

# Preface

## A Brief Overview of Historical Origins of the Theme

Properties of collective excitations in physical systems are determined, in generic situations, by the interplay of a few fundamental ingredients: spatial dimension, external potential acting on the physical fields or wave functions, the number of independent components of the relevant fields (i.e., one may naturally categorize the systems as single-component *scalar* and multi-component *vectorial* ones), and nonlinear self-interactions of the fields. In particular, the shape of the external potentials determines the system's symmetry, two most ubiquitous types of which correspond to periodic lattice potentials and double-well potentials (DWPs) with the symmetry between the wells.

It is commonly known that the ground state in quantum mechanics exactly follows the symmetry of the underlying potential, while excited states may realize other representations of the same symmetry [1]. In particular, the wave function of the ground state of a particle trapped in the one-dimensional DWP potential is even, with respect to the double-well structure, while the first excited state has the opposite parity, being odd. Similarly, the wave function corresponding to the state at the bottom of the lowest Bloch band induced by the periodic potential features the same periodicity.

While the quantum-mechanical Schrödinger equation is linear for the single particle, the description of rarefied gases formed by quantum bosonic particles (i.e., Bose–Einstein condensates, BECs) is provided by the Gross–Pitaevskii equation (GPE), which, in the mean-field approximation, takes into account effects of collisions between the particles through an effective cubic term, added to the Schrödinger equation for the single-particle wave function [2, 27]. The cubic term, which corresponds to repulsive or attractive forces between the colliding particles, gives rise, respectively, to the self-defocusing (SDF), alias self-repulsion, or self-focusing (SF), i.e., self-attraction, of the wave function. A similar model, based on the nonlinear Schrödinger equation (NLSE) with the cubic term accounting for the effective SF or SDF, describes the propagation of the amplitude of electromagnetic waves in nonlinear optical media [3].

As well as their linear counterparts, the GPE and NLSE include external potentials, which often feature the DWP symmetry. However, the symmetry of the ground state in models with the SF nonlinearity (i.e., the state minimizing the energy at a fixed number of particles in the bosonic gas, or fixed total power of the optical beam—in either case, this is represented by a fixed norm of the respective wave function) follows the symmetry of the underlying potential structure only in the weakly nonlinear regime. A generic effect which occurs with the increase of the norm is *spontaneous symmetry breaking* (SSB). In its simplest manifestation, the SSB implies that the probability to find the particle in one well of the DWP structure is larger than in the other. This, incidentally, implies that another commonly known principle of quantum mechanics, according to which the ground state cannot be degenerate, is no longer valid in the nonlinear models: obviously, the SSB which takes place in the presence of the DWP gives rise to a degenerate pair of two mutually symmetric ground states, with the maximum of the wave function pinned to either potential well.

It should be stressed that the same system admits a symmetric state coexisting with the asymmetric ones, but, above the SSB point, the symmetric wave function no longer represents the ground state, being, in fact, unstable against small symmetry-breaking perturbations. Accordingly, in the course of the spontaneous transition from the unstable symmetric state to a stable asymmetric one, the choice between the two mutually degenerate asymmetric states is determined by random perturbations, which “push” the system with the SF nonlinearity to build the maximum of the wave function in the left or right potential well.

In systems with the SDF nonlinearity, the ground state remains symmetric and stable. In this case, the SSB manifests itself in the form of the spontaneous breaking of the *antisymmetry* of the first excited state (the spatially odd one, which has exactly one zero of the wave function, at the central point, in the one-dimensional setting). The state with the spontaneously broken antisymmetry also features a zero, which may be shifted from the central position.

To the best of my knowledge, the concept of the SSB in nonlinear systems of the NLSE type was first formulated in 1979 by Davies [4], although in a rather abstract form, using a “very mathematical” language. In that work, a nonlinear extension of the Schrödinger equation for a pair of quantum particles, interacting via a three-dimensional isotropic potential, was introduced, and the SSB was predicted in terms of the breaking of the rotational symmetry of the ground state.

