fact, those researches never reached the level of a general theory.

does it mean exactly that a white object be said to participate in or copy the form of whiteness? Must we think that the form of whiteness is white itself? How could there be whiteness without any thing which would be white? What can a white object and the form of whiteness be said to have in common? According to Aristotle ([9], I, 9), the forms fail to explain how there could be permanence and order in the world. Far more, in the view of the Stagyrte, they cannot explain anything at all in our material world.

²In this period, that followed the second world war, the growth of information began to necessitate coordination (see, for instance, [130] and [131]). A decade later, information science was still looking for a language (see [350]), and an effective practice (see [163]).

In Europe, especially within the applied fields of library, life sciences and social studies, a number of papers on classification research were published during the next twenty years in *International Classification*, the German journal founded in 1973 by Dr Ingetraut Dahlberg.³

At the same time, research made progress in France with scholars like de Grolier ([211]) who published some master papers in a famous French library review (see [212]) or Z. Dobrowolski (see [132]), a great taxonomist in the field of technology.

However, most of these papers were not actually concerned with a mathematics of classifications and, around the 1990s, the interest in classifications themselves tends apparently to decline, the title of Dahlberg's journal soon being changed into *Knowledge Organization*, a much more general and up-to-date topic. The subsequent arrival of the desktop computer, followed by the growth of networks providing access to an almost incredible quantity and variety of resources, still kept away from a purely algebraic approach to the problem, automatic clustering and analysis of data being essentially reduced—especially in France—to multidimensional applied statistics (see [36–38, 83, 257]).

Curiously, until recently, library scientists were not necessarily very aware of these mathematical developments in classification science (see [386]). Some time ago, most of them, indeed, continued to work with traditional forms of library classifications. For example, in his *Vocabulaire élémentaire des classifications*, the Belgian librarian André Canonne still speaks of the CDU of Otlet and La Fontaine (or their followers), and of the Bibliographic Classification of Bliss, as good classifications (Bliss's one, though not perfect, being for him, under the form of BC2 (the new Bliss classification), the “best in the world”, at least at the time (see [76], 19)). He was even considering that such classifications, thanks to their analytical divisions, anticipate in fact some aspects of Ranganathan's faceted classification.⁴ On the other side, those who were concerned with advanced studies in classification theory tried essentially to convince professionals that it is possible to improve the Colon Classification of Ranganathan (see, for instance, [226, 346]), which tries to go beyond enumerative or semi-enumerative classification schemes already in use,⁵

³For a brief history of classifications, from the viewpoint of library sciences, see the papers of this author, for instance [118] or [119]. For recent transformations of the library domain, see Parrochia [382, 385].

⁴Let us give a simple example: in the UDC, instead of foreseeing, in the same class, indexes denoting permanent or recurrent concepts—for instance, the “gothic” in the Fine-Arts class (7)—, one may apply, except in certain circumstances, the analytical division 033.5 for indicating “gothic”, a concept which never appears, as a matter of fact, in primitive or derived classes. So one will never find “gothic architecture” nor “gothic churches” in the Architecture class (72), but one will be allowed to construct those symbols by adding to the indexes the appropriate analytical division: 72.033.5, “gothic architecture”; 726.6.033.5: “gothic churches” (see [76], 70).

⁵Dewey Decimal Classification (DDC) or Library of Congress Classification (LCC) are enumerative systems which start with a list of possible subjects, each with a corresponding number, whose divisions are often arranged in a hierarchic manner, from most general to most specific. In those systems, each document (book, paper, webpage, etc.) to be cataloged is then assigned one of the numbers in the scheme. Bliss's classification, because of the possible combinations of

but, as we shall explain later, does not seem to have—at least from a mathematical point of view—a very promising future ahead.

Furthermore, until the middle of the 2000s, with the exception of a few papers or books (see, for instance [56, 66, 118, 243, 393] or [245]), no philosophical analysis had been made to expose on what grounds, and especially on what theory of meaning, knowledge representations of librarians are based. In 2004, however, a very interesting paper from Elaine Svenonious put forward that three models might be concerned: Operationalism, the Referential or Picture theory of meaning, and the Contextual or Instrumental theory of meaning (see [477]). Let us say a few words about them.

