Preface

Diffusion processes can be employed to model many phenomena arising in the natural and social sciences, such as biological and financial quantities. When modeling these phenomena, it is often natural to employ a number of driving Wiener or Bessel processes, placing us immediately in a multi- and often high-dimensional setting. The key questions that then typically arise concern a range of functionals for such models. The focus of this research monograph is, therefore, on tractable multidimensional models with functionals that have explicit solutions. After transformations of Brownian motion, as applied for the Black-Scholes model, it will be natural to concentrate in this book on models that are in some sense transformations of squares of Brownian motions, such as Bessel processes, square root processes and affine processes. Additionally, tractable diffusion processes will be studied which have been recently discovered via Lie symmetry methods. Numerical methods will be presented that allow to evaluate efficiently and accurately a wide range of functionals of multidimensional diffusions. The importance of these functionals and methods will be demonstrated in applications to finance. However, the same functionals and numerical methods can be of relevance in many other areas of application.

Given the ubiquitousness of multidimensional Wiener and Bessel processes, it is obvious that particular functionals of Wiener and Bessel processes are of importance in many different areas. However, the multidimensional nature of the processes, especially if non-trivial dependence structures are modeled between their drivers, often means that these functionals are difficult to compute. Especially closed-form solutions are rarely available. Consequently, numerical methods have to be usually employed to compute these important functionals.

The contribution of this monograph is fourfold: Firstly, it collects in a systematic way existing results on functionals of tractable processes from the literature. These results are mostly of closed form and so far often only more widely known for one-dimensional processes or very special multidimensional processes exhibiting a trivial dependence structure. Secondly, the book provides approaches which empower the reader to obtain systematically closed form solutions for various problems of interest. Thirdly, it recalls powerful numerical methods from the literature, and discusses how to apply these to the stochastic processes and functionals studied in this text. Finally, it suggests how to exploit the availability of closed form solutions in finance for particular models when numerically solving more general models even in high-dimensional situations.

Our systematic approach to developing closed form solutions proceeds around the following ideas: In the one-dimensional setting, we recall mathematical methods developed by Craddock and collaborators, see Craddock (2009), Craddock and Lennox (2007), Craddock and Lennox (2009), and Craddock and Platen (2004). In particular, we employ Lie symmetry group methods to compute transition densities of stochastic processes of interest. Furthermore, we study solvable affine models, in the sense introduced by Grasselli and Tebaldi, see Grasselli and Tebaldi (2008), which include a wide range of functionals of affine models for which explicit solutions can be obtained.

As often as we have access to explicit transition densities, we employ Monte Carlo and quadrature methods, which allow us to solve integration problems associated with functionals of interest. To quantify functionals rather generally, we remark that explicit solutions derived in this text serve as a useful check for these methods and allow us to tailor these methods.

Besides considering functionals of multidimensional Wiener and Bessel processes, which can be applied in very different areas, we focus on functionals of multidimensional Wiener and Bessel processes occurring in finance. We show how methods developed in this text can be used to solve typical problems under the benchmark approach. Within this book, we discuss several classes of tractable diffusions, which do not satisfy the classical Lipschitz and linear growth conditions, often assumed to be in force when studying diffusion models. The ability to compute important functionals of these tractable diffusions allows us to access a rich modeling world. In fact, advanced realistic long-term financial modeling under classical Lipschitz and linear growth conditions may potentially turn out to be not realistic enough for typical risk management tasks.

The purpose of Chap. 1 is to demonstrate that the book can be used to solve practical problems arising in finance under the benchmark approach. Chapters 2 and 3 summarize the current literature in the area. Chapter 4 introduces Lie Symmetry Group methods, an important tool that can be used to compute functionals of one- and multidimensional diffusions. Chapter 5 builds on Chap. 4 and we show how to compute explicitly important functionals of diffusions. In Sect. 5.5, we give a first application of these results to problems in finance. Chapter 6 applies the results from Chap. 5 to stochastic volatility models, where we present a simulation method based on Lie Symmetry methods. We then continue our study of multidimensional diffusions and turn to affine processes. We summarize the existing literature on affine processes in Chaps. 7 and 8. Chapter 9 presents the novel approach to affine processes due to Grasselli & Tebaldi. This approach analyzes when functionals of multidimensional affine processes can be computed analytically, and hence is of upmost importance to the topic of this book. As this approach is recent, we illustrate it using several examples in Sects. 9.4, 9.5, 9.6, and 9.7. Finally, we discuss a flexible class of multidimensional affine processes, the Wishart processes. Unlike the classical affine processes discussed in Chaps. 7, 8, and 9, Wishart processes do not assume values in the Euclidean space, but are matrix-valued. To fully appreciate the flexibility of Wishart processes, we provide an introduction to matrix-valued processes in Chap. 10. It seems that introductions to Lie symmetry methods for diffusions, to matrix-valued processes, and to Wishart processes have not been discussed in such a book form before. We hope that this monograph will be a valuable reference for readers interested in these topics. Finally, Chap. 14 integrates the material covered in the preceding chapters and demonstrates how it can be used to solve problems in finance entailing credit risk.

The remaining chapters are supporting chapters. We survey numerical methods, including Monte Carlo and quadrature methods and computational tools, which complement the methods described in Chaps. 4 to 11. Chapters 15, 16 and 17 are self-contained and aimed to summarize key results on stochastic processes, time-homogeneous scalar diffusions, and the distinction between martingales and strict local martingales. The material on stochastic processes is included to make this book more self-contained and easily readable without forcing some readers to rely on other sources. The material on time-homogeneous scalar diffusions is used in a few places in this book. In those cases, the reader is simply referred to the corresponding results in Chap. 16. Finally, Chap. 17 discusses when local martingales are true martingales or strict local martingales, which is an important theme of this book and relevant for the benchmark approach to finance.

The formulas in the book are numbered according to the chapter and section where they appear. Assumptions, theorems, lemmas, definitions and corollaries are numbered sequentially in each section. The most common notations are listed at the beginning and an *Index of Keywords* is given at the end of the book. Some readers may find the *Author Index* at the end of the book useful. *Suggestions for the Reader* are given after the content to give a guidance to the use of the book for different groups of readers. The *basic notation* used in the book is summarized after these suggestions.

We conclude with the remark that the practical application and theoretical understanding of functionals of multidimensional diffusions are an area of ongoing research. This book shall stimulate interest and further work on such methods and their application.

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It is greatly appreciated if readers could forward any suggestions, errors, misprints or suggested improvements to: JanBaldeaux@gmail.com. The interested reader is likely to find in future updated information about functionals of multidimensional diffusions via a link on Jan Baldeaux's homepage.

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