

Chapter 2

Flavor in the Standard Model

2.1 The CKM Matrix

The standard model of particle physics describes the fundamental particles and their interactions, the strong, weak and electromagnetic force, very successfully. An important quantum number in this model is the flavor of quarks, introduced by Gell-Mann in 1953 [1]. The three generations of quark flavor pairs are:

$$\begin{pmatrix} \text{u p} \\ \text{d own} \end{pmatrix} \quad \begin{pmatrix} \text{c harm} \\ \text{s trange} \end{pmatrix} \quad \begin{pmatrix} \text{t op} \\ \text{b ottom} \end{pmatrix} \quad (2.1)$$

Each pair consists of an up-type quark with electric charge $+2/3e$ and a down-type quark with charge $-1/3e$. The generations are distinguished by the different masses, increasing from the first to the third.

In the standard model the flavor quantum number is conserved in strong and electromagnetic interactions. It can only be changed by charged current weak processes, described by the exchange of a W^\pm boson. The neutral current weak interaction (Z^0 boson exchange) is flavor-conserving. Therefore flavor changing neutral currents (FCNC) do not occur in the standard model at tree level. This makes FCNC processes a good candidate to search for deviations from the standard model because new particles or new interactions may introduce flavor changing tree level amplitudes, that are of comparable size or larger than the amplitude of the higher order standard model loop processes.

The W boson couples to good approximation to the pairs of left handed quarks within one generation as shown in Eq. (2.1). The β decay of the neutron is one example for such a process. But as the decay $K^- \rightarrow \pi^0 \mu^- \bar{\nu}_\mu$ demonstrates transitions between generations are possible, too, in this case from a s quark to a u quark. The coupling of the W boson to a us pair, however, is much smaller than to a ud pair. To describe this effect Cabibbo introduced a mixing angle θ_c [2] in order to preserve a common coupling parameter g that is multiplied by the factor $\cos \theta_c$ for ud pairs and

$\sin \theta_c$ for us pairs. This leads to a 2×2 mixing matrix for the first two generations of quarks.

This model describes the experimental results for processes involving the first two generations already quite well, but it turned out to be only an approximation of a more general theory. The decays $B^0 \rightarrow D^- \pi^+$ and $B^0 \rightarrow \pi^+ \pi^-$ show that there are also transitions from the third to the second and first generation, respectively. The coupling of the W to a cb or ub pair, however, is even more suppressed than the us coupling. To describe the different coupling strengths across the three generations the 2×2 Cabibbo matrix is extended to a 3×3 matrix. The Cabibbo-Kobayashi-Maskawa matrix, or short CKM matrix,

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad (2.2)$$

determines the coupling of W bosons to pairs of up- (u, c, t) and down-type quarks (d, s, b). The dynamics of the charged current interaction between left handed quarks can be expressed by the Lagrangian

$$\mathcal{L} = -\frac{g}{2} (\bar{u} \ \bar{c} \ \bar{t})_L \gamma^\mu V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W_\mu^+ + \text{h.c.}, \quad (2.3)$$

where the subscript L denotes the left-handed component of the quark fields, γ^μ are the gamma matrices, W_μ^+ is the gauge field, and h.c. stands for the hermitian conjugated of the preceding expression.

The basis of quark states can be changed from the mass eigenstates to a basis where the CKM matrix vanishes. Usually the down-type quarks are transformed:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \quad (2.4)$$

The coupling of the W boson to quarks can then be described by the coupling to ud' , cs' , and tb' pairs with coupling constant g . The states d' , s' , and b' are therefore called weak eigenstates, but one should keep in mind that there is an infinite number of weak bases, related by arbitrary unitary transformations. E.g. one could change the basis of up-type quarks instead of the down-type quark basis by

$$(\bar{u}' \ \bar{c}' \ \bar{t}') = (\bar{u} \ \bar{c} \ \bar{t}) V_{CKM} \quad (2.5)$$

and then describe the charged current interaction by the coupling of W bosons to $u'd$, $c's$, and $t'b$ pairs.

