Chapter 2
Materials and Device Degradation

Degradation is seemingly fundamental to all things in nature. Often this is described as one of the consequences of the Second Law of Thermodynamics—*entropy* (*disorder*) *of isolated systems will tend to increase with time*.[1] The evidence for degradation is apparently everywhere in nature. A fresh coating of paint on a house will eventually crack and peel. The finish on a new automobile will oxidize with time. The tight tolerances associated with finely meshed gears will deteriorate with time. The critical parameters associated with precision semiconductor devices (threshold voltages, drive currents, interconnect resistances, capacitor leakage, etc.) will degrade with time. In order to understand the useful lifetime of the device, it is important to be able to model how critically important material/device parameters degrade with time.

1 Material/Device Parameter Degradation Modeling

Reliability concerns arise when some critically important material/device parameter (e.g., mechanical strength, capacitor leakage, transistor threshold voltage, brake-lining thickness, etc.) degrades with time. Let S represent a *critically important material/device parameter* and let us assume that S changes monotonically and relatively slowly over the lifetime of the material/device. A Taylor expansion about $t = 0$, produces the Maclaurin Series:

$$S(t) = S_{t=0} + \left(\frac{\partial S}{\partial t}\right)_{t=0} t + \frac{1}{2}\left(\frac{\partial^2 S}{\partial t^2}\right)_{t=0} t^2 + \ldots \tag{1}$$

It will be assumed that the higher order terms in the expansion can be approximated by simply introducing a power-law exponent m and writing the above expansion in a shortened form:

[1] Regardless of how carefully crafted a material/device is at time zero, the material/device will degrade with time.

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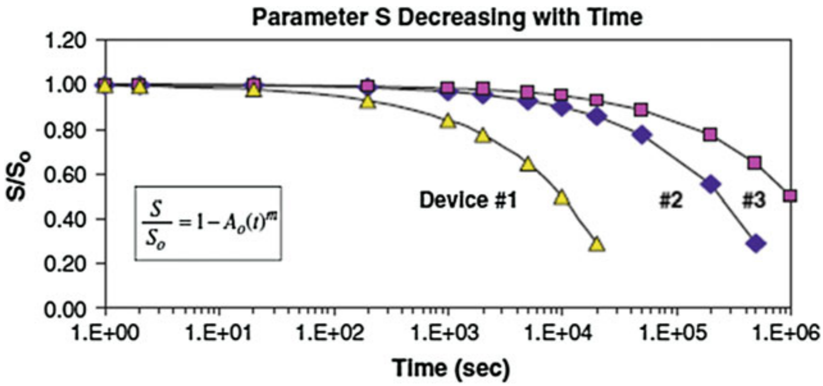

Fig. 1 Critical material/device parameter S is observed to reduce with time

$$S = S_0[1 \pm A_0(t)^m], \tag{2}$$

where A_0 is a material/device-dependent coefficient[2] and m is the power-law exponent. Both A_0 and m are adjustable parameters that can be extracted from observed parameter-degradation data. For $+A_0$, the observed parameter S increases monotonically with time, whereas for $-A_0$ it decreases. Either an increase in a critical device parameter S (increase in threshold voltage of a semiconductor device, increase in leakage of a capacitor, increase in resistance of a conductor, etc.) or a decrease in critical parameter S value (decrease of pressure in a vessel, decrease of spacing between mechanical components, decrease in lubricating properties of a fluid, etc.) can eventually lead to device failure. Since *device failure* can result from either increase or decrease of some critically important material/device parameter S, both cases are discussed.

1.1 Material/Device Parameter Decreases with Time

Shown in Fig. 1 is the observed time dependence of the degradation for a critical device parameter S for three devices.[3] Reduction in a critically important parameter S can be described by:

$$S = S_0[1 - A_0(t)^m]. \tag{3}$$

Taking the logarithm of both sides of Eq. (3) yields,

$$\ln(S^*) = m\ln(t) + \ln(A_0) \tag{4}$$

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[2] Note that A_0 must have the units of reciprocal-time to the mth power.

[3] The term *device* is very general: any apparatus that serves some useful purpose.

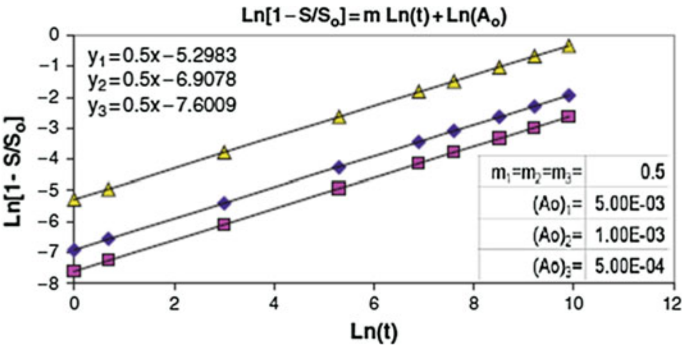


Fig. 2 Logarithmic plots reveal straight lines with equal slopes m (for the three devices) but each device has a different prefactor A_0 (A_0 is said to be materials/device dependent. A_0 variation will result in a distribution of degradations for the devices, as discussed in Chap. 6)

where:

$$S^* = 1 - \frac{S}{S_0} = \frac{S_0 - S}{S_0}. \tag{5}$$

Using Eq. (4), a logarithmic plot for the three devices shown in Fig. 1 is now re-plotted in Fig. 2. Note that the unknown parameters in Eq. (3) can now be easily extracted from such plots.

Example Problem 1

The threshold voltage V_{th} for a semiconductor device was found to degrade with time t, as indicated by the data in the table below.

Time: t (h)	V_{th} (V)
0	0.750
1	0.728
2	0.723
10	0.710

(a) Find the power-law exponent m which best describes the degradation of the threshold voltage V_{th} data versus time.
(b) Find the complete power-law equation which describes the shift in threshold voltage V_{th}.
(c) Estimate the value expected for the threshold voltage after 100 h.

