# Chapter 2
# An Introduction to Field Theory in Magnetic Systems: The Néel Antiferromagnet

The purpose of this chapter is to introduce the main theoretical tool used in this thesis, continuum field theories of magnetic systems that undergo a phase transition into an ordered state. This is a symmetry-based approach, and, in consequence, the same continuum theory can be adapted to describe materials with vastly different microscopic structure, as long as the symmetries broken by the order parameter are the same. Since it is useful to have a physical system in mind when describing these theories, we will concentrate on the 2-dimensional, square-lattice Néel antiferromagnet realised in $La_2CuO_4$. The theories considered include both the non-linear sigma model and the Ginzburg-Landau theory of critical phenomena, along with its extension to include critical dynamics. Before embarking on a discussion of the field theory, there is a brief introduction to spin-wave theory, and how this can be used to understand the low temperature ordered phase of Néel antiferromagnets such as $La_2CuO_4$.

## 2.1 $La_2CuO_4$, the Néel Antiferromagnet and Spin-Wave Theory

Spin models such as $\mathcal{H}_{\text{Heis}}$ [Eq. (1.1)] provide an excellent description of the magnetic states stabilised in Mott insulators at half filling. A good example to keep in mind is the Mott insulator $La_2CuO_4$, which realises a Néel antiferromagnetic order at low temperature [1–5], as shown in Fig. (2.1). In $La_2CuO_4$ a single hole in the $Cu^{2+}$ 3d electron shell acts as a spin-1/2 moment, and superexchange via the $O^{2-}$ ions leads to a significant exchange coupling within the CuO planes. Long-range order is stabilised by a much smaller interplane exchange interaction [6, 7]. The low-temperature, spin-wave excitations are well modelled by the square lattice, nearest-neighbour Heisenberg model $\mathcal{H}_{\text{Heis}}$ [Eq. (1.1)] [2, 3], although for a detailed description of the excitation spectrum it is also necessary to include ring and further-neighbour exchange interactions [4]. As a note of caution, it appears that the spin-waves decay at high energy, possibly into spinon pairs [5].

A. Smerald, *Theory of the Nuclear Magnetic 1/$T_1$ Relaxation Rate in Conventional and Unconventional Magnets*, Springer Theses, DOI: 10.1007/978-3-319-00434-1_2,

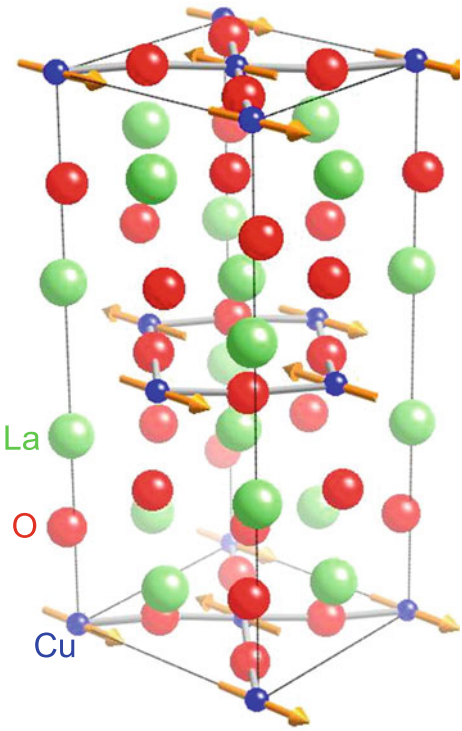


**Fig. 2.1** The crystal structure and magnetic ordering of $La_2CuO_4$. Covalently bonded planes of CuO (Cu *blue*, O *red*) are ionically linked by sheets of La (*green*). Spin-1/2 moments are associated with $Cu^{2+}$ ions and these order antiferromagnetically for $T < T_N \approx 200\,\mathrm{K}$ [1–3]

The classical picture of the Néel antiferromagnetic state realised in $La_2CuO_4$ involves an alternation of 'up' and 'down' spins in a 'g-type' order, shown in Fig. (2.1). The two sublattices can be labelled A and B, and the order parameter is given by,

$$\mathbf{n} = \frac{2}{N}\sum_{i\in\mathsf{A}} \mathbf{S}_i - \frac{2}{N}\sum_{i\in\mathsf{B}} \mathbf{S}_i\,, \tag{2.1}$$

where $N$ is the total number of lattice sites and $\mathbf{n}$ is the staggered magnetisation. However, this classical picture of alternating spins *cannot* be the true ground state of the system, as it is not a $T = 0$ eigenstate of $\mathcal{H}_{\mathsf{Heis}}$ [Eq. (1.1)]. Nevertheless, in the thermodynamic limit the Néel state is approximately realised, and is formed from the superposition of degenerate eigenstates with different total spin moments (see the notes by Lhuillier [8] for a detailed analysis of this point). Even at $T = 0$, the order parameter is reduced from its classical Néel value by quantum fluctuations, and thermal fluctuations further reduce its value at finite $T$.

Spin-wave theory provides a useful, theoretical tool for understanding the magnetic excitation spectrum of low-temperature, ordered magnets [9, 10]. It involves a systematic expansion of the Hamiltonian around the classical Néel ordered state. Quantum and thermal fluctuations are parametrised in terms of bosons, which are

created by the operator $a^\dagger$. The small parameter involved in the expansion is $a^\dagger a/S$, and spin-wave theory is thus valid when the average boson density is small compared to the spin moment.

There are a number of ways of calculating the magnetic excitation spectrum using spin-wave theory. For Néel ordering parallel to the $z$-direction, the simplest is probably to rotate the 'down' sublattice by $\pi$ around the $x$-axis. This results in the 'ferromagnetic' formulation of the Hamiltonian,

$$\mathcal{H}_{\text{Heis}} = -\frac{J}{2}\sum_{i,\delta} S_i^z S_{i+\delta}^z - \frac{1}{2}\left(S_i^+ S_{i+\delta}^+ + S_i^- S_{i+\delta}^-\right), \tag{2.2}$$

where $\delta$ counts nearest-neighbour bonds and $S^\pm = S^x \pm iS^y$. The spin operators can be rewritten using the Holstein-Primakoff boson transformations,

$$S^z = S - a^\dagger a \ , \quad S^+ = \sqrt{2S - a^\dagger a}\ a \ \approx \sqrt{2S}\, a \ , \quad S^- = a^\dagger \sqrt{2S - a^\dagger a}\ \approx \sqrt{2S} a^\dagger . \tag{2.3}$$

