Contents

Introduction and Outline **1**

1 The joint time-space H^∞-calculus **11**
1.1 The joint H^∞-calculus for tuples of operators 12
a) Sectorial and bisectorial operators, $\mathcal{R}$-boundedness 12
b) Joint H^∞-calculus 17
1.2 Vector-valued Sobolev spaces 27
a) Interpolation of Banach spaces 27
b) Retractions and coretractions 34
c) Definition of Sobolev spaces 36
1.3 The time-space derivative 45
a) Fourier multipliers 45
b) Vector-valued space and time derivatives 52
c) Joint space-time H^∞-calculus 58

2 The Newton polygon approach for mixed-order systems **69**
2.1 Inhomogeneous symbols and the Newton polygon 70
a) Inhomogeneous symbols and principal parts 71
b) Newton polygons and order functions 77
2.2 N-parameter-ellipticity and N-parabolicity 91
a) N-parameter-elliptic symbols and $S_N(L_t \times L_x)$ 92
b) Partition of the co-variable space 94
c) Equivalent characterization of $S_N(L_t \times L_x)$ 98
2.3 H^∞-calculus of N-parabolic mixed-order systems 114
a) The H^∞-calculus of N-parabolic symbols 115
b) Mixed-order systems on spaces of mixed scales 123
c) Remarks on the compatibility condition 132

3 Triebel-Lizorkin spaces and the L_p-L_q-setting **143**
3.1 Vector-valued Triebel-Lizorkin spaces and interpolation 144
3.2 Anisotropic Triebel-Lizorkin spaces and representation by intersections 151

3.3 Auxiliary results on Bessel-valued Triebel-Lizorkin spaces 160
a) The joint time-space H^∞-calculus on Bessel-valued Triebel-Lizorkin spaces 161
b) H^∞-calculus of N-parabolic symbols on Bessel-valued Triebel-Lizorkin spaces 164
3.4 Mixed-order systems on Triebel-Lizorkin spaces 166
3.5 Singular integral operators on L_p-L_q 173
a) Singular integral operators 173
b) Extension symbols 179

4 Application to parabolic differential equations 187
4.1 The generalized L_p-L_q Stokes problem on $\Omega = \mathbb{R}^n$ 188
a) Remarks on homogeneous Sobolev spaces 188
b) The generalized Stokes problem 190
4.2 The generalized L_p-L_q thermo-elastic plate equations on $\Omega = \mathbb{R}^n$. 196
4.3 A linear L_p-L_q Cahn-Hilliard-Gurtin problem in $\Omega = \mathbb{R}^n$ 199
4.4 A compressible fluid model of Korteweg type on $\Omega = \mathbb{R}^n$ 202
4.5 A linear three-phase problem on $\Omega = \mathbb{R}^n$ 205
4.6 The spin-coating process 207
4.7 Two-phase Navier-Stokes equations with Boussinesq-Scriven surface and gravity 214
4.8 The L_p-L_q two-phase Stefan problem with Gibbs-Thomson correction 225

List of Figures 229

Bibliography 231

List of symbols 239

Index 247

http://www.springer.com/978-3-319-01999-4

General Parabolic Mixed Order Systems in Lp and Applications

Denk, R.; Kaip, M.

2013, VIII, 250 p. 16 illus., 1 illus. in color., Hardcover
ISBN: 978-3-319-01999-4
A product of Birkhäuser Basel