Solution

(a) Inspecting the data, one can see that the device parameter V_{th} is decreasing with time. Thus, power-law model, Eq. (3), is used:

$$V_{\text{th}} = (V_{\text{th}})_0[1 - A_0(t)^m].$$

Rearranging, one obtains:

$$\frac{(V_{\text{th}})_0 - V_{\text{th}}}{(V_{\text{th}})_0} = A_0(t)^m.$$

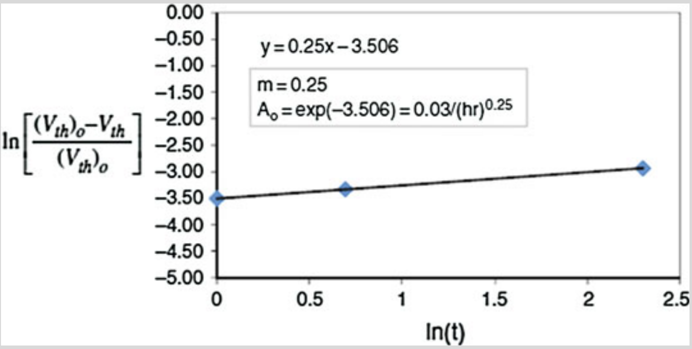
Taking the logarithm of both sides of the above equation, one obtains:

$$\ln\left[\frac{(V_{\text{th}})_0 - V_{\text{th}}}{(V_{\text{th}})_0}\right] = m\ln(t) + \ln(A_0).$$

Using the data in the above table, one can add useful columns to the table as shown below:

Time: t (h)	V_{th}(V)	$\frac{(V_{\text{th}})_O - V_{\text{th}}}{(V_{\text{th}})_O}$	$\ln(t)$	$\ln\left[\frac{(V_{\text{th}})_O - V_{\text{th}}}{(V_{\text{th}})_O}\right]$
0	0.750	0.000		
1	0.728	0.030	0	−3.51
2	0.723	0.036	0.693	−3.33
10	0.710	0.053	2.303	−2.93

Thus, plotting the values in the last two columns on the right of the table, one obtains:



(a) From the above plot, one can see that the slope (power-law exponent m) is given by: $m = 0.25$.

(b) Using Eq. (3), the threshold voltage Vth shift/degradation equation is given by:

$$V_{\text{th}} = (V_{\text{th}})_0(1 - A_0 t^m) = (0.75\,\text{V})\left[1 - \frac{0.03}{(\text{h})^{0.25}}(t)^{0.25}\right].$$

(c) The value of the threshold voltage V_{th}, after $t = 100$ h, is expected to be:

$$V_{th} = (0.75V)\left[1 - \frac{0.03}{(\mathrm{h})^{0.25}}(100\,\mathrm{h})^{0.25}\right] = 0.68\,\mathrm{V}.$$

1.2 Material/Device Parameter Increases with Time

As previously mentioned, degradation is not always associated with a decrease in a critical parameter S. Device failure can result from an increase in a critically important material/device parameter with the increase assumed to be described by:

$$S = S_0[1 + A_0(t)^m], \tag{6}$$

where A_0 is again a material/device-dependent coefficient and m is the time-dependence exponent. Shown in Fig. 3 is the time dependence for the degradation of three devices due to the increase in magnitude of the critical parameter S.

Taking the natural logarithm of both sides of Eq. (6) yields,

$$\ln(\mathrm{S}^*) = m\ln(t) + \ln(A_0), \tag{7}$$

where:

$$\mathrm{S}^* = \frac{S}{S_0} - 1 = \frac{S - S_0}{S_0}. \tag{8}$$

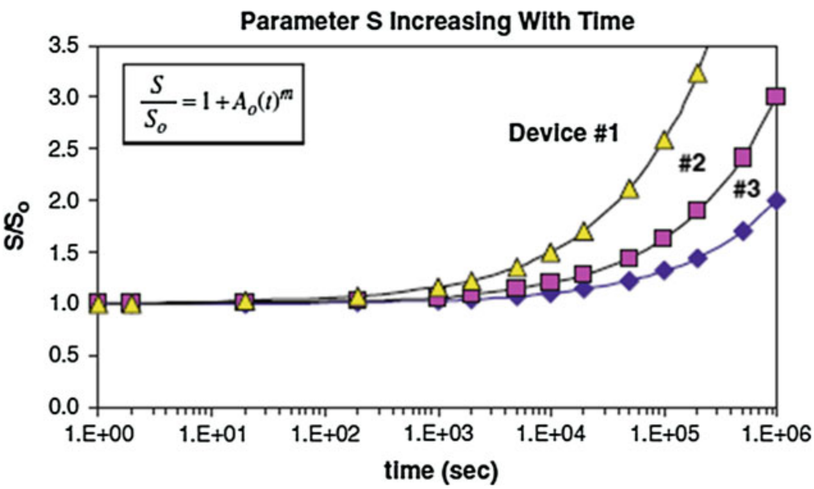


Fig. 3 Critical material/device parameter S is observed to degrade (*increase*) with time

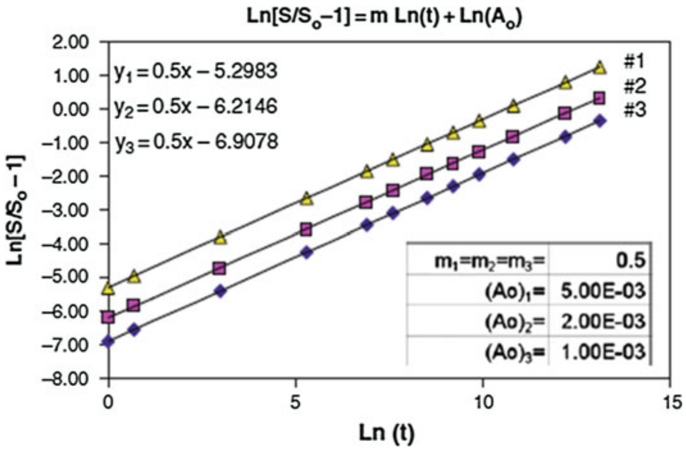


Fig. 4 Ln–Ln plots reveal straight lines with equal slopes m for the three devices, but each device has a different prefactor A_0

Using Eq. (7), the logarithmic plots for the three devices, with increasing critical parameter S as shown in Fig. 3, are now re-plotted in Fig. 4. Note that the unknown parameters in Eq. (6) can be easily extracted from such plots.

