Preface

The following represents a revised and a complete form of the book [1] with the purpose to define, interpret, and use "**the matrix with partial derivatives of the state vector** ($\mathbf{M}_{pdx}$)", designed for some categories of processes with distributed and lumped parameters.

Within this frame, the work elaborates in a unitary systemized form:

- The analogical modeling through ($\mathbf{M}_{pdx}$)
- The numerical simulation through ($\mathbf{M}_{pdx}$) and Taylor Series.

Different stages of analyses and synthesis that are specific to the numerical calculus are included, for processes with distributed (and lumped) parameters such as: the local-iterative linearization, initial conditions for the beginning of the calculus, errors' propagation, case studies, the numerical integration's performances, usually with respect to time (t) obtained in the programs run on the computer, and others [2–14].

In order to ensure the beginning of the calculus for $t = t_0$, the ideal situation consists in knowing the state vector for the initial conditions $\mathbf{x}_{IC} = \mathbf{x}$ (t_0, p, q, r), where the spatial variables (p, q, r) can be associated or not to some coordinate axes (Cartesian, polar, cylindrical, etc.).

In many technical applications ($\mathbf{x}_{IC}$) cannot be precisely known, but there are some methods (for instance expert type) that can approximate the elements (or at least the first element of) the state vector ($\mathbf{x}_{IC}$) in a not very close to (t_0) vicinity. Within this frame, one of the usual variables for approximating the first element has been considered:

$$X_{0000}(t, p, q, r) = K_y \cdot F_{0T}(t) \cdot F_{0P}(P) \cdot F_{0Q}(q) \cdot F_{0R}(r) \cdot u_0(t) \qquad (A)$$

with the ensurance of the continuity conditions for the functions $F_{0T}(t)$, $F_{0P}(p)$, $F_{0Q}(q)$, and $F_{0R}(r)$. The independent variable of the input signal was noted with $u_0(t)$ and (K_y) is a proportionality coefficient. In the composition of these functions there will be different levels of freedom (time and spatial constants) whose convenient choice might contribute to a better approximation for ($\mathbf{x}_{IC}$).

Undoubtedly the solutions expressed by the expression (A), can sometimes be approximated by means of experimental takings too, with respect to time, for different measurable existence domains of the spatial variables (p), (q), and (r). The $F_{0T}(t)$, $F_{0P}(p)$, $F_{0Q}(q)$, and $F_{0R}(r)$ functions, can be ordinarily presented, either with periodical evolutions or nonperiodical evolutions, but damped with respect to (t), (p), (q), respectively, (r), for the initial conditions (IC) at ($t = t_0$), respectively, $x_{0000IC} = x_{0000}$ (t_0 ,p, q, r) , known, or enough precisely approximable.

Because in physics, chemistry, termotechnics, electrotechnics, mechanics, etc., the weight of the partial derivative equations of second order is bigger, in this book more heed was given to these categories of PDE. In this case, the solution (A), could verify an ordinary variant of PDE II · 4 of the form (with details presented starting with Chap. 4):

$$\left.\begin{aligned} &a_{0000}\cdot x_{0000}+a_{1000}\cdot x_{1000}+a_{0100}\cdot x_{0100}+a_{0010}\cdot x_{0010}+a_{0001}\cdot x_{0001}+a_{2000}\cdot x_{2000}\\ &+a_{1100}\cdot x_{1100}+a_{0200}\cdot x_{0200}+a_{0110}\cdot x_{0110}+a_{0020}\cdot x_{0020}+a_{0011}\cdot x_{0011}+a_{0002}\cdot x_{0002}\\ &+a_{1001}\cdot x_{1001}+a_{1010}\cdot x_{1010}+a_{0101}\cdot x_{0101}=K_y\cdot(\varphi_{0.0000}\cdot u_0+\varphi_{1.0000}\cdot u_1+\varphi_{2.0000}\cdot u_2) \end{aligned}\right\} \quad (B)$$

where $a_{....}$ = const. and $\varphi_{0000} = \varphi_{0000}$ (t, p, q, r) .

The quite general form of PDE II · 4 presented in (B) or simpler forms resulted from (B), covers domains of very large applicability, and represents a part of the contents of this book.