The simplest approximation involves retaining terms at first order in $a^\dagger a/S$, and this is known as linear spin-wave theory. Substituting Eq. (2.3) into Eq. (2.2), making the linear approximation, taking the Fourier transform and diagonalising the resulting Hamiltonian while preserving the boson commutation relations results in the dispersion relation,

$$\omega_{\mathbf{q}} \approx JzS\sqrt{1-\gamma_{\mathbf{q}}^2}, \tag{2.4}$$

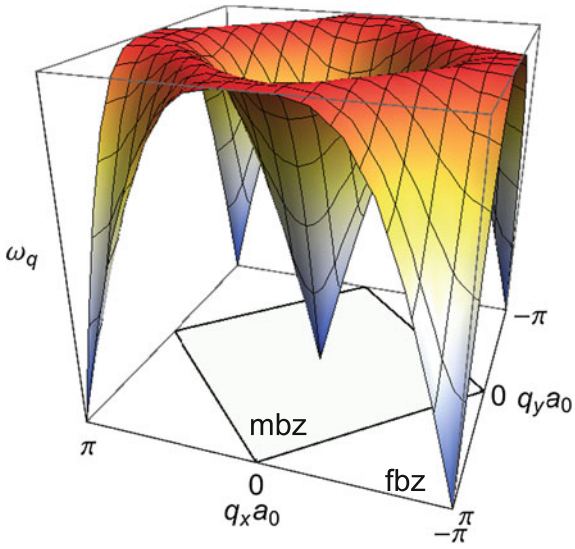


**Fig. 2.2** Linear spin-wave theory predictions for the dispersion of magnetic excitations in the Néel antiferromagnetic state of the Heisenberg model on the square lattice, $\mathcal{H}_{\text{Heis}}$, Eq. (1.1). The dispersion, Eq. (2.4), is plotted in the full Brillouin zone (fbz) of the paramagnet. The magnetic Brilliouin zone (mbz) for 2-sublattice order is shown in green. If plotted entirely within the mbz, the dispersion is twofold degenerate

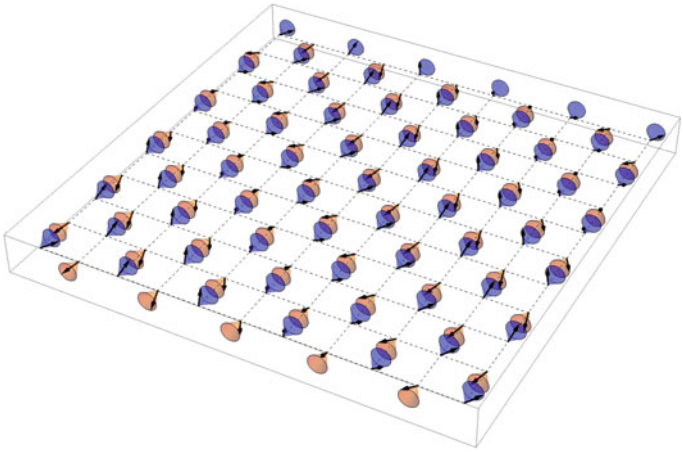

**Fig. 2.3** Spin-waves in a Néel antiferromagnet. Both $\mathcal{H}_{\text{Heis}}$ [Eq. (1.1)] and the non-linear sigma model action [Eq. (2.27)] describe the excitation of coherent spin excitations in the ordered state of the Néel antiferromagnet. The spin moments can be represented semiclassically by staggered *arrows*, and the wavevector, **k**, is chosen such that a single wavelength fits inside the bounding box. The figure shows a freeze-frame of the precessional motion, in which the spins move on the surface of the cones

where, for the square lattice, $z = 4$ is the coordination number and $\gamma_{\mathbf{q}} = 0.5(\cos q_x a_0 + \cos q_y a_0)$, with $a_0$ the lattice constant. The dispersion of magnetic excitations is plotted in Fig. (2.2). At low energies $\omega_{\mathbf{q}} \approx v|\mathbf{q}-\mathbf{q}^*|$, where $v = 2\sqrt{2}JSa_0$ is the spin-wave velocity and $\mathbf{q}^* = (0, 0)$ or $\mathbf{q}^* = (\pi, \pi)$. A semiclassical illustration of a spin wave is shown in Fig. (2.3).

## 2.2 The Quantum Non-Linear Sigma Model as a Description of the Néel Antiferromagnet

As a useful introduction to the methods employed in this thesis, we show how the non-linear sigma model (NLSM) field theory arises from the nearest-neighbour Heisenberg model on the square lattice. We will find that at low temperatures and energies the field theory makes the same predictions as spin-wave theory for the dispersion of magnetic excitations. The derivation is relatively well documented, for example see the books by Fradkin [11], Auerbach [12] and Zinn-Justin [13] or the papers by Chakravarty, Halperin and Nelson [7] and Allen and Loss [14]. References [7] and [13] also contain a detailed description of the renormalisation procedure, which will not be presented here.

The NLSM field theory captures the low-energy spin-wave excitations of materials like $La_2CuO_4$, and can be used to understand the critical fluctuations associated with long-range order for a wide range of temperatures. While we will derive the

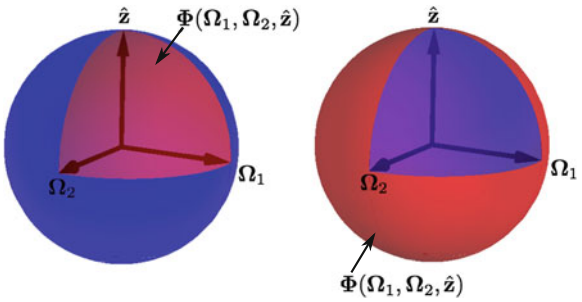


**Fig. 2.4** The spin-coherent state is the quantum mechanical representation of spin that most closely resembles a classical vector on a sphere. It is formed by applying the SU(2) rotation matrix to the maximally polarised $S^z$ eigenstate, Eq. (2.5). Coherent states are parametrised by the spherical-polar angles $(\theta, \phi)$. Two spin coherent states, $\mathbf{\Omega}_1$ and $\mathbf{\Omega}_2$, form a spherical triangle when combined with the $z$-axis, $\hat{\mathbf{z}}$. This spherical triangle has area $\Phi(\mathbf{\Omega}_1, \mathbf{\Omega}_2, \hat{\mathbf{z}})$. The area is defined modulo $4\pi$, since the three vectors define two possible spherical *triangles* (shown in *red*). This ambiguity in the definition of the area is the origin of spin quantisation

NLSM from the square-lattice, nearest-neighbour Heisenberg model, the power of field theory is that it will describe the low-energy excitations of any system with the same order parameter, whether insulating or metallic. In order for the NLSM to be valid, the correlation length, $\xi$, which is the lengthscale over which the order parameter is well defined, must be large compared to the lattice constant, $a_0$. When this assumption fails, which occurs at high energies or temperatures, the field theory is no longer valid.