Example Problem 2

During a fatigue study, *crack propagation* occurred in a metal component. The crack size (CS) was observed to increase with the number of cyclical stress cycles N_{cyc}. The crack-propagation data is shown below in the table.

# Cycles: N_{cyc}	Crack size: CS (μm)
0	1
100	2
200	9
300	28

(a) Find the power-law exponent m which best describes the CS growth versus number of cycles N_{cyc}.
(b) Find the complete power-law equation which describes the CS versus N_{cyc}.
(c) What is the expected CS after 500 cycles?

Solution

(a) Inspecting the data, one can see that the CS is increasing with time. Thus, power-law Eq. (6) is used:

$$CS = (CS)_0[1 + A_0(N_{cyc})^m].$$

Rearranging, one obtains:

$$\frac{CS - (CS)_0}{(CS)_0} = A_0(N_{cyc})^m.$$

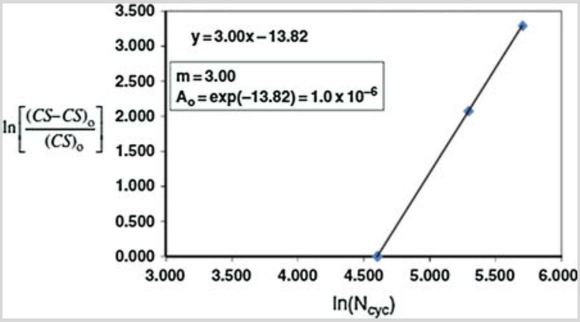
Taking the logarithm of both sides of the above equation, one obtains:

$$\ln\left[\frac{CS - (CS)_0}{(CS)_0}\right] = m\ln(N_{cyc}) + \ln(A_0).$$

Using the data in the above table, one can add useful columns to the table as shown in the table below.

# Cycles: N_{cyc}	Crack-size: CS (μm)	$\frac{CS-(CS)_0}{(CS)_0}$	$\ln(N_{cyc})$	$\ln\left[\frac{CS-(CS)_0}{(CS)_0}\right]$
0	1	0		
100	2	1	4.605	0.000
200	9	8	5.298	2.079
300	28	27	5.704	3.296

Plotting the values in the last two columns on the right of the above table, one obtains:



(a) From the above plot, one can see that the slope (power-law exponent m) is given by: $m = 3$

(b) The CS increase, Eq. (6), is given by:

$$CS = (CS)_0[1 + A_0(N_{cyc})^m] = (1\ \mu m)\left[1 + \frac{1 \times 10^{-6}}{(\text{cycle})^3}(N_{cyc})^3\right].$$

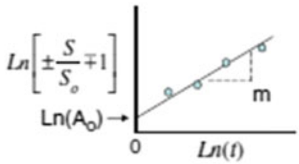
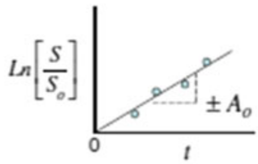
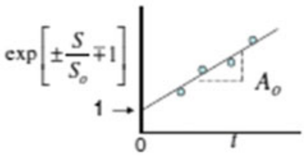
(c) The value of the CS after 500 cycles is expected to be:

$$\mathrm{CS} = (1\,\mu\mathrm{m})\left[1 + \frac{1 \times 10^{-6}}{(\mathrm{cycle})^3}(500\,\mathrm{cycles})^3\right] = 126\,\mu\mathrm{m}.$$

2 General Time-Dependent Degradation Models

There are many time-dependent forms for degradation. However, one of the following three forms is generally used: power-law, exponential, or logarithmic. These three forms were selected because they tend to occur rather frequently in nature. The power-law is clearly the more frequently used. If, however, a power-law model gives a rather poor fit to the degradation data, then perhaps the other two models should be investigated. The three degradation models are shown in Table 1, as well as how the model parameters can be easily extracted from the observed degradation data.

Table 1 Selected time-dependent degradation models

Power-Law $S = S_o[1 \pm A_o(t)^m]$	
Exponential $S = S_o \exp(\pm A_o t)$	
Logarithmic $S = S_o[1 \pm \ln(A_o t + 1)]$	

3 Degradation Rate Modeling

The power-law model is one of the most widely used forms for time-dependent degradation. For this reason, special attention is given to this model. For convenience of illustration, let us assume that the critical parameter S is decreasing with time and that $A_0 = 1$. Then Eq. (3) reduces to:

$$S^* = 1 - \frac{S}{S_0} = (t)^m. \tag{9}$$

In Fig. 5 one can see the usefulness and flexibility of the power-law time-dependent model. Note that for $m = 1$, one will see the expected linear degradation relationship. For $m < 1$, one can see the tendency for the degradation to saturate for long times. However, for $m > 1$, the degradation increases strongly with time and with no evidence of saturation effects.

The degradation rate is better emphasized when the actual degradation rate R equation is used:

$$R = \frac{dS^*}{dt} = m(t)^{m-1}. \tag{10}$$

The degradation rate, for several values of m, is shown in Fig. 6. Note that when $m = 1$, a constant degradation rate is expected. For $m < 1$, a decreasing degradation rate is expected. For $m > 1$, an increasing degradation rate is expected. For a decreasing degradation rate, there is at least some hope that the degradation may saturate before causing material/device failure. For a constant degradation rate, the time-to-failure is easily predicted. For an increasing degradation rate, the degradation is ever increasing, eventually leading to a catastrophic condition. Thus, of the three degradation rate conditions (decreasing, constant, increasing), each of which can produce failure, the increasing degradation rate is clearly the most worrisome.