The CKM matrix has 9 complex elements. The unitarity condition

$$V_{CKM} V_{CKM}^\dagger = V_{CKM}^\dagger V_{CKM} = 1 \quad (2.6)$$

leads to nine independent equations and therefore removes nine of the originally 18 free parameters. Furthermore five phases can be eliminated by absorbing them in the quark fields. This leaves four free parameters. These parameters can be chosen as three mixing angles θ_{12} , θ_{23} , and θ_{13} between the three generations and a phase factor δ :

$$\begin{aligned} V_{CKM} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -s_{23} c_{12} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} \end{pmatrix} \quad (2.7) \end{aligned}$$

where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$ with $ij \in \{12, 23, 13\}$.

Note that the fact which elements are real and which are complex depends on the phase convention. Only phase differences have a physical meaning. The phase differences between certain combinations of CKM matrix elements that arises if δ is not equal to zero or π is responsible for CP violation in the quark sector. The connection between phases and experimental observables will be discussed in Chap. 3. The phase δ is not present in the quark mixing matrix for two generations. The achievement of Kobayashi and Maskawa was to show that this one physical phase occurs if a further generation is added. So far all experimentally observed effects of CP -violation could be explained by this single parameter.

However, the phase and the mixing angles are not predicted by theory. They have to be measured by experiments. The precise determination of these fundamental parameters of nature is one of the main goals of flavor physics research.

The measurements reveal a hierarchy of quark mixings between generations: $1 \gg \theta_{12} \gg \theta_{23} \gg \theta_{13}$. This structure can be clearly seen if the CKM matrix is written in the Wolfenstein parametrization [3]

$$V_{CKM} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4) \quad (2.8)$$

where A , ρ and η are real parameters of order unity and $\lambda := |V_{us}| \approx 0.22$. It can be seen that the diagonal elements are of order one and the off-diagonal elements are of order λ , λ^2 , and λ^3 for transitions between first and second, second and third, and first and third generation, respectively. Furthermore one can see that the phase of all elements is very similar (approximately zero in this parametrization), except for V_{ub} and V_{td} , that are involved in transitions between first and third generation and are

suppressed by λ^3 . This makes hadrons containing b quarks a good place to look for large CP -violating effects.

Why does the CKM matrix have this structure? Well, we do not know. The research on flavor physics still has to answer this question. Maybe future experiments will reveal hints towards a more general theory that can explain the CKM matrix.

2.2 Unitarity Triangles

In principle four measurements are sufficient to determine the four free parameters of the Kobayashi-Maskawa theory. But since there are many more measurements possible this opens a great opportunity to search for physics beyond the standard model. This search for new physics by overconstraining the CKM parameters is one of the main tasks of flavor physics experiments.

If the standard model is valid all measurements must be described by a unitary CKM matrix. In particular Eq. (2.6) implies that the product of different columns vanishes:

$$V_{ud} V_{us}^* + V_{cd} V_{cs}^* + V_{td} V_{ts}^* = 0 \quad (2.9)$$

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0 \quad (2.10)$$

$$V_{us} V_{ub}^* + V_{cs} V_{cb}^* + V_{ts} V_{tb}^* = 0 \quad (2.11)$$

Analog conditions can be obtained from the product of different rows of the CKM matrix. These relations can be represented by triangles in the complex plane and are known as unitarity triangles.

The triangle obtained from the multiplication of the first and third column, corresponding to Eq. (2.10), is shown in Fig. 2.1. Because all three sides have similar length and all angles are well different from zero it gives a visually better perceivable representation of the unitarity condition than the nearly degenerate triangles corresponding to Eqs. (2.9) and (2.11). It is therefore usually referred to as *the* unitarity triangle. As one can see in Fig. 2.1 the angles in this triangle are

$$\alpha = \arg \left(-\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*} \right), \quad \beta = \arg \left(-\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right), \quad \gamma = \arg \left(-\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right). \quad (2.12)$$