It is anticipated the fact that from ($\mathbf{x}_{IC}$) is calculated ($\mathbf{x}_{PQR,IC}$) by successive analytical derivatives, which represents the most laborious part and implicitly the main disadvantage of the method, in order to ensure the beginning of the calculus.

It is also anticipated that from ($\mathbf{x}_T$) we can approximate ($\mathbf{x}_{TPQR}$) through some iterative subroutines of symbolic derivatives through indexes. These iterative procedures can easily be extended for partial derivatives of a superior order, thus ensuring cumulated relative errors in percentages (CREP) of numerical integration that are totally negligible. Some possible generalizations and diversifications in the application of this method thus also results for other expressions (A) for the approximation of the components of the state vector ($\mathbf{x}_{IC}$).

It needs to be mentioned that these methods of analogical modeling through ($\mathbf{M}_{pdx}$), *numerical simulation through* ($\mathbf{M}_{pdx}$) *and Taylor Series*, *as well as the local iterative linearization presented in this paper*, *have not been published by other sources in the literature*, *according to our knowledge*.

The three parts of the book present relatively different approaches, as follows:

Part I: “Processes with Lumped Parameters”

Chapter 1: “Linear Processes Invariant in Time” pursues the extension of the use of the Taylor series (in the complete variants, with odd derivatives and even derivatives) to numerical integration of ordinary differential equations (ODE) with constant coefficients, through the method of local-iterative linearization.

Chapter 2: "Time Varying Linear Processes" pursues the extension of the use of the same Taylor series (in the complete variants, with odd derivatives and even derivatives) to numerical integration of ordinary differential equations (ODE) with at least one time-depending coefficient (the other coefficients being constant), through the method of local-iterative linearization.

Chapter 3: "Non-linear Processes with Lumped Parameters" pursues the same extension of the use of Taylor series (in the complete variants, with odd derivatives and even derivatives) to numerical integration of ordinary differential equations (ODE), nonlinear of the form (3.7), through the method of local-iterative linearization.

The details on the method of local-iterative linearization (LIL) are presented in [15–32] and others.

Part II: "Processes with Distributed Parameters"

Chapter 4: "Linear Processes with Distributed Parameters" refers to the phenomenon modeled by equations with partial derivatives (PDE), with constant coefficients, for which the state variables are defined and that are associated with initial conditions and final conditions. The matrix with partial derivatives of the state vector is defined ($\mathbf{M}_{pdx}$), and the two sequential stages of iterative calculus are explained. The possible extension of ($\mathbf{M}_{pdx}$) is being underlined, for (PDE) with more independent variables, for instance with respect to the time (t) and three spatial Cartesian (p, q, r) coordinates.

Due to the fact that the use of the matrix ($\mathbf{M}_{pdx}$) is also based on the Taylor series, the three variants of these series (complete, with odd derivatives and even derivatives) are taken into account, for the numerical approximation of the state vector ($\mathbf{x}_k$).

The method of the complete Taylor series is being developed by the local-iterative linearization (LIL) for the calculus of the same state vector ($\mathbf{x}_k$).

At the end there are some references on the initial and starting conditions, associated with the matrix ($\mathbf{M}_{pdx}$).

Chapter 5: "Non-linear Processes with Distributed Parameters" underlines the fact that the stages of calculus based on the operator matrix ($\mathbf{M}_{pdx}$) are formally identical to those presented in Chap. 4. Comparatively, the method becomes more laborious, but the general, unitary, and well-systemized character of the stages is maintained.

Chapter 6: "Truncated Errors of the Operator Matrix $\mathbf{M}_{pdx}$" refers to the manner of sequential propagation of the "contamination" with these errors. The importance of the pivotal element for the formation and complete interpretation of the matrix ($\mathbf{M}_{pdx}$) is being underlined.

Part III: "Simulation Examples" groups on the following two chapters:

Chapter 7: "Modeling-Simulation Examples for Processes with Lumped Parameters", presents a number of eight programs, considered quite usual in engineering, respectively, two simple systems of automatic control, a linear process variant in time, Van der Pol, Düffing and Riccati equations, a system of

control with two nonlinear elements and an asynchronous motor, supplied with variable frequency voltage, from an electric railway traction.

