Preface

Let $n \geqslant 4$, and let $B_n(\mathbb{S}^2)$ denote the n-string braid group of the sphere. In [1], we showed that the isomorphism classes of the maximal finite subgroups of $B_n(\mathbb{S}^2)$ comprise cyclic, dicyclic (or generalised quaternion) and binary polyhedral groups. In this book, we study the infinite virtually cyclic groups of $B_n(\mathbb{S}^2)$, which are in some sense, its 'simplest' infinite subgroups. As well as helping to understand the structure of the group $B_n(\mathbb{S}^2)$, the knowledge of its virtually cyclic subgroups is a vital step in the calculation of the lower algebraic K-theory of the group ring of $B_n(\mathbb{S}^2)$ over $\mathbb{Z}$, via the Farrell-Jones fibred isomorphism conjecture [2].

The main result of this manuscript is to classify, with a finite number of exceptions and up to isomorphism, the virtually cyclic subgroups of $B_n(\mathbb{S}^2)$. As corollaries, we obtain the complete classification of the virtually cyclic subgroups of $B_n(\mathbb{S}^2)$ when n is either odd, or even and sufficiently large. Using the close relationship between $B_n(\mathbb{S}^2)$ and the mapping class group $\mathcal{MCG}(\mathbb{S}^2, n)$ of the n-punctured sphere, another consequence is the classification (with a finite number of exceptions) of the isomorphism classes of the virtually cyclic subgroups of $\mathcal{MCG}(\mathbb{S}^2, n)$.

The proof of the main theorem is divided into two parts: the reduction of a list of possible candidates for the virtually cyclic subgroups of $B_n(\mathbb{S}^2)$ obtained using a general result due to Epstein and Wall to an almost optimal family $\mathbb{V}(n)$ of virtually cyclic groups; and the realisation of all but a finite number of elements of $\mathbb{V}(n)$. The first part makes use of a number of techniques, notably the study of the periodicity and the outer automorphism groups of the finite subgroups of $B_n(\mathbb{S}^2)$, and the analysis of the conjugacy classes of the finite order elements of $B_n(\mathbb{S}^2)$. In the second part, we construct subgroups of $B_n(\mathbb{S}^2)$ isomorphic to the elements of $\mathbb{V}(n)$ using mainly an algebraic point of view that is strongly inspired by geometric observations, as well as explicit geometric constructions in $\mathcal{MCG}(\mathbb{S}^2, n)$ which we translate to $B_n(\mathbb{S}^2)$.

In order to classify the isomorphism classes of the virtually cyclic subgroups of $B_n(\mathbb{S}^2)$, we obtain a number of results that we believe are interesting in their own right, notably the characterisation of the centralisers and normalisers of the maximal cyclic and dicyclic subgroups of $B_n(\mathbb{S}^2)$, a generalisation to $B_n(\mathbb{S}^2)$ of a result due to Hodgkin for the mapping class group of the punctured sphere concerning conjugate powers of torsion elements, the study of the isomorphism classes of those virtually cyclic groups of $B_n(\mathbb{S}^2)$ that appear as amalgamated products, as well as an alternative proof of a result due to [3, 4] that the universal covering of the nth configuration space of $\mathbb{S}^2, n \geqslant 3$, has the homotopy type of $\mathbb{S}^3$.

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http://www.springer.com/978-3-319-00256-9

The Classification of the Virtually Cyclic Subgroups of
the Sphere Braid Groups
Lima Goncalves, D.; Guaschi, J.
2013, X, 102 p. 26 illus., Softcover
ISBN: 978-3-319-00256-9