In order to simplify the display of experimental constraints on the sides and angles of the triangle it is often normalized to $V_{cd} V_{cb}^*$ so that the lower side is fixed to the real axis in the range $[0,1]$ (see Fig. 2.2). After this transformation the coordinates of the apex are given by

$$\bar{\rho} + i\bar{\eta} = -\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}. \quad (2.13)$$

Fig. 2.1 Unitarity triangle for the product of first and third column of the CKM matrix

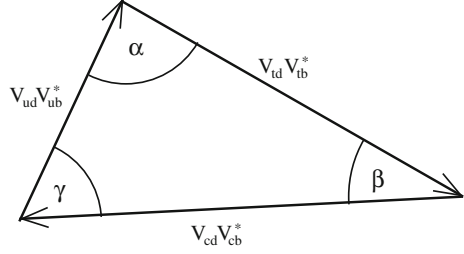


Fig. 2.2 Unitarity triangle for the product of first and third column of the CKM matrix normalized to $V_{cd} V_{cb}^*$

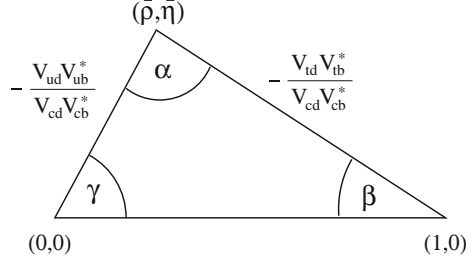
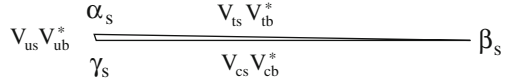


Fig. 2.3 Unitarity triangle for the product of second and third column of the CKM matrix



Since this triangle is constructed from CKM matrix elements for d and b quarks, $b \rightarrow d$ transitions, as occurring in the mixing of B^0 and \bar{B}^0 mesons which are composed of these two quarks, are ideal to study it. The large angles, corresponding to large phase differences between the involved CKM matrix elements, illustrate that large CP violating effects can be expected in the B^0 - \bar{B}^0 system. The situation is different for the triangle obtained from the product of the second and third column, quoted in Eq. (2.11). As can be seen in Fig. 2.3 this triangle, involving the CKM matrix elements for s and b quarks, is nearly degenerate. It gives a visual representation of the small expected CP violation in B_s^0 - \bar{B}_s^0 mixing. Quantitatively this is expressed by the small value of the angle β_s defined as

$$\beta_s = \arg \left(-\frac{V_{ts} V_{tb}^*}{V_{cs} V_{cb}^*} \right). \quad (2.14)$$

Although the unitarity triangles have quite different shapes they all have the same area. The area is a phase convention independent measure of CP -violation and equal to half of the Jarlskog invariant J [4] defined by

$$\text{Im}[V_{ij} V_{kl} V_{il}^* V_{kj}^*] = J \sum_{m,n} \varepsilon_{ikm} \varepsilon_{jln}, \quad (2.15)$$

where ε_{ikm} is the antisymmetric permutation tensor. E.g. for $i = 1$, $j = 2$, and $k = l = 3$ one obtains

$$J = -\text{Im}[V_{us}V_{tb}V_{ub}^*V_{ts}^*] \approx -\text{Im}[\lambda \cdot 1 \cdot A\lambda^3(\rho + i\eta) \cdot (-A\lambda^2)] = \lambda^6 A^2 \eta. \quad (2.16)$$

In case of CP -violation J is finite, corresponding to a non-zero value of η in the Wolfenstein parametrization and leading to non-degenerate unitarity triangles. The world average value of the Jarlskog invariant is $J = (2.96_{-0.16}^{+0.20}) \times 10^{-5}$ [5].

The unitarity triangles provide a good visual representation of the consistency checks of the standard model. The sides and angles must match to form a closed triangle if the standard model is correct. As often only *the* unitarity triangle for the product of first and third column of the CKM matrix is shown it should be noted that this gives a necessary, but not a sufficient condition for the standard model consistency. Even if *the* unitarity triangle shows nice consistency there can still be significant new physics contributions in processes involving CKM matrix elements of the second column. To cover as many new physics scenarios as possible, the high precision measurements of B^0 and B^+ mesons at the B factories have to be accompanied by measurements of the B_s^0 system at the Tevatron. In particular the angle β_s is a promising quantity because its value is approximately zero in the standard model and any measurement of a significant non-zero value would be an indication for new physics.