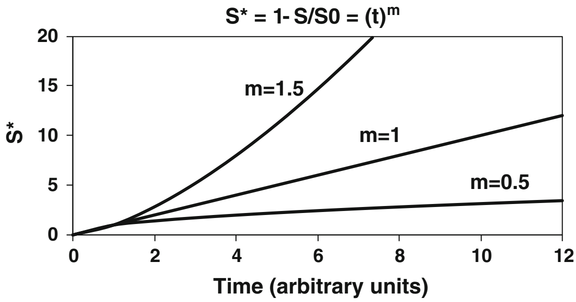


Fig. 5 Power-law time-dependent degradation model: (**a**) for $m = 1$, (**b**) $m < 1$, and (**c**) for $m > 1$

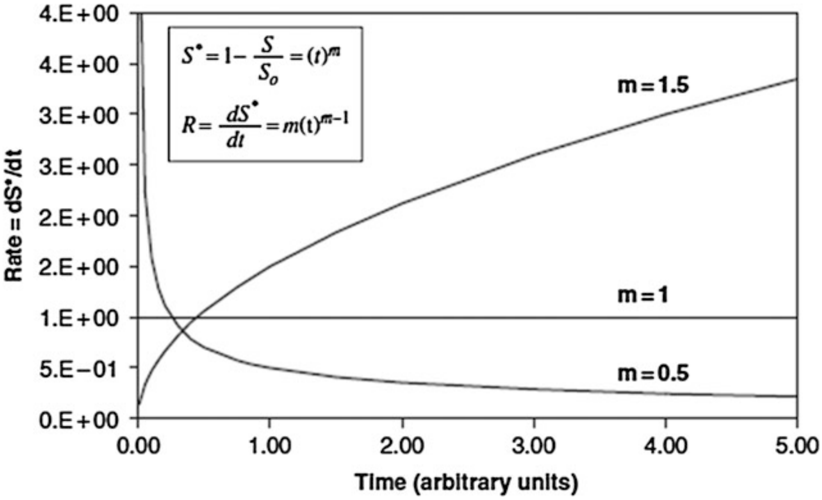


Fig. 6 Degradation rate as predicted by the power-law model: (**a**) $m = 1$, (**b**) $m < 1$, and (**c**) $m > 1$. One can see that $m = 1$ produces a constant degradation rate. $m < 1$ produces a decreasing degradation rate. Whereas, $m > 1$ produces an increasing degradation rate

Example Problem 3

(a) In Example Problem 1, it was determined that the threshold-voltage parameter for a semiconductor device was degrading (decreasing) with time. Is the *degradation rate*, for the threshold-voltage parameter V_{th}, increasing or decreasing with time?
(b) In Example Problem 2, it was determined that the CS parameter for a metal component was degrading (increasing) with the number of cyclical stress cycles. Is the *degradation rate*, for the CS parameter, increasing or decreasing with the number of cycles?

Solution

(a) It was determined, in Example Problem 1, that the threshold voltage V_{th} parameter was decreasing with time according to the equation:

$$V_{th} = (V_{th})_0(1 - A_0 t^m) = (0.75\,\mathrm{V})\left[1 - \frac{0.03}{(\mathrm{h})^{0.25}}(t)^{0.25}\right].$$

Since the exponent for the degradation is $m = 0.25$ (less than 1) then, according to Eq. (10), or Fig. 6, the *degradation rate* for the threshold-voltage parameter is decreasing with time.

(b) It was determined in Example Problem 2 that the CS parameter was increasing with time according to the equation:

$$\text{CS} = (\text{CS})_0\left[1 + A_0(N_{\text{cyc}})^m\right) = (1\,\mu\text{m})\left[1 + \frac{1\times10^{-6}}{(\text{cycle})^3}(N_{\text{cyc}})^3\right].$$

Since the exponent for the degradation is $m = 3$ (greater than 1), then according to Eq. (10), or Fig. 6, the *degradation rate* (crack growth rate) is increasing (in fact, strongly increasing) with time.

4 Delays in the Start of Degradation

Sometimes materials/devices will be remarkably stable for a period of time t_0 and then show relatively rapid degradation with time. Examples of this include: a tire that holds stable pressure until a nail punctures the tire; the resistance of a metal conductor is stable until a void starts to form; the fuel efficiency of an engine until the fuel injector starts to clog; an air-conditioner compressor that works fine until a leak in the coolant system develops; etc. Sometimes, it can be extremely important to be able to identify precisely when the degradation started.[4]

If a time-delay t_0 exists, before the start of degradation for the important material/device parameter S, then one can write the degradation equation as

$$\begin{aligned} &S = S_0 &&(\text{for}\, t \le t_0) \\ &S = S_0[1 \pm A_0(t - t_0)^m] &&(\text{for}\, t \ge t_0) \end{aligned}. \tag{11}$$

In the above equation, the + sign is used when S increases with time, whereas the − sign is used when S decreases with time. Equation (10) is very useful in determining the precise time that the instability started. The degradation rate R equation can be used to help pinpoint t_0, the time at which degradation actually started. Taking the derivative of Eq. (11) one obtains:

$$\begin{aligned} &R_1 = \frac{\mathrm{d}S}{\mathrm{d}t} = 0 &&(\text{for}\, t \le t_0) \\ &R_2 = \frac{\mathrm{d}S}{\mathrm{d}t} = (\pm)mS_0A_0(t - t_0)^{m-1} &&(\text{for}\, t \ge t_0) \end{aligned}. \tag{12}$$

Note that:

(a) If $m > 1$, then R_2 goes to zero at $t = t_0$;
(b) if $m = 1$, then R_2 is a constant; and
(c) if $m < 1$, then R_2 goes to infinity at $t = t_0$.