(1) Operationalism, the theory of meaning emanating from the philosophy of logical positivism (particularly the one developed in the first decades of the XXth century in the Vienna Circle), is the attempt to define scientific concepts in terms of specifically described operations of measurement and observation. This means, in the case of library science, that one aims to construct objective evaluations of retrieval operations⁶ and operational definitions (of entity works, subjects, etc.) so that one can unambiguously define the ontology of a retrieval language, i.e. its entities, attributes, and relationships. But as such operational definitions always lack validity to some degrees, there is a chance they might often be ineffective.

(2) Coming from the founding fathers of logic (Frege, Russell) the Referential or Picture theory of meaning, though it derives also from an empiricist view of knowledge, takes for granted that the extensional meaning of a word is its *referent*. Words are contained in propositions, and these propositions, deriving from sensory experience, express properties and relationships of objects. In this way, empirical propositions picture reality in some sense. It does not mean necessarily what Elaine Svenonious says, i.e. that a proposition has empirical meaning if and only if it corresponds to (or “pictures”) reality.⁷ In addition to propositions that picture the world (in the

indexes mentioned before, may appear as a semi-enumerative system. But, in fact, all such systems happened to be more or less limited, because they all require some kind of “pigeon holing” documents into preconceived fixed categories. Ranganathan’s classification, as we shall see, tries to escape from these limits.

⁶For Elaine Svenonious, “an example of a productive operational definition is the precision-recall measure, which was developed to measure the degree to which a given retrieval system does or does not achieve its discrimination and collocation objectives” (see [90, 91]). Precision (number of relevant documents retrieved divided by the total number of documents retrieved) measures the degree to which the system delivers only relevant documents. Recall (number of relevant documents retrieved divided by the total number of relevant documents) measures the degree to which the system delivers all relevant documents. As Svenonious shows, “the use of these measures makes it possible to generalize about the impact of various factors on retrieval effectiveness. One of the earliest factors studied was indexing depth, the number of index terms assigned to a document. The more index terms assigned—or, alternatively, the more access points a document admits of—the higher the recall, the lower the precision”. For more recent references on the subject of the formal aspects of information retrieval, see [290] and [109].

⁷This is only the strong interpretation of the “picture condition” (the Aristotelian ontological view). In fact, this one can reduce to the existence of a one-to-one correspondence within some language, or, in a more formal way, to the existence of a correspondence between what we call an “object

sense of Wittgenstein's *Tractatus logico-philosophicus*), the empiricists recognized also the existence of *tautologies*, which are special propositions that express purely logical relationships among propositions, and are always true. Now the fact that true propositions could be formulated within a logical calculus, had a lot of consequences in the bibliographic area. Since Poincaré observed in his *Last Essays*, it was well-known that the extensional view of the logical syllogism was but a kind of hierarchical classification. This probably inspired Feibleman and made him develop a theory of integrative levels of knowledge, based on logic (see [166]). It also led Ranganathan himself to build his classification on the analogy with a *meccano set*, assuming thereby that all knowledge could be based on a standard set of concepts and relations among them (see [414], 20). As we have seen, one of the first librarians to promulgate the notion of a "mathematics of classifications", R. Fairthorne (see [160]), tried to prove that a classification of knowledge could be formulated as a lattice (indeed, a boolean one), and Wojciechowski (see [508], 17–18) concluded that the chance of survival for a classification was proportional to the degree to which it was formalized. When computers and the discipline of Artificial Intelligence came on the scene, the Referential or Picture theory of meaning laid ready at hand. For their most part, the data structures IA researchers used to represent knowledge (see, for instance, Winograd's micro-worlds [505], or Minsky's frames [347]) were founded upon a referential theory of meaning. Always true or essential properties, like always true relationships, are viewed as context-free expressions, and, thus, via inheritance or by hierarchical force, apply to particular instances. An example of such frame-based systems is the one constructed by Humphrey and Miller (see [253]) for machine-assisted indexing in the medical field. In that system, as Svenonious said, "if we assume a document is assigned the term "bone neoplasm", then we see that "bone neoplasm" is an instance of "neoplasm by site", a term that has associated with it two attributes: histologic type and disease process. By hierarchical force, bone neoplasm can be characterized by histologic type and disease process. The indexer, thus, is prompted to supply values for these attributes". That is the way the system works.

