Preface

This monograph is an introduction to the stochastic analysis of self-similar processes both in the Gaussian and non-Gaussian case.

The text is mostly self-contained and should be accessible to graduate students and researchers with a basic background in probability theory and stochastic processes. Although Part II of the monograph is based on the Malliavin calculus, the tools used are basic and consequently readers who are not familiar with the theory will nevertheless be able to follow the exposition.

The majority of these notes were completed during my research visits to several university and research centers such as Purdue University, Keio University, Universidad de Valparaíso, Humboldt Universität zu Berlin, Centre Interfacultaire Bernoulli at Lausanne, Ritsumeikan University, University of Trento, Charles University, University of Sydney and Centre de Recerca Matemàtica in Barcelona. I would like to thank my colleagues Frederi Viens, Makoto Maejima, Soledad Torres, Peter Imkeller, Robert Dalang, Marco Dozzi, Francesco Russo, Arturo-Kohatsu-Higa, Stefano Bonaccorsi, Bohdan Maslowski, Qiying Wang, Xavier Bardina and Marta Sanz-Solé for their kind invitations.

A part of the material presented in this book is contained in the doctoral theses of my former and present students Khalifa Es-Sebaiy, Solesne Bourguin, Jorge Clarke De la Cerda and Alexis Fauth.

Introduction

Self-similar processes are stochastic processes that are invariant in distribution under a suitable scaling of time and space. This property is crucial in applications such as network traffic analysis, mathematical finance, astrophysics, hydrology and image processing. For this reason, their analysis has long constituted an important research direction in probability theory. Several monographs, such as [75] or [160], provide a complete analysis of the properties of this class of stochastic processes and many other research papers and monographs focus on the practical aspects of self-similarity. A bibliographical guide to the applications of self-similar processes is provided in [191]. In the last few decades, new developments in self-similarity have been obtained, including the appearance of new classes of (Gaussian or non-Gaussian) self-similar processes and new techniques to study their behavior, related to the stochastic calculus (especially the Malliavin calculus). The aim of this text is to survey these new developments.

This monograph comprises two parts, each of them divided into several chapters, and Appendices A, B, C.

In Part I we discuss the basic properties of several classes of (Gaussian or non-Gaussian) self-similar stochastic processes. This part is divided into four chapters. Chapter 1 focuses on fractional Brownian motion and related processes. Fractional Brownian motion is the most well known self-similar process with stationary increments. It includes standard Brownian motion as a particular case. The applications of this process are now widely recognized. We survey the basic properties of the process and several other related processes that have recently emerged in scientific research, such as bifractional Brownian motion and subfractional Brownian motion. Chapter 2 treats the Gaussian solutions to stochastic heat and wave equations and in Chap. 3 we introduce some non-Gaussian self-similar processes which are known as *Hermite processes*. Chapter 4 contains some examples of multi-parameter self-similar processes and their basic properties.

Part II is dedicated to the study of quadratic (and other) variations of several self-similar processes. The variations of a stochastic process play a crucial role in its probabilistic and statistical analysis. Best known is the quadratic variation of a semi-martingale, which is crucial for its Itô formula; quadratic variation also has a direct utility in practice, in estimating unknown parameters, such as volatility in financial models, in the so-called "historical" context. For self-similar stochastic processes, the study of their variations constitutes a fundamental tool in constructing good estimators of their self-similarity parameters. These processes are well suited to modeling various phenomena where scaling and long memory are important factors (internet traffic, hydrology, econometrics, among others, see [191]). The most important modeling task is then to determine or estimate the self-similarity parameter, because it is also typically responsible for the process's long memory and its regularity properties. Studying such processes is thus an important research direction both in theory and in practice. The approach we use is based on the so-called Malliavin calculus and multiple Wiener-Itô integrals. Part II comprises two chapters. In the first we study the asymptotic behavior of various types of variations of fractional Brownian motion, of the Hermite process and of the solution to the linear heat equation. In the second chapter we study other types of variations for stochastic processes, including Hermite-type variations for self-similar processes and fields and so-called Spitzer's and Hsu-Robbins type results.

Each chapter concludes with a collection of exercises.

Paris Ciprian A. Tudor
January 2013

Springer
http://www.springer.com/978-3-319-00935-3

Analysis of Variations for Self-similar Processes
A Stochastic Calculus Approach
Tudor, C.A.
2013, XI, 268 p., Hardcover
ISBN: 978-3-319-00935-3