The action of the NLSM is derived from $\mathcal{H}_{\mathsf{Heis}}$, Eq. (1.1), by forming a path integral for the partition function using spin-coherent states. The SU(2) spin-coherent state representation [see Fig. (2.4)] is the way of representing quantum mechanical spin that most resembles the classical notion of an $SO(3)$ symmetric vector on a sphere. The states are created by applying the $SU(2)$ rotation operator, $\mathcal{R}(\chi, \theta, \phi)$, to the maximally polarised spin state in the basis where $S^z$ is diagonal [12],

$$|\mathbf{\Omega}(\chi, \theta, \phi)\rangle = \mathcal{R}(\chi, \theta, \phi)|S, S\rangle = e^{iS^z\phi}e^{iS^y\theta}e^{iS^z\chi}|S, S\rangle, \tag{2.5}$$

where $\chi$ is an (often unimportant) phase factor and the angles $(\theta, \phi)$ define a familiar spherical-polar coordinate system. After fixing the value of $\chi$, the spin-coherent state is parametrised by the angles $(\theta, \phi)$, and $\mathbf{\Omega} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ is simply a vector of fixed origin that sweeps out a unit sphere. The angles $(\theta, \phi)$ are continuous variables, and the spin-coherent states therefore define an overcomplete basis, since a finite set of states in the $S^z$ basis has been mapped onto an infinite set of spin-coherent states. The spin coherent states can also be expanded in terms of the complete set of $S^z$ eigenstates as [11],

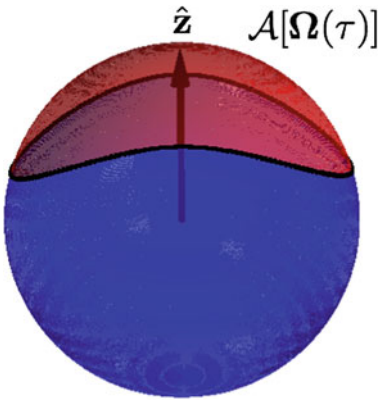


**Fig. 2.5** A closed orbit of spin-coherent states on the surface of the sphere. The spin-coherent state basis is overcomplete and is not orthogonal [see Eq. (2.9)]. In consequence, at each imaginary time step in the path integral, Eq. (2.11), there is a finite probability of tunnelling into a different spin-coherent state. As a result the spin vector traces out a path on the surface of the sphere (shown as a *black line*). The semiclassical approximation is that this path is smooth and continuous [see Eq. (2.19)]. The partition function considers closed orbits, and these define an area $\mathcal{A}[\boldsymbol{\Omega}(\tau)]$ (shown in *red*), modulo $4\pi$

$$|\boldsymbol{\Omega}\rangle = \sum_{M=-S}^{S} D^{(S)}(\boldsymbol{\Omega})_{M,S}|S, M\rangle, \tag{2.6}$$

where $D^{(S)}(\boldsymbol{\Omega})_{M,S}$ are a set of matrix elements (labelled by $M$, $S$) that satisfy the matrix equation,

$$\mathbf{D}^{(S)}(\boldsymbol{\Omega}_1) \cdot \mathbf{D}^{(S)}(\boldsymbol{\Omega}_2) = \mathbf{D}^{(S)}(\hat{\mathbf{z}}) e^{i\Phi(\boldsymbol{\Omega}_1, \boldsymbol{\Omega}_2, \hat{\mathbf{z}})S^z}, \tag{2.7}$$

and $\Phi(\boldsymbol{\Omega}_1, \boldsymbol{\Omega}_2, \hat{\mathbf{z}})$ is the area of the spherical triangle defined by the three vectors $\hat{\mathbf{z}}$ (the unit vector parallel to the $z$-axis), $\boldsymbol{\Omega}_1$ and $\boldsymbol{\Omega}_2$, shown in Fig. (2.5). The definition of this spherical triangle is ambiguous, since three vectors on a sphere define two different triangles. The requirement that physical observables are equivalent whichever triangle is chosen, means that the area need only be defined modulo $4\pi$ [see Fig. (2.5)]. This leads to the equation,

$$e^{i4\pi M} = 1, \tag{2.8}$$

which expresses the requirement that spin should be quantised in integer or half-integer steps.

Since the states $|S, M\rangle$ are not simple vectors, the spin-coherent states do not form an orthogonal basis, and their overlap is given by,

$$\langle \mathbf{\Omega}_1 | \mathbf{\Omega}_2 \rangle = e^{i\Phi(\mathbf{\Omega}_1, \mathbf{\Omega}_2, \hat{\mathbf{z}})S} \left( \frac{1 + \mathbf{\Omega}_1 . \mathbf{\Omega}_2}{2} \right)^S, \tag{2.9}$$

while the resolution of the identity is,

$$I = \int \left( \frac{2S+1}{4\pi} \right) d^3\mathbf{\Omega} \delta(\mathbf{\Omega}^2 - 1) |\mathbf{\Omega}\rangle\langle\mathbf{\Omega}|. \tag{2.10}$$

It is possible to use spin-coherent states to write the partition function for a spin system as a path integral. To do this it is useful to rewrite the partition function as,

$$\mathcal{Z} = \mathrm{Tr}\left( e^{-\beta\mathcal{H}} \right) = \lim_{N\to\infty} \left( e^{-\delta\tau\mathcal{H}} \right)^N, \tag{2.11}$$

where $N\delta\tau = \beta$ and $\delta\tau$ is interpreted as a timestep in imaginary time. Inserting unity, Eq. (2.10), at each imaginary timestep results in,

$$\mathcal{Z} = \lim_{N\to\infty} \left( \frac{2S+1}{4\pi} \right)^N \left( \prod_{j=1}^{N} \int d^3\mathbf{\Omega}_j \delta(\mathbf{\Omega}_j^2 - 1) \langle \mathbf{\Omega}(\tau_j) | e^{-\delta\tau\mathcal{H}} | \mathbf{\Omega}(\tau_{j+1}) \rangle \right). \tag{2.12}$$

Expanding the matrix element to first order in $\delta\tau$ and taking the limit $\delta\tau \to 0$ leads to the partition function,

$$\mathcal{Z} = \int \mathcal{D}\mathbf{n} \delta(\mathbf{\Omega}^2 - 1) e^{-\mathcal{S}[\mathbf{\Omega}]}, \tag{2.13}$$

where the Euclidean action is,

$$\mathcal{S}[\mathbf{\Omega}] = \mathcal{S}_{\mathsf{kin}} + \mathcal{S}_{\mathcal{H}} = -iS \sum_i \mathcal{A}[\mathbf{\Omega}_i(\tau)] + \int_0^\beta d\tau \sum_i \langle \mathbf{\Omega}_i(\tau) | \mathcal{H} | \mathbf{\Omega}_i(\tau) \rangle. \tag{2.14}$$

Here $i$ labels lattice sites and $\mathcal{A}_i$ is the spherical area swept out by the spin vector as it performs a closed orbit on the unit sphere, shown in Fig. (2.5). This area arises from summing over the area of many infinitesimal spherical triangles, each of area $\Phi_j$ where $j$ is the timestep label, and using the periodic boundary conditions demanded by the trace. This term is unexpected from the view of classical mechanics. It is a geometrical 'Berry' phase term that arises from the non-orthogonal nature of the spin-coherent states, and it will become clear below that it is essentially the kinetic energy of the spin. The second term, the Hamiltonian term, is expected classically and describes the potential energy associated with a given spin configuration.

### 2.2.1 The Hamiltonian Term

First consider $\mathcal{S}_{\mathcal{H}}$, the Hamiltonian term in the action, Eq. (2.14), and substitute in the spin Hamiltonian, Eq. (1.1), to find,

$$\mathcal{S}_{\mathcal{H}} = \frac{JS^2}{2} \int_0^\beta d\tau \sum_{lm} \left[ \left( \mathbf{\Omega}_{l,m} + \mathbf{\Omega}_{l+1,m} \right)^2 + \left( \mathbf{\Omega}_{l,m} + \mathbf{\Omega}_{l,m+1} \right)^2 \right], \tag{2.15}$$

where $l$ labels the $x$-coordinate and $m$ the $y$-coordinate on the square lattice.