One can see from above equations that the rate/slope R of the degradation can be used to find the time delay t_0. If one plots the observed degradation rate R versus

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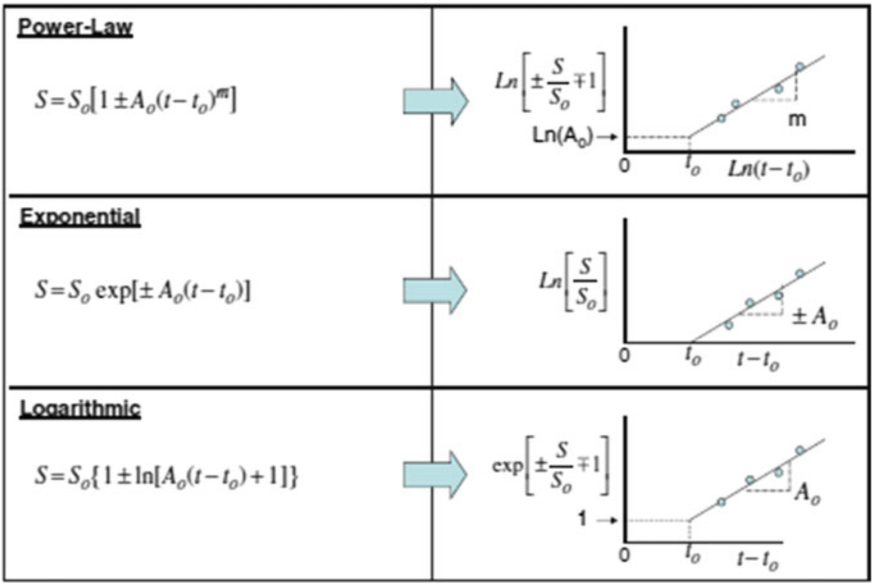
[4] If significant degradation (but not failure) started in the warranty period, does one have a claim? Sometimes, it can be very important to be able to identify the onset of degradation.

Table 2 Delayed start (t_0) degradation models



time t, then the time at which the rate R goes to zero, or R goes to infinity, is $t = t_0$. If R goes to zero, or infinity, at $t = 0$, then $t_0 = 0$ and a time delay is not needed in the degradation equation. The power-law model with a time delay t_0, as well as other models, are shown in Table 2.

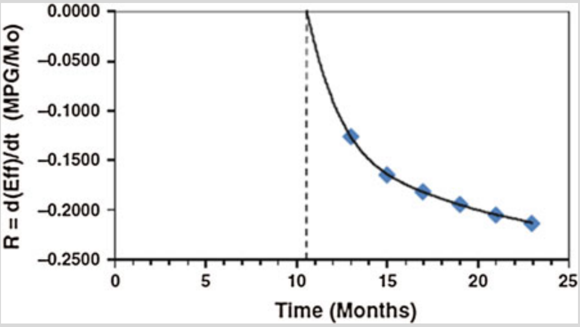
Example Problem 4

The fuel efficiency for a new auto remained very stable during the first 12 months of use. However, after about 1 year of use, a measureable degradation occurred in the efficiency (Eff) as shown in the table below.

Time (Mo)	Efficiency (MPG)	Time (Mo)	R = d(Eff)/dt (MPG/Mo)
0	22.00		
2	22.00		
4	22.00		
6	22.00		
8	22.00		
10	22.00		
12	22.00		
14	21.75	13	−0.1264
16	21.42	15	−0.1639
18	21.06	17	−0.1819
20	20.67	19	−0.1947
22	20.26	21	−0.2048
24	19.83	23	−0.2132

(a) Pinpoint the time t_0 that the degradation actually started.
(b) Determine the power-law equation which best fits the efficiency versus time for the full 24 months of use.

Solution
The observed degradation rate R is shown in the graph below.



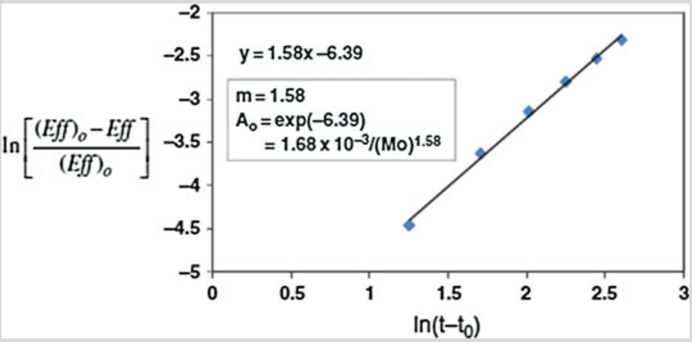
(a) One can see from the above plot of degradation rate R versus time that degradation rate R goes to zero at $t = t_0 = 10.5$ months.[5]
(b) Now that the value of the time-delay $t_0 = 10.5$ months has been determined, then one can proceed with finding the best fitting parameters (m, A_0) as follows:

$$\mathrm{Eff} = (\mathrm{Eff})_o[1 - A_0(t - 10.5\mathrm{Mo})^m] \quad (\text{for}\, t \geq 10.5\mathrm{Mo})\,.$$

Rearranging and taking the logarithm of both sides, one obtains:

$$\ln\left[\frac{(\mathrm{Eff})_0 - \mathrm{Eff}}{(\mathrm{Eff})_0}\right] = m\ln(t - 10.5) + \ln(A_0).$$

The plot of the data is shown in the graph below.

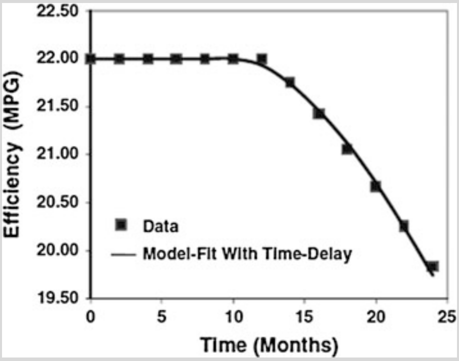


[5] Note that even though the degradation started at 10.5 months, the degradation at 12 months is so small that it went undetected by the measuring instrument.