(3) The basic tenet of the Instrumental theory of meaning is that we know what a word means when we know how to use it. Thus, what a word conveys is in part variable, in part fixed. The fixed part is the dictionary or context-free meaning. Some of them, e.g., words used in scientific discourse, reduce to that: their meanings are not negotiable. But words used in, say, the social sciences, are regularly used with changeable meanings, which depend on the context. To avoid the extreme consequences of instrumentalism, that would lead to solipsism, the second Wittgenstein, since the publication of *Philosophical Investigations*, tries to make explicit some rules that govern our words' use, and which are embedded in what he calls "language games".

language" and a higher "metalanguage": for A. Tarski, for instance, the sentence "snow is white" is true if and only if snow is white, and this requisite would persist even if there had never existed any snow in the world. As Quine (see [407]) has shown, the Aristotelian theory of truth-correspondence is then reduced to a kind of "dequotation" mechanism, which yields a deflationary conception of truth.

The main idea of the philosopher is that knowledge representations are not descriptive of things and relations in the real world, but rather of linguistic behavior as such. Against Picture theory, which assumes a universal form of language, implies fixity of reference and wants to represent knowledge of the world as the conjunction of knowledge of independent micro-worlds, the instrumental theory of meaning supposes that pictures can be differently interpreted, that many words have fluid boundaries and that it is not possible to isolate micro-worlds. For example, the Cyc project, a database constructed by Lenat and his team, consists of over a million propositions, but, in addition to this, it contains information about the use of hundreds of thousands of root words, names, descriptions, and abstract concepts. As Elaine Svenonious told us, “a Cyc robot knows that anthrax can mean the heavy metal band, a bacterium, or a disease. More significantly it knows that while a piece of wood can be broken into smaller pieces, a table cannot be broken into smaller tables. Knowing rules for the use of words, it “understands” language behavior” (see [305, 306]).

In Aristotle classifications, classes were mutually exclusive and totally exhaustive, because there could not exist cross-classifications in nature.⁸ Membership in a given class was defined in terms of essential properties; that is, two members belong to a class if they share the same essential property(ies) to a sufficient degree. But, according to Wittgenstein, it is doubtful there exist common essences. In fact, the meaning of a word does not describe reality but is a function of language behavior, the instances of its use being similar to one another in different ways, and so, only share what the philosopher calls “family resemblances” (as in nature, where the members of, say, a family of animals, have the same nose, some the same eyes, some the same tail, etc., but where there is not a single property they all share).

This way of thinking has led to adoption of the methods of numerical taxonomy, which had a spectacular success in challenging the traditional biological dichotomies. It has led also to fuzzy set theory in the field of logic and computer science, and to the use of ambiguity operators in bibliographic classifications. As Elaine Svenonious has shown, Information retrieval operational definitions, using similarity matrices of family resemblance-type categories, have here advanced some sound techniques of automatic classification. “An example is the U.S. Census

⁸Indeed, in the course of his philosophy, Aristotle had to recognize the existence of what we call now “nonsubstantial particulars”. These are not primary substances (individuals) nor secondary substances (which is predicated on a substance and forms the *genera* and *species* in a “natural” classification), nor simple accidents (which is in a substance as a non essential predicate). With those nonsubstantial particulars, Aristotle speaks of predicates that are in a substance but does not belong to the category to which the substance belongs, so that they constitute, besides ordinary predication (predication within a substance), a kind of cross-categorical predication. For example “a certain white”, “a particular white” (*to ti leukon*), i.e. what Aristotle called a *trope*: something unique to the individual substance in which it inheres and not repeatable elsewhere (see [10, 1a 24]). Formally speaking, it is a certain x , which is in some y , with the meaning: x belongs to y , and x is not a part of y , and x cannot exist separately from y (or cannot exist on its own, or cannot exist independently of y). There is a vast literature on this dispute (see [96]). Overlapping classifications (see, below, Chap. 5, Sect. 5.3) might be a solution to this problem.

(PACE) system, an expert coding system developed to analyze U.S. census response forms (see [108]). Employing a vocabulary consisting of 800 industry and occupation categories, the system assigns terms to a candidate response form by comparing it with other forms that have been manually indexed—a large number of them. At a rate of 10 responses per second, the system was able to classify 22 million responses in three months—a task that, if done manually, would have cost 15 million dollars in labor costs. The system was reported to perform with an accuracy rate of 86 percent” (see [477]).