So far $\mathcal{H}_{\mathsf{Heis}}$, Eq. (1.1), has simply been rewritten in a path integral form, without any loss of degrees of freedom. The idea now is to reparametrise the degrees of freedom so as to partition them into those that leave intact the order parameter and those that reduce the value of the order parameter. For a pair of neighbouring spins, the coherent-state vector is reparametrised as,

$$\mathbf{\Omega} = \frac{\pm \mathbf{n} + \mathbf{l}}{\sqrt{1 + \mathbf{l}^2}}, \qquad \mathbf{n} \cdot \mathbf{l} = 0, \qquad \mathbf{n}^2 = 1, \tag{2.16}$$

where the fields $\mathbf{n}$ and $\mathbf{l}$ are defined at the centre of the two sites and are orthogonal to one another. The continuous field $\mathbf{n}$ is closely related to the lattice order parameter, given in Eq. (2.1), and the $\pm$ encodes the staggering of the spin moments between neighbouring sites. The field $\mathbf{l}$ is known as the canting field and is approximately equal on neighbouring sites. It describes the degree of ferromagnetic alignment between the two spins.

In order to encode the assumption that the system is ordered at short lengthscales, even if it is disordered at macroscopic lengthscales, the fields $\mathbf{n}(\mathbf{r}, \tau)$ and $\mathbf{l}(\mathbf{r}, \tau)$ are taken as slowly varying functions of $\mathbf{r}$ and $\tau$. At low energies it is assumed that the behaviour of the system is only dependent on long-range spin fluctuations, and that these do not couple strongly to short-range excitations. In consequence, it is reasonable to take the continuum limit by letting the lattice spacing, $a_0$, tend to zero. This has the effect of 'smearing out' the microscopic fluctuations of the system. Finally, the assumption of a well formed short-range order means that the field $\mathbf{l}^2 \ll 1$, and therefore the action can be expanded in a power series in $\mathbf{l}$, which is truncated at Gaussian order.

The result of substituting Eq. (2.16) into Eq. (2.15) and making the above continuum and small $\mathbf{l}$ approximations is,

$$\mathcal{S}_{\mathcal{H}} \approx \frac{JS^2}{2a_0^2} \int d\tau d^2r \left( 8\mathbf{l}^2 + a_0^2 \left( \delta_x \mathbf{n} \right)^2 + a_0^2 \left( \delta_y \mathbf{n} \right)^2 \right). \tag{2.17}$$

### 2.2.2 The Kinetic Term

Now consider the kinetic term in the action, Eq. (2.14), given by,

$$\mathcal{S}_{\mathsf{kin}} = -iS \sum_i \mathcal{A}[\mathbf{\Omega}_i(\tau)]. \tag{2.18}$$

Unlike the Hamiltonian term, which takes into account the interaction between neighbouring spins, the kinetic term is a property of individual spins. We now make a semiclassical approximation that the spin moments move in periodic orbits that are smoothly connected to the classical path, and never tunnel to distant points on the surface of the sphere [see Fig. (2.5)]. The classical path is controlled by the classical Hamiltonian dynamics and is described by $\mathbf{\Omega}_{\mathsf{cl}}(\tau)$. The area swept out by the spin can therefore be approximated as,

$$\mathcal{A}[\mathbf{\Omega}(\tau)] \approx \mathcal{A}[\mathbf{\Omega}_{\mathsf{cl}}(\tau)] + \frac{\delta \mathcal{A}[\mathbf{\Omega}(\tau)]}{\delta \mathbf{\Omega}(\tau)} \cdot \delta \mathbf{\Omega}\Big|_{\mathbf{\Omega}_{\mathsf{cl}}}. \tag{2.19}$$

The first term in this expansion is the area swept out by a classical spin and results in a term in the action that is simply a (topology-dependent) multiple of $2\pi i$. By considering small deformations of the classical path it is possible to show the geometrical result,

$$\delta \mathcal{A}[\mathbf{\Omega}(\tau)] = \mathcal{A}[\mathbf{\Omega}(\tau) + \delta\mathbf{\Omega}(\tau)] - \mathcal{A}[\mathbf{\Omega}(\tau)] = -\int_0^\beta d\tau\, \delta\mathbf{\Omega}(\tau) \cdot [\mathbf{\Omega}(\tau) \times \partial_\tau \mathbf{\Omega}(\tau)]. \tag{2.20}$$

Substituting in Eq. (2.16) for the fields $\mathbf{n}$ and $\mathbf{l}$, and making the same approximations as in the case of the Hamiltonian term, $\mathcal{S}_{\mathcal{H}}$, results in,

$$\mathcal{S}_{\mathsf{kin}} \approx -\frac{iS}{a_0^2} \int d\tau d^2 r\, [\mathbf{l}.(\mathbf{n} \times \partial_\tau \mathbf{n})]. \tag{2.21}$$

In 1-dimension there is also a topological term that leads to soliton solutions, but this can be shown to be unimportant in 2 or more dimensions [11].

### 2.2.3 Eliminating Ferromagnetic Fluctuations

Combining the Hamiltonian and kinetic terms of the action results in,

$$\mathcal{S}_{\mathsf{AFM}}[\mathbf{n}, \mathbf{l}] \approx \frac{1}{a_0^2} \int d\tau d^2 r \left[ -iS\, \mathbf{l}.(\mathbf{n} \times \partial_\tau \mathbf{n}) + \frac{JS^2}{2} \left( 8\mathbf{l}^2 + a_0^2 \left(\delta_x \mathbf{n}\right)^2 + a_0^2 \left(\delta_y \mathbf{n}\right)^2 \right) \right], \tag{2.22}$$

and the partition function is,

$$\mathcal{Z}_{\mathsf{AFM}} \approx \int \mathcal{D}\mathbf{n}\mathcal{D}\mathbf{l}\, \delta(\mathbf{n}^2 - 1)\, \delta(\mathbf{n}.\mathbf{l})\, e^{-\mathcal{S}_{\mathsf{AFM}}[\mathbf{n},\mathbf{l}]}. \tag{2.23}$$

The canting field $\mathbf{l}$ enters Eq. (2.22) at a Gaussian level. As a result it can be eliminated by integration, or equivalently by the saddle point approximation,

$$\frac{\delta \mathcal{S}_{\mathsf{AFM}}[\mathbf{n}, \mathbf{l}]}{\delta \mathbf{l}} \approx 0, \tag{2.24}$$

which leads to,

$$\mathbf{l} = \frac{i}{8JS}(\mathbf{n} \times \partial_\tau \mathbf{n}) = i\frac{\chi_\perp}{S}(\mathbf{n} \times \partial_\tau \mathbf{n}), \tag{2.25}$$

where $\chi_\perp = 1/8J$ is the perpendicular susceptibility and measures the response to a magnetic field applied perpendicular to the ordering direction. The canting field is dependent on the time derivative of $\mathbf{n}$, which means that fluctuations of the field $\mathbf{n}$ lead to a finite $\mathbf{l}$. This is a physically obvious point, since fluctuations of the order parameter field lead to a local reduction in the staggering of the spins, and therefore a local increase in the degree of ferromagnetic alignment.

