Preface

This exercise textbook is the result of several years of experience in teaching courses on Quantitative Finance at our Universities. Although part of the content was already published in an Italian edition a few years ago (2007), this new version has been substantially modified: the exercise collection is much more consistent and three new chapters have been included: one chapter on Arbitrage Theory for discrete-time models, which is a relevant issue in many courses on Mathematical Finance, another on Risk Measures, and one on option pricing in models beyond the classical Black-Scholes framework. The subjects covered by the textbook include option-pricing methods based both on Stochastic Calculus and Partial Differential Equations, and some basic notions about portfolio optimization and risk theory. Many textbooks on option pricing focus *either* on stochastic methods *or* on PDE methods. The aim of the present collection of exercises is to provide examples of both approaches and try to illustrate their mutual interplay with the aid of the fundamental notions. Some of the exercises can be solved by students with a rather basic mathematical background (essentially, we mean an introductory course on Calculus and a basic course in Probability), while others require a more developed mathematical knowledge (a course on Differential Equations, and an introductory course on stochastic processes). The book can be used in courses with mathematical finance contents aimed at graduate students of Engineering, Economics and Mathematics.

The material proposed has been organized in 12 chapters. The first chapter reviews the basic notions of probability and stochastic processes through specific examples, while the second chapter illustrates the fundamental results of (static) portfolio optimization. Chapter 3 presents applications of the basic option-pricing techniques in a discrete-time framework, mainly in the binomial setting. Chapter 4 provides examples of the fundamental results of arbitrage theory in a discrete-time setting. Chapter 5 reviews basic notions on Itô's integration and on SDEs, while Chapter 6 presents exercises on some standard methods in partial differential equations, which turn out to be helpful in solving valuation problems for derivatives in continuous-time models. Chapter 7 focuses on the main results of the Black-Sholes model related to European option-pricing and hedging; examples involving both static and dynamic hedging strategies are illustrated. Chapter 8 deals with more complex derivative products, namely American options: valuation and hedging problems for these options do not admit explicit solutions, except in very few (somehow trivial) cases; approximate solutions can be found by applying suitable numerical techniques. We present a few examples in a discrete-time setting and simple applications of the basic notions. Chapter 9 deals with valuation and hedging of Exotic options; a huge number of different kind of derivative contracts belong to this class, but we focused on the most common type of contracts, in particular those for which an explicit solution for the valuation problem exists in a diffusion setting: Barrier, Lookback, geometric Asian options; moreover, we provide several examples of valuation in a binomial setting. Chapters 10 provides applications of the valuation results for interest rate derivatives: the concern is mainly on short-rate models, but a few examples on the so-called *change of numéraire* technique are included. Chapter 11 attempts to illustrate how the derivatives valuation problem can be attacked in models that drop some of the main assumptions underlying the Black-Scholes model: a few examples, mainly involving affine stochastic volatility models, are provided together with simple examples of jump-diffusion models; all these are typically incomplete market models, and some specific assumptions about the risk-neutral measure adopted by the market in order to assign an arbitrage-free price to contingent claim must be made; the theoretical issues arising in this framework go far beyond the purpose of the present textbook, so we decided to limit our description to the most basic (and popular) models, in which these problems can be avoided by making simple, but reasonable assumptions. Chapter 12 presents some applications of the most important notions related to risk measures; these seem to play an increasingly relevant role in many introductory courses in mathematical finance, so we decided to include some examples in our exercise collection.

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