Formal Ontologies, Formal Concept Analysis

Elsewhere, especially in the USA, the need to get an unbiased domain and application-independent view on the divisions of a more and more complex scientific reality, yields the development of the large domain of *formal ontologies*, now carefully explored from the viewpoint of cognitive science.

Of course, theories on how to conceptualize reality date back as far as Plato and Aristotle, and the term “ontology” (or *ontologia*) itself is a very old one. Often used by philosophers as a synonym of “metaphysics”, it refers in fact to what Aristotle called “first philosophy”. As Barry Smith (after Ingarden) reminded us, “the term was coined in 1613, independently, by two philosophers, Rudolf Göckel (Goclenius), in his *Lexicon philosophicum* and Jacob Lorhard (Lorhardus), in his *Theatrum philosophicum*” (see [458], 155). Its first occurrence in English appears in Bailey’s dictionary of 1721, defining ontology as “an Account of being in the Abstract”. Classification is concerned with ontology because “ontology seeks to provide a definitive and exhaustive classification of entities in all spheres of being” (see [458], 155).

It is with the German XIXth century philosopher Edmund Husserl that this project began to be a scientific one. For Husserl, logic was a theory of science which took seriously “the idea that scientific theories are constituted by the mental acts of cognitive subjects” (see [455], 29). As such, logic, for Husserl, relates not only to meaning categories such as truth and proposition, subject and predicate, but also to object categories such as object and property, relation and relatum, manifold, part, whole, state of affairs, and so on.

But nowadays, in the context of a very complex world, upper ontology had to be divided into a lot of practical formal ontologies. Such a project has been carried out in a very competent way by Barry Smith in more than 450 papers, and a lot of collaborative experiments have established not only a set of principles for ontology development but real open ontologies in biological and biomedical domains (genes, proteins, infectious diseases, embryology, anatomical information. . .), in geospatial sciences (geographical categories, spatial objects, tools for geographic representation. . .) and also in social and cognitive sciences (information artifact, theory of the act, documents, naive physics, cognitive geometry of war, etc.).

This attempt, much like the approach of the German mathematician Rudolf Wille—known as “conceptual analysis” clearly borrows some concepts and structures from mathematics, but makes use of them to enhance the common way we define and employ them in order theory.

To conclude this section we shall observe that there still does not exist a general theory that tries to compare the numerous advances that have led, in the last few decades, and in several areas (data mining, computer science, taxonomy, DNA analysis, concept lattices in cognitive science and management), to some very sophisticated theories and techniques in particular kinds of classifications, while active branches of today’s mathematics have strong associations with them (lattice theory, model theory, descriptive set theory). So we need an overview that would help to unify several parts of this wide field, or at least provide a survey of it at a reasonably deep level.

Classification Research: from Philosophy to Mathematics

The idea of a basic theory of pure classifications, indeed, is not new. Such a project, anticipated by Kant’s logic (see [269]) at the end of the XVIIIth century, and followed by many attempts to classify sciences (see [272]) at the beginning of the XIXth, had been posed by Auguste Comte in his *Cours de philosophie positive* (see [98], lessons 36, 40 and 42) as a general theory based on the study of symmetries in nature (see [365]). In the same way, Augustin-Pyramus de Candolle (see [75]), in his *Théorie élémentaire de la Botanique*, published in 1813, a book in which he introduces the term of “*taxonomia*”, used in this work for the first time, had shown that Botany had to leave “artificial” methods for “natural” ones. In front of the big number of classifications in this science since Enlightenment, he intended to classify the classifications themselves (see [134], 154; [135], 255), delineating a difference between empirical and rational classifications, the first being independent from the nature of objects, the second ones having real relations to them. These latter ones were divided in three groups according to their aim: usual or practical classifications (based, for instance, on therapeutic properties of plants), artificial classifications (chosen simply for identification) and natural ones (which were the only ones that can reveal the true relations between species).

These projects were only concerned with finite classifications (particularly, biological ones), but a higher and more general view came into light around the 1960s with the Belgian logician Leo Apostel (see [8]). At the time, his theory was thought of as a more concrete version of set theory, including classes actually existing in the world and rejecting some well-known axioms of ZFC. A few years later, the views of Apostel were formalized by two polish mathematicians, Seweryna Luszczyńska-Romahnowa and Tadeusz Batog, who published a “generalized theory of classifications” in the famous review *Studia Logica* (see [322, 323]).