After eliminating the canting field, $\mathbf{l}$, the partition function is,

$$\mathcal{Z}_{\mathsf{AFM}} \approx \int \mathcal{D}\mathbf{n}\, \delta(\mathbf{n}^2 - 1)\, e^{-\mathcal{S}_{\mathsf{AFM}}[\mathbf{n}]}, \tag{2.26}$$

where,

$$\mathcal{S}_{\mathsf{AFM}}[\mathbf{n}] \approx \frac{1}{2a_0^2} \int d\tau d^2 r \left[ \chi_\perp (\partial_\tau \mathbf{n})^2 + \sum_{\lambda=\mathsf{x},\mathsf{y}} \rho_{\mathsf{s}} (\partial_\lambda \mathbf{n})^2 \right], \tag{2.27}$$

and,

$$\chi_\perp = \frac{1}{8J}, \qquad \rho_{\mathsf{s}} = JS^2 a_0^2. \tag{2.28}$$

Physically $\rho_{\mathsf{s}}$ is the spin stiffness, and measures the energy cost of twisting the order parameter.

### *2.2.4 Spontaneous Symmetry Breaking*

In the 2-dimensional Heisenberg model on a square lattice, Eq. (1.1), there is a long-range Néel antiferromagnetic order at $T = 0$. This is an example of spontaneous symmetry breaking, since the system spontaneously picks a direction for the order

parameter, $\mathbf{n}$. In $La_2CuO_4$ a small interplane coupling increases the critical temperature of the ordered phase from $T_N = 0$, which is the case for a purely 2-dimensional system, to $T_N \approx 200$ K [1–3]. Collinear magnets, such as $La_2CuO_4$, do not break the full $SU(2)$ symmetry of the Hamiltonian, as there remains a $U(1)$ degree of freedom associated with rotations around the collinear axis. Instead they break the smaller symmetry group $SU(2)/U(1)$. In a classical picture this is equivalent to breaking $SO(3)/SO(2)$.

Goldstone's theorem [15] requires that for every spontaneously broken symmetry, there must be a gapless, Goldstone mode associated with the generator of that symmetry. In the case of the Néel antiferromagnet there are two broken symmetries, with generators corresponding to the two rotations that change the orientation axis of the order parameter. The simplest way to see this is to parametrise the order parameter as,

$$\mathbf{n} = \left(\phi_y, \phi_x, \sqrt{1-\phi_x^2-\phi_y^2}\right) = \left(\phi, \sqrt{1-\phi^2}\right), \qquad \phi = (\phi_y, \phi_x), \tag{2.29}$$

and linearise the theory by Taylor-expanding the square root, $\mathbf{n} \approx (\phi, 1-\phi^2/2)$. The partition function becomes,

$$\mathcal{Z}_{\mathsf{AFM}} \approx \int \mathcal{D}\phi\, e^{-\mathcal{S}_{\mathsf{AFM}}[\phi]}, \tag{2.30}$$

where the action is,

$$\mathcal{S}_{\mathsf{AFM}}[\phi] \approx \frac{1}{2a_0^2}\int d\tau d^2r \left[\chi_\perp(\partial_\tau\phi)^2 + \sum_{\lambda=\mathsf{x},\mathsf{y}} \rho_{\mathsf{s}}(\partial_\lambda\phi)^2\right]. \tag{2.31}$$

This approximation is only justified for small fluctuations around a long-range ordered state, and results in a simple Gaussian field theory. The fields $\phi_y$ and $\phi_x$ are generators of rotations around the $y$ and $x$ axes, and correspond to the Goldstone modes. The Euler-Lagrange equations of motion are,

$$\left(\chi_\perp\partial_\tau^2 + \rho_{\mathsf{s}}\partial_{\mathsf{x}}^2 + \rho_{\mathsf{s}}\partial_{\mathsf{y}}^2\right)\phi(\mathbf{x},\tau) = 0, \tag{2.32}$$

and these can be solved with the ansatz,

$$\phi(\mathbf{x},\tau) = \mathbf{c}e^{i\mathbf{k}.\mathbf{r}+\omega_{\mathbf{k}}\tau}. \tag{2.33}$$

where $\mathbf{r} = (x, y)$. The resulting dispersion relation is given by,

$$\omega_{\mathbf{k}} = v|\mathbf{k}|, \qquad v = \sqrt{\frac{\rho_{\mathsf{s}}}{\chi_\perp}} = 2\sqrt{2}JSa_0, \tag{2.34}$$

where $v$ is the spin wave velocity. This is the low-energy magnetic dispersion of the coherent spin-wave excitations [see Fig. (2.3) for a semiclassical illustration of the spin-wave excitations]. At low energies it is identical to the dispersion predicted by linear spin-wave theory, Eq. (2.4) [see Fig. (2.2)]. At higher energies the field theory diverges from spin-wave theory.

### 2.2.5 The Spin Moments and Correlation Functions

Experimentally measurable quantities, such as the $1/T_1$ relaxation rate or the inelastic neutron scattering cross section, are determined by correlation functions. In particular the dynamic spin susceptibility,

$$\chi^{\alpha\beta}(\mathbf{q}, i\omega_n) = \left\langle \delta S^{\alpha}(\mathbf{q}, i\omega_n) \delta S^{\beta}(-\mathbf{q}, -i\omega_n) \right\rangle, \tag{2.35}$$

where $\omega_n$ are Matsubara frequencies, $S^{\alpha}(\mathbf{q}, i\omega_n)$ is the Fourier tranformed spin moment and $\delta S^{\alpha}(\mathbf{q}, i\omega_n) = S^{\alpha}(\mathbf{q}, i\omega_n) - \langle S^{\alpha}(\mathbf{q}, i\omega_n) \rangle$ measures the spin fluctuations. In order to calculate the dynamic spin susceptibility, it is first necessary to express the spin field, $\mathbf{S}(\mathbf{r}, \tau)$, in terms of the fields $\mathbf{n}$ and $\mathbf{l}$,

$$\mathbf{S}(\mathbf{r}, \tau) = S \frac{\pm \mathbf{n}(\mathbf{r}, \tau) + \mathbf{l}(\mathbf{r}, \tau)}{\left[1 + \mathbf{l}(\mathbf{r}, \tau)^2\right]^2} \approx S \left( e^{i\mathbf{Q}.\mathbf{r}} \mathbf{n} + i \frac{\chi_{\perp}}{S} (\mathbf{n} \times \partial_{\tau} \mathbf{n}) \right), \tag{2.36}$$

where $\mathbf{Q}$ is the ordering vector. This provides a general scheme for calculating experimental responses.