Therefore, the power-law equation, with time delay, that best fits the fuel efficiency data is:

$$\begin{aligned} &\mathrm{Eff} = 22.0 && (\text{for}\, t \le 10.5\mathrm{Mo}) \\ &\mathrm{Eff} = (22.0)\left[1 - \frac{1.68 \times 10^{-3}}{(\mathrm{Mo})^{1.58}}(t - 10.5\mathrm{Mo})^{1.58}\right] && (\text{for}\, t \ge 10.5\mathrm{Mo}) \end{aligned}.$$

The plot of the data, and the modeled fit to the data, are shown in the graph below.



Example Problem 5

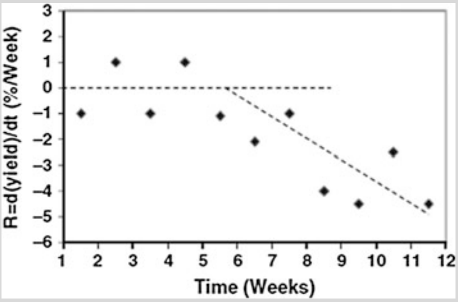
In semiconductor processing, yield (number of electrically good chips on a Si-wafer divided by the total number of chips on a wafer) is a key manufacturing parameter. The yield data is shown below for several weeks. Using the yield-degradation rate, pinpoint when the yield degradation started.

Time (Week)	Yield (%)
1	68.2
2	67.2
3	68.2
4	67.2
5	68.2
6	67.1
7	65.0
8	64.0
9	60.0
10	55.5
11	53.0
12	48.5

Solution
To find the rate of yield degradation, additional columns are added to the table as shown below.

Time (Wk)	Yield (%)	Time (Wk)	R = d(Yield)/dt (%/Wk)
1	68.2		
2	67.2	1.5	−1.00
3	68.2	2.5	1.00
4	67.2	3.5	−1.00
5	68.2	4.5	1.00
6	67.1	5.5	−1.10
7	65.0	6.5	−2.10
8	64.0	7.5	−1.00
9	60.0	8.5	−4.00
10	55.5	9.5	−4.50
11	53.0	10.5	−2.50
12	48.5	11.5	−4.50

The plot of the yield-degradation rate is shown below.



One can see that while the degradation rate shows fluctuation from week to week, the average degradation rate was nearly constant through the first five weeks. Between weeks 5 and 7, the average degradation rate changed with time. This helps to pinpoint the time that some process step(s) started to go out of control.

5 Competing Degradation Mechanisms

Competing mechanisms can also occur (one mechanism is driving an increase in the critical parameter S while the other mechanism is driving a reduction in S). This can be described by the equation:

$$S = S_0[1 + A_0(t)^{m_1}][1 - B_0(t)^{m_2}], \tag{13}$$

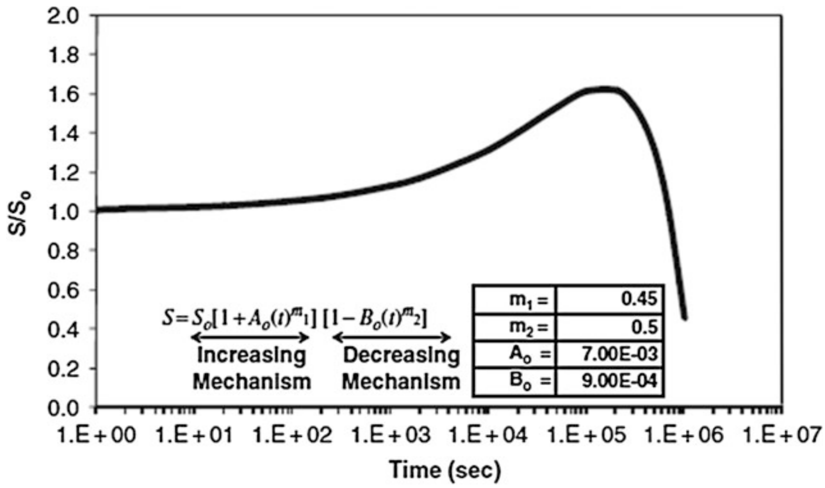


Fig. 7 Maximum (or minimum) in the degradation parameter S/S_0 is generally indicative of competing mechanisms: one mechanism driving an increase in S and the other driving a decrease in S

Table 3 Model parameter extraction method (competing mechanisms)

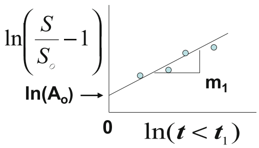
$$S = S_o[1 + \underbrace{A_o(t)^{m_1}}_{\text{Increasing Term}}][1 - \underbrace{B_o(t)^{m_2}}_{\text{Decreasing Term}}]$$

Limiting Conditions:

1. <u>During Early Stages</u> ($t<t_1$):

$$S \cong S_o[1 + A_o(t)^{m_1}]$$

$$\Rightarrow \ln\left(\frac{S}{S_o} - 1\right) = \ln(A_o) + m_1 \ln(t)$$

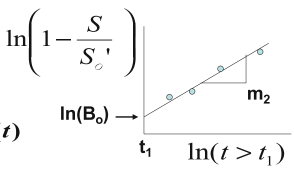


2. <u>During Later Stages</u> ($t>t_1$)

$$S \cong S_o'[1 - B_o(t)^{m_2}]$$

$$where: S_o' = S_o[1 + A_o(t)^{m_1}]$$

$$\Rightarrow \ln\left(1 - \frac{S}{S_o'}\right) = \ln(B_o) + m_2 \ln(t)$$



where the first term on the right of the above equation is tending to increase the parameter S while the second term on the right is trying to decrease the parameter S. These mechanisms are competing and can produce either a maximum or minimum in the degradation, as illustrated in Fig. 7.