Against all expectations, nothing went any further. Despite many studies in automation of storage and retrieval, clustering analysis or data mining, set theory, order theory, theory of partition lattices (see [5]), combinatorics (see [1]), theory of

relations (see [179]), graph theory (see [129]), or classification theory (see [448], more recently [251, 523]) during the last fifty years, no unified summary has been proposed until now.

Interesting as these results may be, as indeed all the results obtained in automata theory in connection with infinite trees (see [69, 340, 408]), infinite objects (see [482]), orders (see [483]), μ -calculus (see [12]), Markov chains (see [429]) or information theory (see [335]), they have not been integrated into a global view.

The Viewpoint of This Book

To tell the truth, this situation can easily be explained: the least we can say is that the field of a mathematical theory of classifications is not a completely stable domain, and one is led to think, finally, that a vast side of it is still to be developed. Another reason for the lack of a general theory, close to the previous one, is that scientists are faced with a very difficult problem (finding a formalism enough general to apply to any kind of classifications), for which no complete solution is known at the present.

All the same, we think that the research we have carried on for more than thirty years, might be of some interest for librarians, logicians, and also for scientists in the different fields of empirical science, all of whom need to devise their own classifications.

But this book has a deeper stake. In fact, pure mathematics wants also a general theory of classifications to take over from classic (and too limited) versions of set theory. It is why, though aware of not being exhaustive, we have not only tried to make good use of numerical taxonomy, but also to derive some advantage from partial orders, algebra, logic and set theoretic approaches, including some views on infinite classifications (generally a neglected subject in our field).⁹

In this context, it will appear that a mathematical theory of classifications is, in fact, a new construction of the continuum. This is an original view on the subject, initiated by one of us (see [362, 363]) and very different from computing as well as from classic model theoretic approaches.

Some Comments About the Contents

We hasten to add that we do not blame scientists for using computers to build upon less than optimal empirical classifications when these are the only ones to which they have access. We quote numerous results obtained in the field of clustering

⁹Perhaps one may consider it a limitation of the book to gather so many different approaches. But we think it is also its main interest. Moreover, it must be known that no unified language exists at the present in the domain of classification theory and that it is not certain that any such global view will ever be reached: in fact, the concept of “classification” is probably too complex to be studied within one and the same formalism.

(Chap. 3) and even try to extend some of them to weak forms of classifications (Chap. 5), as a recent work of Brucker and Barthélemy suggests (see [68]). Proofs of existence of locally finite generalized classifications, based on minimal covering structures are notably given for experimental sciences (Chap. 6). But we have shown previously (Chap. 4) that the true solution of the classification problem, when classifications are defined as hierarchies of classes or tree-structures, would probably lie in a non-associative algebra of classification trees, if it could be found. Yet, such an algebra, as we shall see, is still to come, even if, since two famous papers of Loday (see [314] and [317]), there exist, among a lot of different types of k -algebras described together with their properties under operadic and homological points of view, some good candidates (like the Dendriform algebras) which must be carefully studied.

It would also be unreasonable to deny the great interest of model theory, which is an indisputable field of pure mathematical knowledge. However, we would be unable to use in our own work the part of model theory called “classification theory” (see [19, 229]). Born in 1965 with M. Morley (see [353]) and famous since the massive work of S. Shelah (see [448]) and his numerous papers (>800 at the present), classification theory aims at classifying mathematical structures according to the type of combinatorial object (infinite order, infinite tree, ...) they refer to, with a particular view to using this classification for developing a dimension theory attached to definable sets. As we shall see, such a project, however, has a very slight intersection with our problem (the algebra of classifications), even if, after B. Zil’ber (see [523]) and E. Hrushovski (see [250]), classification theory, extended into a theory of geometrical stability, is dealing with finite dimension sets and the classifications one may construct over them, according to the type of algebraic structures (group, field...) they actually possess.