### 2.2.6 The Phase Transition and Renormalisation

Field theory is also useful for describing the critical region close to a phase transition. In the critical paramagnet the order parameter is well defined on lengthscales shorter than the correlation length, but there is a slow twist at longer lengthscales, and therefore no long-range order. As the phase transition is approached, the correlation length increases, and, exactly at $T_{\mathrm{N}}$, it diverges. Thermodynamic parameters, such as the heat capacity and the susceptibility, also diverge, and their critical exponents can be measured experimentally. This drives a divergence in experimentally measurable quantities, such as the $1/T_1$ relaxation rate.

The renormalisation approach provides a powerful way to understand critical behaviour, including critical divergences (see in particular Nelson and Pelcovits' [16] discussion of the classical NLSM and Chakravarty, Halperin and Nelson [7] for the quantum NLSM). The idea is to write down a set of scale transformations that relate the critical region, where fluctutations are highly correlated and calculation

is difficult, to either the low-temperature ordered region or the high-temperature disordered region, where perturbation theory works and calculation is comparatively easy. One of the powerful findings is the idea of universality: critical behaviour is controlled by symmetry class and dimensionality, *not* by microscopic details. This is the heart of the unifying principle provided by the field theory approach.

The NLSM shown in Eq. (2.27) has dimension $d = 2$, but one can imagine generalising the model to live on a $d$-dimensional hypercubic lattice. In $d = 2$ there is a phase transition at $T = 0$, and therefore this is the lower critical dimension. In higher dimensions the phase transition occurs at finite $T$, and the low-energy approximation begins to break down. For this reason the renormalisation procedure works best close to $d = 2$. In order to gain some insight into the behaviour in $d > 2$, the standard method is to perform a perturbative expansion in $\epsilon$, where $d = 2 + \epsilon$. Conversely, the Ginzburg-Landau approach, which is the subject of Sect. 2.3, works best at the upper critical dimension $d = 4$, where mean-field theory becomes exact. Physically realistic dimensions can be approached perturbatively by expanding in $\epsilon$ where $d = 4 - \epsilon$. The two approaches find qualitative agreement in the physically interesting dimension $d = 3$ [13].

## 2.3 The Ginzburg-Landau Model and Critical Dynamics

The Néel antiferromagnet, and in particular the critical region, can also be understood within Ginzburg-Landau theory. The idea is to make a power-series expansion of the free energy around the critical point, using the order parameter as the small variable. At first sight this appears to be a terrible idea, since the free energy is known not to be analytic at the critical point. However, despite this reservation, the theory has been very successful at describing continuous phase transitions. In its standard form, Ginzburg-Landau theory is time-independent, and in order to treat critical dynamics it is necessary to add time dependence by hand. The critical dynamics will be the main focus of this introduction, and the ideas will later be generalised to spin-nematic states. For an introduction to Ginzburg-Landau theory and the associated critical dynamics and renormalisation see the books by Chaikin and Lubensky [17] and Ma [18] and the papers by Halperin and Hohenberg [19–21].

It is possible to derive the Ginzburg-Landau (GL) model from effective microscopic models such as $\mathcal{H}_{\mathsf{Heis}}$, Eq. (1.1), and this approach results in a detailed, microscopic understanding of the expansion coefficients [13]. However, the GL model takes a far less microscopic view than the NLSM. In the NLSM, paramagnetic behaviour results from slowly twisting the otherwise well-formed order parameter. The local order parameter field $\mathbf{n}(\mathbf{r}, \tau)$ is of fixed length, and therefore does not fluctuate longitudinally. In the GL model the order parameter field $\mathbf{m}_{\mathsf{S}}(\mathbf{r})$ does fluctuate longitudinally, and paramagnetic behaviour is signalled by $\langle \mathbf{m}_{\mathsf{S}} \rangle = 0$. However, it is important to note that this does not imply longitudinal fluctuation of the physical spin moments. It is instead a symptom of the coarse-graining procedure, in which an average is taken over many physical spins to create the field $\mathbf{m}_{\mathsf{S}}(\mathbf{r})$. The linear size over

which the physical spins are averaged to form the order parameter field $\mathbf{m_s}(\mathbf{r})$ is given by the correlation length, $\xi$. Longitudinal fluctuations in $\mathbf{m_s}(\mathbf{r})$ therefore arise from transverse fluctuations of the fixed-length, physical spin moments on the scale of $\xi$.

The NLSM and GL model descriptions of the Néel antiferromagnet are in many ways complementary. The NLSM captures the low-temperature phase transition experienced by systems close to the lower critical dimension (in the case of the Néel antiferromagnet $d = 2$), while the GL model describes the high-temperature phase transition associated with the upper critical dimension ($d = 4$). Using the dimension as a perturbation parameter, the two models find qualitative agreement in $d = 3$ [13].

As a concrete example, consider the Néel antiferromagnet. Spin excitations can be described phenomenologically using a Ginzburg-Landau Hamiltonian, constructed from invariant combinations of the order parameter $\mathbf{m_s}(\mathbf{r})$, and its associated canting field $\mathbf{l}(\mathbf{r})$,

$$\mathcal{H}_{\mathsf{AFM}} \approx \int d^3r \left[ \frac{\alpha \tilde{t}}{2} \mathbf{m_s}^2 + \frac{K}{2} (\nabla \mathbf{m_s})^2 + \frac{u}{4} \mathbf{m_s}^4 + \frac{1}{2\chi_\perp} \mathbf{l}^2 \right], \tag{2.37}$$

where the coefficient $\tilde{t} = T - T_{\mathsf{N}}$ is the reduced temperature[1], $\chi_\perp$ is the transverse susceptibility, $\alpha > 0$ and $u > 0$ are positive constants and $K$ is a generalised elastic constant.

The order parameter field, $\mathbf{m_s}$, contains 3 degrees of freedom and these can be parametrised in terms of a unit vector, $\hat{\mathbf{n}}$ and a length $\eta$ according to,

$$\mathbf{m_s}(\mathbf{r}) = \eta(\mathbf{r})\hat{\mathbf{n}}(\mathbf{r}) = [\langle \eta \rangle + \delta\eta(\mathbf{r})]\hat{\mathbf{n}}(\mathbf{r}), \quad \hat{\mathbf{n}}^2 = 1. \tag{2.38}$$

Substituting this into the Hamiltonian, Eq. (2.37), results in,

$$\mathcal{H}_{\mathsf{AFM}} \approx \int d^3r \left[ \frac{\alpha \tilde{t}}{2} \eta^2 + \frac{K}{2} (\nabla \eta)^2 + \frac{u}{4} \eta^4 + \frac{\rho_{\mathsf{s}}(\eta)}{2} (\nabla \hat{\mathbf{n}})^2 + \frac{\mathbf{l}^2}{2\chi_\perp} \right], \tag{2.39}$$

where the spin stiffness is defined by $\rho_{\mathsf{s}}(\eta) = K\eta^2$. The final two terms in this Hamiltonian look very much like the static, Hamiltonian part of the NLSM action, Eq. (2.17). The difference is that the GL model is far more coarse-grained, and in consequence $\rho_{\mathsf{s}}$ is no longer a constant, but a function of $\eta$. This part of the Hamiltonian could be derived by performing a block-scaling, renormalisation procedure on the static part of the NLSM, Eq. (2.27). Also the unit length constraint present in the NLSM has been softened, and is now controlled by an anharmonic potential in terms of $\eta$. Assuming that $\eta$ is spatially uniform, this potential is minimised for,

$$\eta = \begin{cases} 0 & \tilde{t} > 0 \\ \sqrt{-\alpha \tilde{t}/u} & \tilde{t} < 0 \end{cases}, \tag{2.40}$$

and there is a phase transition at $\tilde{t} = 0$ as expected.