In Fig. 7, a maximum occurs in the critical parameter S/S_0 due to the dominance of the increasing mechanism initially, then due to the dominance of *decreasing* *mechanism* during the later stages.[6] If the roles are reversed, the decreasing term dominates initially, but the increasing term dominates during the later stages of parameter degradation, then a minimum will be observed.

Table 3 indicates a method that is sometimes useful in separating the problem into early stages of degradation versus the later stages of degradation. If the strengthening term dominates the early stages while the later stages are dominated by the weakening term, then the model parameters can easily be extracted.

Problems

1. The threshold voltage V_{th} for a semicondutor device was observed to degrade with time. The degradation data is shown in the table.

Time (h)	V_{th} (V)
0	0.40
1	0.42
10	0.44
100	0.48

(a) Find the power-law equation which best fits the threshold voltage V_{th} versus time data.
(b) What is the expected value of the threshold voltage V_{th} after 1,000 h?
(c) Is the *degradation rate* increasing or decreasing with time?

Answers:

a) $V_{th} = 0.40V\left[1 + \frac{0.05}{(\mathrm{h})^{0.3}}(t)^{0.3}\right]$

b) $V_{th}(t = 1{,}000 \text{ h}) = 0.56\ V$
c) Since $m = 0.3(<1)$, then degradation rate is decreasing with time.

[6] An example of competing mechanism comes from the joining/bonding of dissimilar materials. During the bonding of dissimilar metals at high temperatures, interdiffusion of the two materials is usually required in order to establish good bonding. Initially, this interdiffusion of materials will cause an increase in bonding strength. However, often during the later stages of interdiffusion, the bond strength can start to weaken due to Kirkendall voiding.

2. The pressure P in a tire was found to degrade with time according to the table shown.

Time (day)	P (lb/in^2)
0	32.00
1	30.72
2	30.06
3	29.53

(a) Find the power-law equation which best-fits the pressure P versus time data.
(b) What is the expected value of the pressure P after 10 days?
(c) Is the *degradation rate* for the pressure P increasing or decreasing with time?

Answers:

(a) $$P = 32(\text{lb/in}^2)\left[1 - \frac{0.04}{(\text{day})^{0.6}}(t)^{0.6}\right]$$

(b) $P(t = 10\ \text{day}) = 26.9\ \text{lb/in}^2$
(c) Since $m = 0.6(<1)$, then *degradation rate* is decreasing with time.

3. As current flowed through a precision resistor, it was noted that the value of resistance R for the resistor degrades with time according to the data in the table.

Time (h)	R (ohm)
0	10.00
1	10.02
5	10.22
10	10.63

(a) Find the power-law equation which best fits the resistance R versus time.
(b) What is the expected value of the resistance R after 100 h?
(c) Is the *degradation rate* for the resistance R increasing or decreasing with time?

Answers:

a) $$R = 10.00(\Omega)\left[1 + \frac{0.002}{(\text{h})^{1.5}}(t)^{1.5}\right]$$

b) $R(t = 100\ \text{h}) = 30.00\ \Omega$
c) Since $m = 1.5(>1)$, then *degradation rate* is increasing with time.

4. A metal component was corroding/oxidizing with time. The metal-oxide thickness with time is shown in the table.

Time (year)	Oxide thickness T_{ox}(μm)
0	1.00
1	1.90
2	2.27
3	2.56

(a) Find the power-law equation which best fits the oxide thickness T_{ox} versus time data.
(b) What is the expected value of the oxide thickness T_{ox} after 10 years?
(c) Is the *degradation rate* for the oxide thickness T_{ox} increasing or decreasing with time?

Answers:

(a) $T_{ox} = 1.00\,\mu\text{m}\left[1 + \frac{0.9}{(\text{year})^{0.5}}(t)^{0.5}\right]$

(b) T_{ox} $(t = 10\text{ year}) = 3.85\ \mu\text{m}$
(c) Since $m = 0.5(<1)$, then *degradation rate* is decreasing with time.

5. The prostate-specific antigen (PSA) test is routinely used to detect the possibility of prostate cancer. The absolute level of the PSA is expected to be <4.0 ngm/ml, but the rate of change is also important. Below are the hypothetical PSA levels for a patient over a 3-year period. The absolute PSA level is less than 4.0 ngm/ml, but is the rate a concern?
(a) Find the power-law model which best fits the increase in PSA versus time data.
(b) Is the increase in PSA occurring at an increasing or decreasing rate?

Time (year)	PSA (ngm/ml)
0	1
1	1.1
2	1.4
3	1.9

Answers:

a) $\text{PSA} = (\text{PSA})_0[1 + A_0(t)^m] = (1.0\,\text{ngm/ml})\left[1 + \frac{0.1}{(\text{year})^2}(t)^2\right]$

b) Since $m = 2$ (>1), the rate of increase for the PSA is very strong and should be noted to the physician.

6. For our nervous system to work properly, the nerve cell must be able to generate a potential difference of about 50 mV. This is done through the differential diffusion rates of sodium (Na-ions) and potassium (K-ions). The ratio of the Na to K in our blood is typically (Na/K) = 31.93. If this ratio drops to 25.47, then health issues can sometimes occur.

Time (year)	(Na/K) ratio
0	31.93
1	31.61
2	31.51
3	31.43

(a) Find the power-law model which best fits the reduction in (Na/K) ratio versus time data.
(b) Is the decrease of the (Na/K) ratio occurring at an increasing or decreasing rate?

Answers:

a) $$\left(\frac{\text{Na}}{\text{K}}\right) = \left(\frac{\text{Na}}{\text{K}}\right)_0 [1 - A_0 t^m] = (31.93)\left[1 - \frac{0.01}{(\text{year})^{0.4}}(t)^{0.4}\right]$$

b) Since $m = 0.4$ (<1), the rate of reduction is decreasing with time.