2.3 Theoretical Tools

In flavor physics we are interested in the quark level transitions. However, in nature all quarks are confined in hadrons (except for the top quark that decays before it can hadronize). Therefore we cannot observe quark level processes directly in experiments. We can only measure decays of hadrons containing quarks. This means we have to calculate the transition amplitude, $\mathcal{M}(B \rightarrow f)$, of a hadron B to a final state f taking into account the interactions of quarks inside hadrons in order to connect the quark level transitions to the observed hadron decay rates.

Different scales are involved in this calculation that allow to apply a factorization approach and certain approximations. In the standard model flavor changing processes are mediated by a W boson. Because the mass of the W is much larger than the mass of quarks inside the hadron the W exchange takes place at a much shorter distance scale, corresponding to a higher energy scale, than the hadronic interaction. Therefore the “hard” process involving a W or other heavy particles, like the top quark or potential new physics particles, can be factorized and treated as a point-like interaction. This approach is used in the operator product expansion (OPE) [6]. In this framework the decay matrix element \mathcal{M} is given by

$$\mathcal{M}(B \rightarrow f) \approx -\frac{4G_F}{\sqrt{2}} V \sum_i C_i(\mu) \langle f | O_i(\mu) | B \rangle. \quad (2.17)$$

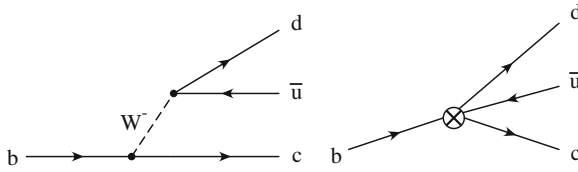


Fig. 2.4 The $b \rightarrow c\bar{u}d$ transition described by a W boson exchange (*left*) and by an effective 4-quark operator (*right*)

By convention the constant $4G_F/\sqrt{2}$, where G_F is the Fermi constant, and the involved CKM matrix elements, denoted V , are factored out. The Wilson coefficients $C_i(\mu)$ contain the hard, and the application of the local operators $O_i(\mu)$ to the initial and final states the soft processes. The separation in hard and soft is arbitrary and determined by the introduced scale μ . In principle the sum has an infinite number of terms, but their size is suppressed by powers of the ratio of quark and W masses squared, m_q^2/m_W^2 , with increasing dimension. So only a finite number has to be considered in order to get a sufficiently accurate result.

Figure 2.4 illustrates the approximation of the W exchange in a $b \rightarrow c\bar{u}d$ transition (left diagram) by a point-like interaction (right diagram). For this tree level process the local operators are given by

$$O_1 = \bar{d}^\alpha \gamma_\mu (1 - \gamma_5) u^\beta \bar{c}^\beta \gamma_\mu (1 - \gamma_5) b^\alpha, \quad (2.18)$$

$$O_2 = \bar{d}^\alpha \gamma_\mu (1 - \gamma_5) u^\alpha \bar{c}^\beta \gamma_\mu (1 - \gamma_5) b^\beta \quad (2.19)$$

where α and β are color indices. The local operators basically provide a projection of the initial and final state hadrons to the quark states, that enter the hard interaction. The operator O_2 describe the case where b and c quarks have the same color charge. The corresponding hard process with Wilson coefficient C_2 is calculated in leading order from the Feynman diagram in Fig. 2.4. The color-rearranged operator O_1 stems from a diagram with an additional gluon connecting the two quark lines.

An advantage of the OPE is that the Wilson coefficients are universal, meaning they are independent of the hadronic final state f . Once they are calculated or measured in one decay mode they can be applied in the prediction for other decay modes. Often the Wilson coefficients are considered effective coupling constants for the point-like interactions described by the local operators. Thus one can use them to define an effective Hamiltonian.

