Foreword

This book is the twelfth in a series of Proceedings for the *Séminaire Poincaré*, which is directed towards a broad audience of physicists, mathematicians, and philosophers of science.

The goal of this Seminar is to provide up-to-date information about general topics of great interest in physics. Both the theoretical and experimental aspects of the topic are covered, generally with some historical background. Inspired by the *Nicolas Bourbaki Seminar* in mathematics, hence nicknamed *"Bourbaphy"*, the Poincaré Seminar is held twice a year at the Institut Henri Poincaré in Paris, with written contributions prepared in advance. Particular care is devoted to the pedagogical nature of the presentations, so as to be accessible to a large audience of scientists.

This new volume of the Poincaré Seminar Series, *"Chaos"*, corresponds to the fourteenth such seminar, held on June 5, 2010. It presents a complete and interdisciplinary view of the concept of chaos, both in classical mechanics in its deterministic version, and in quantum mechanics. This volume describes recent developments related to the mathematical, physical, theoretical, and experimental aspects of this fascinating concept. It expounds some of the most challenging questions in science, including predictions on the future of our own planetary system.

The first survey, by Étienne Ghys, entitled *"The Lorenz Attractor, a Paradigm for Chaos"*, offers a broad description of the fundamental mathematical issues at play with **deterministic chaos**, thereby serving in effect as an introductory chapter to the book. After having invoked Laplace, Maxwell, Poincaré and Hadamard's pioneering standpoints on determinism and chaos, the author first presents two past paradigms for chaos which have been superseded by the Lorenz attractor: "(quasi-)periodic dynamics" and "hyperbolic dynamics". He then focusses on describing Lorenz's fundamental 1963 article, which bears the technical title *"Deterministic non periodic flow"*, and stayed largely unnoticed by mathematicians for almost a decade. The 1972 conference by Lorenz, entitled: *"Predictability: does the flap of a butterfly's wings in Brazil set off a tornado in Texas?"*, made the butterfly effect so famous that it finally disseminated to society at large. The reader fascinated by the paradigmatic power contained in this "little ordinary differential equation", originally motivated by meteorological phenomena, will find here the Lorenz butterfly as it is understood today. This includes topological aspects, such as the "Lorenz knots and links", the relation to Hadamard's study of hyperbolic geodesics, as well as the statistical aspects, such as the Sinai–Ruelle–Bowen (SRB) measures which provide a quantitative description of this type of chaotic system. A general picture of dynamical systems finally emerges from this very complete article.

The second article, *"Chaotic Dynamos Generated by Fully Turbulent Flows"*, by Stéphan Fauve, describes a masterpiece experiment, the von Kármán Sodium or VKS experiment, which for the first time established in 2007 the spontaneous generation of a magnetic field in a strongly turbulent sodium flow. The interplay between deterministic chaos in low-dimensional dynamical systems and the many degrees of freedom of hydrodynamic turbulence is magisterally explained, together with the earlier Karlsruhe and Riga experiments. The author then turns to the VKS experimental setup and its spectacular results, reporting the dynamics of the observed magnetic fields, including their reversal, which pertain to the modeling of Earth's magnetic field. These results are compared both to a theoretical model and to numerical simulations.

The research in **quantum chaos** attempts to uncover the fingerprints of classical chaotic dynamics in the corresponding quantum system. In the third contribution, *"Discrete Graphs – A Paradigm Model for Quantum Chaos"*, Uzy Smilansky describes a simple toy model – the discrete Laplacian on finite d-regular expander graphs – which allows one to grasp the essential ingredients of quantum chaos. Following the pioneering work by Jakobson *et al.*, he reviews the numerical evidence that illustrates the excellent agreement between the spectral statistics of the discrete Laplacian (or its magnetic analogue) on a large d-regular graph, and the predictions of Random Matrix Theory (RMT) for the Gaussian Orthogonal Ensemble (or the Gaussian Unitary Ensemble (GUE)). Theoretical attempts to justify the famed Bohigas–Gianonni–Schmit (BGS) conjecture were mostly based on trace formulae *à la* Gutzwiller, which establish a link between the (quantum) spectral information on one side, and the periodic trajectories of the underlying classical dynamics on the other side. In the present context, the connection between spectral statistics and periodic orbits is directly demonstrated by using explicit trace formulae for d-regular graphs, leading to interesting yet difficult questions on the combinatorics of closed cycles on these graphs. The author takes a "reverse" point of view: assuming that the spectral statistics agrees with RMT, he obtains new (conjectural) expressions for these combinatorics, which are actually well supported by numerical simulations. As the author remarks, this "reverse" viewpoint is similar with that of Keating and Snaith (see the following contribution), who deduced the asymptotic behavior of the high moments of Riemann's ζ-function on the critical line, by assuming that the Riemann zeros satisfy the GUE statistics.