Chapter 8: "Modeling-Simulation Examples for Processes with Distributed Parameters", presents a number of 18 programs, for PDE I.2, PDE I.3, PDE I.4, PDE II.2, PDE II.3, PDE II.4, PDE III.2, PDE III.3, PDE IV.2, PDE IV.3, and PDE IV.4. All these (PDE) have been considered with constant coefficients.

Furthermore, we illustrate a nonlinear PDE II.2 for modeling a column of isotopic separation N^{15}, then two control systems of some processes defined by PDE II.2 and PDE II.3, and finally a system made of two (PDE), each of second order. The chapter ends with two examples containing an ODE II, as well as a PDE II.2, solved with the methods Taylor series and the Taylor series-LIL, underlining some comparative aspects that practically certify the same results.

Chapter 9: "Cases for Establishing the $\mathbf{M_{pdx}}$ Matrix" refers to a usual and simple control system, with a PID controller for three distributed parameters processes, defined by PDE II.4, then PDE II.3, and PDE II.2. The systemized and unitary character is being underlined for the stages of establishing the $\mathbf{M}_{pdx}$ matrix.

Chapter 10: "Equations with Partial Derivatives in the Cartesian Area" after presenting some variants for approximating the solutions $y_{0000}(t, p, q, r)$ and $y_{00}(t, s)$ details the stages of analogical modeling and numerical simulation for PDE I.2, PDE II.2, PDE III.2, and PDE IV.2, with the associated truncated errors. Some preliminaries for simple control loops are also mentioned, which include PDE, as well as some references for establishing the matrix ($\mathbf{M}_{pdx}$) for serial, parallel, and with feed-back connections of PDE II.2.

Chapter 11: "Serial, Parallel and with Feedback Connections for the Processes Modeled Through PDE", describes three case studies, for two PDE, respectively, PDEA II · 2 and PDEB II · 2 connected in parallel, in series, and with feedback.

Based on the elaborated programs, is attested the fact that the method of numerical simulation with ($\mathbf{M_{pdx}}$) and Taylor series, can be extended to four systems with (PDE) connected in parallel, in series and with feedback too, with remarkable possibilities of using them in specific automation systems.

Chapter 12: "Control Loops with Distributed and Lumped Parameters in the Cartesian Area" presents in detail, based on programs run on the computer, the stages, and the results of the analogical modeling through ($\mathbf{M}_{pdx}$) and numerical simulation through ($\mathbf{M}_{pdx}$) and Taylor Series for two case studies that include PDE II.2 in simple control loops.

It needs to be noted that all (ODE) and (PDE) have proved either general solutions, or particular solutions, and the singular solutions have not been taken into consideration. The particular solutions have been considered in exponential or polynomial variants, used in technique. With these solutions we were able to establish the initial conditions and the final conditions. Also, we were able to establish the performances of numerical integration, using the indicator called "cumulative relative error in percent" (CREP), which in most examples was between the limits $(10^{-6} \div 10^{-2})$ %, a fact that certifies the accuracy of the method and the programs.

Chapter 13: "Numerical Simulation Using Partial Differential Equations, for Propagation and Control in Discontinuous Structures Processes" presents a possible study variant of propagation and control of some specific phenomena, in the areas with discontinuous structures. The numerical simulation is based on "**the matrix with partial derivatives of the state vector** ($\mathbf{M}_{pdx}$) **method**, **associated with Taylor series**". The theoretical preliminaries of this method with a general character are exposed in a compendious way, in order to be used in case studies, with heterogeneous applications, for example in chemistry, thermoenergetics, biomedical processes, etc.

Chapter 14: "Conclusions" presents the importance of studying the analogical modeling and numerical simulation through ($\mathbf{M}_{pdx}$) and Taylor Series.

Appendices AI, AII, and AIII help the reader to understand all material presented in Chaps. 1–14.

The entire work has been elaborated by Tiberiu Coloşi. This book could not be published without the very qualified and collegiate support of all authors.

Some examples and programs have been elaborated and included, throughout many years, in the projects and diploma papers of the students of the Faculty of Automation and Computer Science within the Technical University of Cluj-Napoca.

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