
Preface

This book is a result of a long-term project which originated in courses we taught to undergraduate students who specialize in mathematics. These students had δ - ε calculus before, but there did not seem to be a suitable comprehensive textbook for a follow-up course in analysis.

We wanted to write such a textbook based on our courses, but that was not the only goal. Teaching bright students is about introducing them to mathematics. Therefore, we wanted to write a book which the students may want to keep after the course is over, and which could serve them as a bridge to higher mathematics. Such a book would necessarily exceed the scope of their courses.

We start with standard material of second year analysis: multi-variable differential calculus, Lebesgue integration, ordinary differential equations and vector calculus. What makes all this go smoothly is that we introduce some basic concepts of point set topology first. Since our aim is to be completely rigorous and as self-contained as possible, we also include a Preliminaries chapter on the basic topic of one-variable calculus, and two Appendices on the necessary concepts of linear algebra. This pretty much comprises the first part of our book.

With the foundations covered, it is possible to venture much further. The common theme of the second part of our book is the interplay between analysis and geometry. After a second installment of point set topology, we are quickly able to introduce complex analysis, and after some multi-linear algebra, also manifolds, differential forms and the general Stokes Theorem. The methods of manifolds and complex analysis combine in a treatment of Riemann surfaces. Basic methods of the calculus of variations are applied to a theory of geodesics, which in turn leads to basic tensor calculus and Riemannian geometry. Finally, infinite-dimensional spaces, which have already made an appearance in multiple places throughout the text, are treated more systematically in a chapter on the basic concepts of functional analysis, and another on a few of its applications.

The total amount of material in this book cannot be covered in any single year course. An instructor of a course based on this book should probably aim for covering the first part, and take his or her picks in the second part. As already mentioned, we hope to motivate the student to hold on to their textbook, and use it for further study in years to come. They will eventually get to more advanced books in analysis and beyond, but here they can get, relatively quickly, their first glimpse of a big picture.

Because of this, the aim of our book is not limited to undergraduate students. This text may equally well serve a graduate student or a mathematician at any career stage who would like a quick source or reference on basic topics of analysis. A scientist (for example in physics or chemistry) who may have always been using analysis in their work, can use this book to go back and fill in the rigorous details and mathematical foundations. Finally, an instructor of analysis, even if not using this book as a textbook, may want to use it as a reference for those pesky proofs which usually get skipped in most courses: we do quite a few of them.

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