In the thoroughly written article, *"Quantum Chaos, Random Matrix Theory, and the Riemann ζ-function"*, Paul Bourgade and Jonathan P. Keating illuminate step by step the connections between the distribution of zeros of the Riemann zeta function and the statistics of eigenvalues of random unitary matrices. The classical results on the ζ-function and its relation to the distribution of prime numbers, from Riemann's fundamental idea to apply complex analytic methods, which led to the prime number theorem by Hadamard and de la Vallée-Poussin, to Weil's explicit formula for the zeros of the ζ-function (or "zeta zeros"), are brilliantly summarized. The authors then expound the basics of Dyson's classical ensembles of random matrices. The seminal discovery by Montgomery that, assuming Riemann's Hypothesis, the gaps between the zeta zeros show the same "fermionic" repulsion law as the GUE eigenvalues was the starting point of a flurry of analytic and numerical studies trying to rigorously establish these fascinating connections. The next part provides a digest of spectral statistics in *quantum chaos*: the Berry–Tabor or BGS conjectures, regarding the spectral statistics of quantized integrable or chaotic systems, are analyzed from the viewpoint of Gutwiller's trace formula. The paper ends with a discussion on recent work, in particular by Keating and collaborators, on the macroscopic statistics of zeta zeros and the corresponding conjectures for the moments of the zeta function on the critical axis. This brilliant review convincingly shows how further progress could ultimately provide a *spectral interpretation* for the zeros of the Riemann ζ-function, thus a proof of the celebrated Riemann Hypothesis itself.

Hans-Jürgen Stöckmann is a pioneer of experimental quantum chaos. In his contribution, *"Chaos in Microwave Resonators"*, he first provides a list of possible physical applications of quantum chaos: nuclear, atomic or mesoscopic physics, but also cavity electromagnetism, acoustics, seismology – in the latter cases one should rather speak of *wave chaos* since the waves are purely classical. This broad field of applications comes from the identity between the time independent Schrödinger equation for a quantum particle in a cavity, and the Helmholtz equation describing stationary (classical) waves in the cavity. The classical motion is the ray dynamics inside the cavity, which may be chaotic for certain very simple shapes (like the famous *stadium billiard*). Such "quantum billiards" are among the simplest and best understood quantum chaotic systems, very much studied at the theoretical and numerical level. After recalling some theoretical predictions of quantum chaos, the author describes in detail the experiments on the propagation of microwaves in 2D or 3D chaotic cavities; these delicate *physical* experiments verified these predictions. This contribution also explains some recent applications of the same type of analysis, e.g., in the framework of constructing efficient microlasers, or towards understanding the "freak (or rogue) wave" phenomenon in oceanic waves.

The contribution of Stéphane Nonnenmacher, entitled *"Anatomy of Quantum Chaotic Eigenstates"*, offers a panoramic view of our present knowledge of the *eigenmodes* of quantized chaotic systems, dubbed as "chaotic eigenmodes". Due to the nonseparability of the dynamics, we have no analytic expression at hand for these eigenmodes; as a result, we can only describe them through indirect, unprecise means, or by using statistical methods. The main mathematically rigorous description of chaotic eigenmodes addresses their *macroscopic* localization properties. The central result of this approach, the Quantum Ergodicity theorem (originally formulated by Schnirelman), states that *almost all* the high-energy eigenmodes are *macroscopically equidistributed* over the energy surface, a property viewed as a quantum analogue of the classical ergodicity property. The Quantum Unique Ergodicity conjecture, according to which this asymptotic flatness suffers no exception, has been proven only for very specific systems enjoying extra symmetries, like the case of arithmetic hyperbolic surfaces recently solved by Lindenstrauss. This "macroscopic flatness" does not prevent the eigenfunctions from strongly fluctuating at the *microscopic* scale. Studying these fluctuations on a rigorous footing represents a formidable challenge. A Random Wave Model (RWM) was heuristically proposed to describe these eigenmodes, viewing them locally as random combinations of isoenergetic plane waves. Although a rigorous justification of this statistical model seems totally out of reach, the RWM represents an interesting object by itself, and has recently attracted a lot of attention from probabilists. For instance, one of the most vivid questions consists in understanding the *nodal set* of the random wave, which can be seen as its "microscopic skeleton".

Last, but not least, in *"Is the Solar System Stable?"*, Jacques Laskar studies the fundamental question of the stability, hence the fate, of the Solar planetary system we live in, a subject where he made groundbreaking contributions. He offers here a rather complete perspective, both historically and scientifically. From Newton to Laplace to Poincaré, this fascinating problem drove fundamental progress in mathematics and physics throughout the 18th and 19th centuries. Today's scientists, and preeminently the author, assisted by supercomputers, are running extremely long simulations that allow them to establish that the Solar System is indeed chaotic, and has to be dealt with by using a probabilistic approach. Planetary orbits cannot be accurately predicted after a time scale of a few tens of millions of years, and significant probabilities for major planetary collisions or ejections taking place before the end of life of the Sun can be evaluated. These stunning recent results thereby offer a grand ending to this volume, if not to our Solar System!

This book, by the depth of the topics covered in the subject of Chaos, should be of broad interest to mathematicians, physicists and philosophers of science. We further hope that the continued publication of this series of Proceedings will serve the scientific community, at both the professional and graduate levels. We thank the Commissariat à l'Énergie Atomique et aux Énergies Alternatives (Division des Sciences de la Matière), the Daniel Iagolnitzer Foundation, the Triangle de la Physique Foundation and the École polytechnique for sponsoring this Seminar. Special thanks are due to Chantal Delongeas for the preparation of the manuscript.

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