

# Contents

<b>1</b>	<b>Introduction</b>	1
1.1	Background	1
1.2	Set-Theoretic Preliminaries	3
1.3	Two Basic Negative Results	4
1.4	One Basic Positive Result: The $\mu$ -Predictor	6
1.5	A Preview of What Is to Come	7
<b>2</b>	<b>The Finite Setting</b>	11
2.1	Background	11
2.2	Minimal Predictors	12
2.3	Optimal Predictors	14
2.4	The Role of the Tutte-Berge Formula	14
2.5	A Variable Number of Hat Colors	16
2.6	Variations on the Standard Hat Problem	17
2.7	Open Questions	18
<b>3</b>	<b>The Denumerable Setting: Full Visibility</b>	19
3.1	Background	19
3.2	The Gabay-O'Connor Theorem	20
3.3	Lenstra's Theorem and Sequential Guessing	21
3.4	The Role of the Axiom of Choice	24
3.5	The Role of Square Bracket Partition Relations	26
3.6	Open Questions	27
<b>4</b>	<b>The Denumerable Setting: One-Way Visibility</b>	29
4.1	Background	29
4.2	Optimal and Minimal Predictors for Transitive Graphs	30
4.3	Characterizing Graphs Yielding Finite-Error Predictors	31
4.4	ZFC Results for the Parity Relation	33

4.5	Independence Results for the Parity Relation.....	34
4.6	The Role of P-Point and Ramsey Ultrafilters .....	37
4.7	$\mathcal{U}$ -Predictors .....	40
4.8	Blass's Evasion and Prediction Setting .....	44
4.9	Open Questions .....	46
<b>5</b>	<b>Dual Hat Problems, Ideals, and the Uncountable .....</b>	<b>49</b>
5.1	Background.....	49
5.2	Dual Hat Problems .....	50
5.3	Hat Problems and Ideals .....	51
5.4	The Role of Non-regular Ultrafilters .....	55
5.5	A Hat Problem Equivalent to the GCH .....	57
5.6	Open Questions .....	59
<b>6</b>	<b>Galvin's Setting: Neutral and Anonymous Predictors .....</b>	<b>61</b>
6.1	Background.....	61
6.2	Applications to Logic and Set Theory .....	63
6.3	Neutral and Anonymous Predictors .....	64
6.4	Neutralizing Predictors .....	66
6.5	Combining with Robustness.....	67
6.6	Robust Neutral Predictors and the Axiom of Choice.....	68
<b>7</b>	<b>The Topological Setting .....</b>	<b>71</b>
7.1	Background.....	71
7.2	The Scattered Sets Result.....	72
7.3	Corollaries.....	75
7.4	Guessing the Future.....	76
7.5	The Philosophical Problem of Induction.....	77
7.6	Proximity Schemes .....	78
7.7	Anonymity in $\mathbf{R}$ .....	81
7.8	Open Questions .....	82
<b>8</b>	<b>Universality of the <math>\mu</math>-Predictor .....</b>	<b>83</b>
8.1	Background.....	83
8.2	Scattered Sets.....	84
8.3	Dynamics of Scattered-Error Predictors .....	85
8.4	Getting an Ordering from a Predictor .....	87
8.5	Visibility Graphs .....	88
8.6	Variations on the $\mu$ -Predictor.....	90
8.7	Results Without the Axiom of Choice .....	91
8.8	Open Questions .....	92
<b>9</b>	<b>Generalizations and Galois-Tukey Connections .....</b>	<b>93</b>
9.1	Background.....	93
9.2	Galois-Tukey Connections.....	94

Contents	xi
9.3 Two-Agent Problems and Morphisms .....	96
9.4 Norms .....	98
9.5 Applications of the Metaphor .....	98
9.6 Scattered-Error Predictors .....	99
9.7 Pseudo-scattered Sets .....	100
<b>Bibliography</b> .....	103
<b>Index</b> .....	107

The Mathematics of Coordinated Inference

A Study of Generalized Hat Problems

Hardin, C.S.; Taylor, A.D.

2013, XI, 109 p., Hardcover

ISBN: 978-3-319-01332-9