[1] We label the reduced temperature $\tilde{t}$ rather than the more usual $t$ in order to distinguish it from the time-variable $t$.

Within mean-field theory the correlation length is given by

$$\frac{K}{\xi^2} = \alpha\tilde{t} + 3u\eta^2 = \begin{cases} \alpha\tilde{t} & \tilde{t} > 0 \\ -2\alpha\tilde{t} & \tilde{t} < 0 \end{cases}, \tag{2.41}$$

and therefore diverges as $\xi \propto 1/\sqrt{\tilde{t}}$. The spin stiffness is given by,

$$\rho_{\mathsf{s}} = \eta^2 K \approx \begin{cases} 0 & \tilde{t} > 0 \\ -K\alpha\tilde{t}/u & \tilde{t} < 0 \end{cases}, \tag{2.42}$$

and therefore, in mean-field theory, $\rho_{\mathsf{s}} \propto \xi^{-2}$. In the paramagnet $\rho_{\mathsf{s}} = 0$, and there are no spin-wave excitations. This is not the case in the NLSM where spin-wave excitations exist on lengthscales smaller than $\xi$, and this is an example of the different basic lengthscales over which the theories apply. Mean-field theory can be used to derive critical exponents for all the static thermodynamic quantities, and, while not in quantitative agreement with experiment, gives the correct qualitative picture [17].

In this thesis we will be particularly interested in dissipative processes, in particular the $1/T_1$ relaxation rate, and this requires an understanding of the critical dynamics. Unlike in the NLSM, in GL theory it is necessary to add damping terms phenomenologically to the equations of motion. The damping arises both from interaction between long-wavelength degrees of freedom, and from interaction between long and short-wavelength degrees of freedom [17, 18]. The equation of motion for $\eta$ can be written as [17–20],

$$\partial_t \delta\eta(\mathbf{r}, t) = -\Gamma K(\xi^{-2} - \nabla^2)\delta\eta(\mathbf{r}, t) + \zeta(\mathbf{r}, t), \tag{2.43}$$

where time is measured in units of $\hbar/k_{\mathsf{B}}T_{\mathsf{N}}$, $\Gamma^{-1}$ sets the rate of damping, $u \to 0$, and $\zeta(\mathbf{r}, t)$ is a (white) noise term with correlation function,

$$\langle \zeta(\mathbf{r}, t)\zeta(\mathbf{r}', t')\rangle = 2k_{\mathsf{B}}T\Gamma\delta(\mathbf{r} - \mathbf{r}')\delta(t - t'). \tag{2.44}$$

It follows that the correlation time is given by $\tau_\eta(\mathbf{q}) \approx \Gamma^{-1}\chi_{\mathbf{q}}$, with $\chi_{\mathbf{q}} = K^{-1}(\xi^{-2} + \mathbf{q}^2)^{-1}$. Approaching $T_{\mathsf{N}}$, $\tau_\eta(0)$ diverges as $\tau_\eta(0) \propto \xi^z$ with dynamical exponent $z = 2$. The critical divergence of $\xi$ is thus accompanied by a critical slowing down in fluctuations of AFM order.

Taking the Fourier transform of Eq. (2.43) results in,

$$\delta\eta(\mathbf{q}, \omega) = \frac{\zeta(\mathbf{q}, \omega)}{\Gamma\chi_{\mathbf{q}}^{-1} - i\omega}. \tag{2.45}$$

The Green's function for this differential equation is defined by,

$$\delta\eta(\mathbf{q}, \omega) = G_0^{\eta\eta}(\mathbf{q}, \omega)\zeta(\mathbf{q}, \omega), \tag{2.46}$$

and therefore,

$$G_0^{\eta\eta}(\mathbf{q},\omega) = \frac{1}{\Gamma\chi_{\mathbf{q}}^{-1} - i\omega}. \tag{2.47}$$

The fluctuation-dissipation theorem allows dissipative phenomena to be described in terms of the imaginary part of the dynamic susceptibility, and this is given by [17],

$$\frac{k_{\mathsf{B}}T}{\hbar\omega}\Im m\{\chi(\mathbf{q},\omega)\} \approx (g_{\mathsf{l}}\mu_{\mathsf{B}})^2 \frac{k_{\mathsf{B}}T}{(k_{\mathsf{B}}T_{\mathsf{N}})^2} \frac{\Gamma}{\Gamma^2\chi_{\mathbf{q}}^{-2} + \omega^2}. \tag{2.48}$$

and for $\omega = 0$ and $\mathbf{q} = 0$ this diverges as $\xi^4$. A more sophisticated treatment, in which the case $u \neq 0$ is treated by a dynamic generalisation of the static renormalisation theory [18–20], results in different critical exponents, but does not change the qualitative picture.

In the ordered state it is also interesting to consider the role of spin-wave damping. For an antiferromagnet whose order parameter is parallel to the $\mathsf{z}$-direction, the unit-vector field $\hat{\mathbf{n}}$ can be parametrised as $\hat{\mathbf{n}} \approx (\phi_{\mathsf{y}}, \phi_{\mathsf{x}}, 1)$, while the canting field is approximately given by $\mathbf{l} \approx (l_{\mathsf{x}}, l_{\mathsf{y}}, 0)$. Adding phenomenological damping terms to the linearised equations of motion, Eq. (2.43), results in,

$$\begin{aligned}
\partial_t l_{\mathsf{x}} &= \frac{\rho_{\mathsf{s}}}{\hbar}\nabla^2\phi_{\mathsf{x}} + \frac{\lambda}{\chi_\perp}\nabla^2 l_{\mathsf{x}} + \zeta_{\mathsf{l}} \\
\partial_t \phi_{\mathsf{x}} &= \frac{l_{\mathsf{x}}}{\hbar\chi_\perp} + \rho_{\mathsf{s}}\gamma\nabla^2\phi_{\mathsf{x}} + \zeta_\phi,
\end{aligned} \tag{2.49}$$

where $\lambda$ and $\gamma$ are damping parameters and $\zeta_{\mathsf{l}}$ and $\zeta_\phi$ are noise terms with correlation function,

$$\begin{aligned}
\langle\zeta_{\mathsf{l}}(\mathbf{r},t)\zeta_{\mathsf{l}}(\mathbf{r}',t')\rangle &= 2\lambda k_{\mathsf{B}}T\nabla^2\delta(\mathbf{r}-\mathbf{r}')\delta(t-t') \\
\langle\zeta_\phi(\mathbf{r},t)\zeta_\phi(\mathbf{r}',t')\rangle &= 2\gamma k_{\mathsf{B}}T\delta(\mathbf{r}-\mathbf{r}')\delta(t-t').
\end{aligned} \tag{2.50}$$

Equivalent equations apply on making the substitution $x \to y$. The order parameter field, $\hat{\mathbf{n}}$, is not conserved, whereas the canting field, $\mathbf{l}$, which describes the total magnetisation, is conserved. The $\mathbf{q} = 0$ modes of conserved fields cannot lead to relaxation, and therefore the first relaxational term in the phenomenological expansion of the damping parameter is proportional to $q^2$.