7. The size of an inoperable brain tumor was monitored for 3 months preceding the use of an experimental drug and for 3 months post drug use. The data are shown below.
(a) What is the power-law equation that describes tumor growth prior to experimental drug use?
(b) What is the power-law equation that describes tumor growth versus time after experimental drug use?
(c) Take the ratio of the two growth rates to see if the experimental drug was effective at reducing the tumor growth rate.

Time (Mo)	Tumor size: S (cm)
0	1.00
1	1.10
2	1.20
3	1.30
Drug Introduction	
3	1.30
4	1.43
5	1.48
6	1.52

Answers:

(a)
$$S_{\text{before drug}} = (1.00\text{cm})\left[1 + \frac{0.1}{(\text{Mo})}t\right] \quad (t \leq 3\text{Mo})$$

(b)
$$S_{\text{after drug}} = (1.30\,\text{cm})\left[1 + \frac{0.1}{(\text{Mo})^{0.5}}(t - 3\text{Mo})^{0.5}\right] \quad (t \geq 3\text{Mo})$$

(c)
$$\frac{R_{\text{after}}}{R_{\text{before}}} = \frac{\text{d}S_{\text{after}}/\text{d}t}{\text{d}S_{\text{before}}/\text{d}t} = \frac{0.65(\text{Mo})^{0.5}}{(t - 3\text{Mo})^{0.5}} \quad (t > 3\text{Mo})$$

Note that in the 4th month, after the drug was introduced, the tumor growth rate was 65 % of what it would have been without the drug. In the 5th month, the tumor growth rate was 46 % of what it would have been if no drug was introduced.

8. The pressure P of a toxic gas, in a very large storage vessel, was monitored every month during its 12-month storage and the results are shown below.
(a) Pinpoint the month that a leak started to occur, causing a gradual release of the gas.
(b) What is the power-law equation that best fits the degradation data?
(c) Children, in a nearby school, had a mysterious illness in month 3. Could this have been due to the gas leak?

Time (Mo)	Pressure: P (atm)
0	5.0
1	5.0
2	5.0
3	5.0
4	5.0
5	5.0
6	5.0
7	5.0
8	4.9
9	4.7
10	4.4
11	4.0
12	3.5

Answers:

(a) $t_0 = 6.6$ Months.

(b)
$$P = 5.0\,\text{atm} \qquad (t \leq 6.6\,\text{Months})$$
$$P = 5.0\,\text{atm}\left[1 - \frac{1.03 \times 10^{-2}}{(\text{Mo})^{2.0}}(t - 6.6\text{Mo})^{2.0}\right] \quad (t \geq 6.6\,\text{Months})$$

(c) The gas leak did not start until month 6.6. The illness of the children at the local school occurred in month 3.

9. Nuclear decay from a radioactive material exhibited the decay characteristics:

$$\frac{N}{N_0} = \exp\left[-\left(\frac{6.93 \times 10^{-3}}{\text{h}}\right)t\right]$$

(a) Plot the exponential decay function through the first 100 h.
(b) Find the best fitting power-law model to this exponential function through the first 100 h.
(c) Plot both the exponential and the best fitting power-law model and compare the plots through 100 h.

Answer:

$$\frac{N}{N_0} = 1 - \frac{0.00941}{(\text{h})^{0.871}}(t)^{0.871}$$

10. In semiconductor processing, yield (number of electrically good chips on a wafer divided by the total number of chips on a wafer) is a key manufacturing parameter. The yield data is shown below for several weeks. Using the yield-degradation rate, when did the yield start to degrade?

Time (Wk)	Yield (%)
1	52.1
2	52.6
3	52.7
4	51.6
5	52.2

(continued)

(continued)

Time (Wk)	Yield (%)
6	51.7
7	52.2
8	51.9
9	51.3
10	50.4
11	49.2
12	47.7

Answer: Yield started to degrade between weeks 6 and 8.

11. Thermo-sonic Au ball-bonding to aluminum pads is a common attachment process for silicon chips. If these bonds are stored at high temperatures (>150 °C), one can observe competing mechanisms: interdiffusion of the two elements tends to strengthen the bonds initially but Kirkendall voiding tends to weaken the bonds during longer storage times. The bond strength *S* data is shown versus the storage time at high temperature in the below table.

Determine the degradation equation for the ball bonds shown.

Time (s)	Bond strength: *S* (gm-f)
0.00E+00	20.00
1.00E+00	20.01
1.00E+01	20.03
1.00E+02	20.10
1.00E+03	20.31
1.00E+04	20.90
1.00E+05	22.16
2.00E+05	22.47
4.00E+05	22.32
6.00E+05	21.75
8.00E+05	20.94
1.00E+06	20.00
2.00E+06	14.14
2.20E+06	12.83
2.30E+06	12.17
2.40E+06	11.49
2.50E+06	10.81
2.60E+06	10.12
2.70E+06	9.43
2.80E+06	8.73
3.00E+06	7.32
4.00E+06	0.00

Answer:

$$S = (20.00\,\text{gm} \cdot f)\left[1 + \frac{1 \times 10^{-3}}{(\text{sec})^{0.5}}(t)^{0.5}\right]\left[1 - \frac{5 \times 10^{-4}}{(\text{sec})^{0.5}}(t)^{0.5}\right]$$

12. A metal-oxide thickness T_{ox} was found to take a logarithmic growth functional form:

$$\frac{T_{ox}}{(T_{ox})_0} = 1 + \ln\left[\left(\frac{1 \times 10^{-2}}{\text{h}}\right)t + 1\right]$$

(a) Plot the logarithmic growth function through the first 100 h.
(b) Find the best fitting power-law model to this logarithmic growth function through the first 100 h.
(c) Plot both the logarithmic and the best fitting power-law model and compare the fits through the first 100 h.

Answer:

(b) $\dfrac{T_{ox}}{(T_{ox})_0} = 1 + \dfrac{0.0138}{(\text{h})^{0.858}}\,t^{0.858}$

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