We must remember also that model theory carries on a particular viewpoint on mathematics, which puts them in a specific light. Historically, it is well known that its starting point—the Löwenheim-Skolem theorem—has initiated, through the further works of Tarski (in the 1920s) and then of Abraham Robinson (in the 1950s), a relativistic point of view in mathematical logic (see [241]), leading to a non standard analysis vision of the set $\mathbb{R}$ as a structure embedded in some proper elementary extension (see [84], 276). Yet, more than a half century later, after Diener and Reeb (see [128]), Harthong (see [230]) and Nelson (see [357, 358]) results, it is not absolutely obvious that some of the alternative models we can construct are more than poor versions of the continuum (see [396, 504])—known, indeed, for a long time: an ordered field structure over the theory of algebraic numbers (see [452]). So, one of us (P. Neuville) did not go so far as to conclude that the special infinity of $\mathbb{R}$, as Cohen was thinking (see [95]), is beyond any model. In this respect, we would have only to face the undecidability results and, particularly, the improvability of the Continuum Hypotheses within ZFC. However, we had not the right to neglect alternative views. So, some of them—and the consequences they yield on trees and classifications—are examined in Chap. 9.

We have also admitted, in this book, that it should be useful to try to set out an axiomatic theory of classifications (Chap. 8), even if we have to choose special

axioms sometimes different from the ones to be found in classical set theory. In fact, some version of set theory with the universal set $V = \{x : x = x\}$ seemed to be a good start for our main purpose.

Furthermore, there also exists what one of us—Pierre Neuville—has called “metaclassification” (Chap. 7), which is a way of considering the real numbers as some kind of indexes on which sequences of open sets are convergent. Then, if we try to formalize the actual operation of taxonomy (and not only use of computers to perform clustering analysis), we can reach some representation of the whole set of possible classifications. Then, connecting it to the different circles of knowledge as they appear on the surface of a sphere (associated to cones through which they may be seen by some taxonomists), one gives rise to a classification of knowledge which could help most scientists, especially those working in the domain of library sciences.

Let us finish by expressing what may be a “wishful thought”: as one of the main features of this book is the narrowness of the set of prerequisites necessary to understand it, we hope it will also be useful for all students and searchers in the fields of logic, classification theory and even philosophy of science.

Instructions to the Reader

Our notation, whose main symbols are listed in the “index of notations” at the end of the book, are quite common, except on a few points: for instance, we have reserved the symbol “ $\vdash$ ” for a logical use and the axiomatic presentation of our theory. So, the left and right products in a dialgebra, which are commonly symbolized by “ $-$ ” and “ $\vdash$ ” are represented here respectively by the symbols “ $\lrcorner$ ” and “ $\jrcorner$ ”.

The numeration of our headings is also quite simple and follows the indications of sections and subsections. For example, Definition 1.6.1 means the definition 1 in Chap. 1, Sect. 1.6. In the same way, Theorem 2.4.3 means the theorem 3 of the Chap. 2, Sect. 2.4. Figures and tables are numbered according to the chapter. Figure 3.4 means the figure 4 of Chap. 3. Table 7.1 means the table 1 of Chap. 7.

Acknowledgments

We want to address, first, all our thanks to Professor Luisa Iturrioz (University Claude-Bernard—Lyon I, Department of mathematics, Camille Jordan Institute) who read very carefully some chapters of the whole text and helped us to improve it.

We are very grateful to A. Miladi (I.N.P.G.) and to Pierre Anglès (University of Toulouse III, Institute of mathematics, Laboratory E. Picard), for their first review of the book, their penetrating observations and their judicious comments, which have also contributed to make it better.

We must also thank a lot Jean-Marc Drouin for his kindness and quite interesting references in biology and history of taxonomy.

We are particularly indebted to Mr André Siramy, who illustrated our text with beautiful drawings, which are exact non numerical mathematical models.

We have also to address special thanks to Ivan Lavallée (University of Paris 8) for his help in solving some problems in Latex.

Finally, Daniel Parrochia is pleased to remember that Jean-Pierre Ginisti and Jean-François Pabion were his masters in logic, that François Dagognet has initiated and encouraged his work in classification theory, and that several workshops, in France and Switzerland, in the decade 1985–1995 (with Marcel Brissaud, Djamel Zighed, Gilbert Ritschard and others), have much contributed to develop his interest in clustering analysis and interdisciplinary research.

In the end, the authors thank all the persons who have worked with them and all the institutions that have supported their research during all those years.

Towards a General Theory of Classifications

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2013, XXIII, 304 p. 72 illus., 1 illus. in color., Softcover

ISBN: 978-3-0348-0608-4

A product of Birkhäuser Basel