Fourier transforming Eq. (2.49) results in,

$$\begin{pmatrix} i\omega - \rho_{\mathsf{s}}\gamma q^2 & \frac{1}{\hbar\chi_\perp} \\ -\frac{\rho_{\mathsf{s}}}{\hbar}q^2 & i\omega - \frac{\lambda}{\chi_\perp}q^2 \end{pmatrix} \begin{pmatrix} \phi_{\mathsf{x}}(\mathbf{q},\omega) \\ l_{\mathsf{x}}(\mathbf{q},\omega) \end{pmatrix} = \begin{pmatrix} \zeta_\phi(\mathbf{q},\omega) \\ \zeta_{\mathsf{l}}(\mathbf{q},\omega) \end{pmatrix}, \tag{2.51}$$

and therefore the dispersion is given by,

$$\omega_{\mathbf{q}}^{\pm} \approx \pm vq - \frac{iDq^2}{2} + \mathcal{O}(q^3), \quad v = \sqrt{\frac{\rho_{\mathsf{s}}}{\chi_{\perp}}}, \quad D = \frac{\lambda}{\chi_{\perp}} + \rho_{\mathsf{s}}\gamma. \tag{2.52}$$

The first term in $\omega_{\mathbf{q}}^{\pm}$ clearly describes the coherent spin-wave dynamics, while the second term is imaginary and describes dissipation.

Inverting Eq. (2.51) results in,

$$\begin{pmatrix} \phi_{\mathsf{x}}(\mathbf{q},\omega) \\ l_{\mathsf{x}}(\mathbf{q},\omega) \end{pmatrix} \approx \frac{1}{(\omega-\omega_{\mathbf{q}}^{+})(\omega-\omega_{\mathbf{q}}^{-})} \begin{pmatrix} -i\omega + \frac{\lambda}{\chi_{\perp}}q^2 & \frac{1}{\hbar\chi_{\perp}} \\ -\frac{\rho_{\mathsf{s}}}{\hbar}q^2 & -i\omega + \rho_{\mathsf{s}}\gamma q^2 \end{pmatrix} \begin{pmatrix} \zeta_{\phi}(\mathbf{q},\omega) \\ \zeta_{\mathsf{l}}(\mathbf{q},\omega) \end{pmatrix}, \tag{2.53}$$

and it follows that,

$$\begin{aligned} \frac{k_{\mathsf{B}}T}{\hbar\omega}\Im m\{\chi^{xx}_{\phi,1\mathsf{p}}(\mathbf{q},\omega)\} &= \frac{k_{\mathsf{B}}T}{\hbar\omega}\Im m\{\chi^{yy}_{\phi,1\mathsf{p}}(\mathbf{q},\omega)\} \approx \frac{\hbar k_{\mathsf{B}}T}{(k_{\mathsf{B}}T_{\mathsf{N}})^2}\frac{2\langle\eta\rangle^2(\gamma\rho_{\mathsf{s}}\omega^2\mathbf{q}^2 + \frac{\lambda}{\chi_{\perp}}v^2\mathbf{q}^4)}{\rho_{\mathsf{s}}\mathbf{q}^2|(\omega-\omega_{\mathbf{q}}^{+})(\omega-\omega_{\mathbf{q}}^{-})|^2} \\ \frac{k_{\mathsf{B}}T}{\hbar\omega}\Im m\{\chi^{xx}_{l,1\mathsf{p}}(\mathbf{q},\omega)\} &= \frac{k_{\mathsf{B}}T}{\hbar\omega}\Im m\{\chi^{yy}_{l,1\mathsf{p}}(\mathbf{q},\omega)\} \approx \frac{\hbar k_{\mathsf{B}}T}{(k_{\mathsf{B}}T_{\mathsf{N}})^2}\frac{2(\gamma\rho_{\mathsf{s}}^2\mathbf{q}^4 + \lambda\mathbf{q}^2\omega^2)}{|(\omega-\omega_{\mathbf{q}}^{+})(\omega-\omega_{\mathbf{q}}^{-})|^2}, \end{aligned} \tag{2.54}$$

where $\chi^{xx}_{\phi,1\mathsf{p}}(\mathbf{q},\omega)$ is the 1-particle susceptibility associated with fluctuations of the order parameter field in the $x$-direction, and $\chi^{xx}_{l,1\mathsf{p}}(\mathbf{q},\omega)$ is associated with fluctuations of the canting field.

In the limit $\lambda, \gamma \to 0$, which is valid at low $T$, this reduces to,

$$\frac{T}{\omega}\Im m\{\chi^{xx}_{\phi,1\mathsf{p}}(\mathbf{q},\omega)\} = \frac{T}{\omega}\Im m\{\chi^{yy}_{\phi,1\mathsf{p}}(\mathbf{q},\omega)\} \propto \frac{T}{\omega}\frac{1}{\chi_{\perp}vq}\left[\delta(\omega-vq) + \delta(\omega+vq)\right], \tag{2.55}$$

in agreement with the undamped case.

In the limit of $T \to T_{\mathsf{N}}$ with finite $\gamma$, the spin stiffness is small and therefore $\rho_{\mathsf{s}} \ll D$. In this approximation,

$$\begin{aligned} \frac{T}{\omega}\Im m\{\chi^{xx}_{\phi,1\mathsf{p}}(\mathbf{k},\omega)\} &\propto \frac{2T(\chi_{\perp}^2\gamma\omega^2 + \lambda k^2)}{\left(\chi_{\perp}^2\omega^2 + \lambda^2k^4/4\right)^2}|T - T_{\mathsf{N}}| \\ \frac{T}{\omega}\Im m\{\chi^{xx}_{l,1\mathsf{p}}(\mathbf{k},\omega)\} &\propto \frac{2T\chi_{\perp}^2\lambda k^2\omega^2}{\left(\chi_{\perp}^2\omega^2 + \lambda^2k^4/4\right)^2}, \end{aligned} \tag{2.56}$$

where Eq. (2.42) has been used. The susceptibility measures the correlation between spin fluctuations at different spatial positions, and it therefore seems reasonable to identify $|T - T_{\mathsf{N}}|$ with the correlation length, $\xi^{-2}$. In consequence the imaginary part of the dynamic, order-parameter susceptibility tends to zero as $\xi^{-2}$ as the

critical point is approached. The canting-field susceptibility, which is related to the total magnetisation, has no critical dependence, and is continuous across the phase transition. This point will be important in relation to spin-nematic phases in Chap. 6.

Having introduced some of the theoretical methods used in this thesis, we now turn to the NMR $1/T_1$ relaxation rate.

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