Because of the high scale the Wilson coefficients can be calculated with perturbative methods and are in general well controlled. However, the calculation of the low energetic part is more challenging as non-perturbative methods have to be used.

Often the entire hadronic part is factored out and parametrized by a decay constant or a kinematics dependent form factor. For example the rate of the semileptonic decay $\bar{B}^0 \rightarrow D^+ \ell^- \bar{\nu}_\ell$ is given by the CKM matrix element V_{cb} from the coupling of the

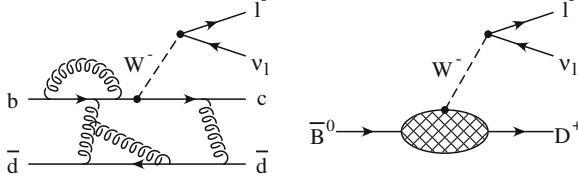


Fig. 2.5 The decay $\bar{B}^0 \rightarrow D^+ \ell^- \bar{\nu}_\ell$ at quark level (*left*) and at hadron level described by a form factor (*right*)

W to the b and c quarks and the transition matrix elements between the hadronic systems:

$$\frac{d\Gamma_{\bar{B}^0 \rightarrow D^+ \ell^- \bar{\nu}_\ell}(q^2)}{dq^2} \sim |V_{cb}|^2 |\langle D^+ | \bar{c} \gamma^\mu (1 - \gamma_5) b | \bar{B}^0 \rangle|^2. \quad (2.20)$$

Here q^2 determines the kinematics and is defined by the four-momenta for the \bar{B}^0 and D^+ meson as $q^2 = [\mathbf{p}(\bar{B}^0) - \mathbf{p}(D^+)]^2$. When the well-known phase-space factor in the hadronic term in Eq. (2.20) is factored out as $\kappa(q^2)$, the remaining part is described by the form factor $f_{\bar{B}^0 \rightarrow D^+}(q^2)$:

$$\frac{d\Gamma_{\bar{B}^0 \rightarrow D^+ \ell^- \bar{\nu}_\ell}(q^2)}{dq^2} \sim |V_{cb}|^2 \kappa(q^2) |f_{\bar{B}^0 \rightarrow D^+}(q^2)|^2. \quad (2.21)$$

Figure 2.5 illustrates the form factor.

Several approaches exist for the calculation of hadronic matrix elements. The best choice of method depends on the case at hand. A powerful handle to simplify theoretical calculations are symmetries. The QCD Lagrangian is invariant under charge conjugation and parity operation so that the exact symmetries of C and P can be exploited. Strong isospin and $SU(3)$ flavor provide further useful symmetries. As these are only approximately valid there can be small corrections to the ideal case of an exact symmetry.

In the case of bottom and charm quarks, when the mass of the heavy quark, m_Q , is much larger than the QCD scale parameter, Λ_{QCD} , another approximate symmetry arises, the heavy quark symmetry (HQS) [7, 8]. In the limit of an infinite mass, the heavy quark can be treated as a source of a static color field. It becomes independent of the flavor, leading to the heavy quark flavor symmetry. This allows to relate quantities in the charm and bottom sector. Another consequence of an infinite quark mass is the decoupling of the spin, leading to heavy quark spin symmetry.

As the b and even more the c quark have finite mass there are corrections to the HQS limit. They can be calculated in the framework of the Heavy Quark Effective Theory (HQET) [9–11] by an expansion in terms of Λ_{QCD}/m_Q . For the calculation of inclusive decay rates the relation $m_Q \gg \Lambda_{QCD}$ is exploited by the Heavy Quark Expansion (HQE) [12–16].

Further techniques include perturbative QCD methods for exclusive decays, QCD sum rules and models of QCD. Another interesting approach is the numerical calculation of hadronic matrix elements in a discretized space-time, called Lattice QCD [17]. In addition to the progress in the theoretical understanding this method profits from the advances in computing technologies.

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