Another early work, which predicted the SSB in a relatively simple form, dealt with the *self-trapping model*, which is based on a system of linearly coupled ordinary differential equations (ODEs) with SF cubic terms [5]. This model finds applications to some types of molecular dynamics. In fact, it was work [5] which had made the research community aware of the SSB concept, and helped to initiate a broad work on this topic.

Another important article which studied in detail the SSB in a physically relevant model described by an ODE system addressed the propagation of CW (continuous-wave) optical beams in dual-core nonlinear optical fibers (alias *nonlinear directional couplers*) [6]. In a scaled form, the corresponding system of equations is

$$\begin{aligned}
i \frac{du_1}{dz} + f(|u_1|^2)u_1 + \kappa u_2 &= 0, \\
i \frac{du_2}{dz} + f(|u_2|^2)u_2 + \kappa u_1 &= 0,
\end{aligned} \tag{1}$$

where  $u_1$  and  $u_2$  are the CW amplitudes in the two cores (“CW” implies, in this case, that the amplitudes do not depend on the temporal variable),  $z$  is the propagation distance,  $\kappa$  the coefficient of the linear coupling between the two cores, due to the mutual penetration of evanescent fields from each core into the parallel one, and  $f(|u_{1,2}|^2)$  is a function of the intensity which accounts for the intrinsic nonlinearity of each core. The study of SSB *bifurcations*, which occur with the increase of the intensity of the symmetric mode ( $u_1 = u_2$ ), has demonstrated that, in the simplest case of the Kerr SF nonlinearity, which corresponds to  $f(u^2) = |u|^2$  (in an appropriately normalized form), the symmetry-breaking bifurcation is of the simplest *supercritical*, alias *forward*, type [7], which destabilizes the symmetric state and, simultaneously, gives rise to a pair of stable asymmetric states, with  $|u_1|^2 \neq |u_2|^2$ . The latter states are mutually symmetric, i.e., one is obtained from the other by the interchange,  $u_1 \rightleftharpoons u_2$ . On the other hand, the saturable nonlinearity, in the form of  $f(|u|^2) = |u|^2 / (I_0 + |u|^2)$ , where  $I_0$  is a positive constant (the nonlinearity of this type can be induced by dopants added to the material of the dual-core fiber) gives rise to a *subcritical*, alias *backward*, symmetry-breaking bifurcation. In that case, the branches of asymmetric states, which originate at the point of the destabilization of the symmetric mode, originally go backward (in terms of the total power,  $|u_1|^2 + |u_2|^2$ ), as unstable solutions, and then turn forward, getting stable precisely at the turning point. This scenario implies that the pair of stable asymmetric states actually emerge *subcritically*, at a value of the total power smaller than that at which the symmetric mode loses its stability. In terms of statistical physics, the super- and subcritical bifurcations may be identified as symmetry-breaking phase transitions of the second and first kinds, respectively.

The next essential step in the studies of the SSB phenomenology in dual-core nonlinear optical fibers and similar systems was the consideration of the fields depending on the temporal variable,  $\tau$ . In that case, assuming the anomalous sign of the group-velocity dispersion in the fiber, Eq. (1) are replaced by linearly coupled partial differential equations (PDEs), in the form of a system of NLSEs, which is usually considered with the Kerr nonlinearity:

$$\begin{aligned}
i \frac{\partial u_1}{\partial \tau} + \frac{1}{2} \frac{\partial^2 u_1}{\partial \tau^2} + f(|u_1|^2)u_1 + \kappa u_2 &= 0, \\
i \frac{\partial u_2}{\partial \tau} + \frac{1}{2} \frac{\partial^2 u_2}{\partial \tau^2} + f(|u_2|^2)u_2 + \kappa u_1 &= 0.
\end{aligned} \tag{2}$$

The same system, but with  $\tau$  replaced by transverse coordinate  $x$ , models the spatial-domain evolution of time-independent electromagnetic fields in a dual-core nonlinear planar waveguide.

A commonly known fact is that uncoupled NLSEs give rise to solitons (temporal or spatial solitary waves, in the temporal or spatial domain, respectively) [3]. Accordingly, the SSB bifurcation may destabilize obvious symmetric soliton solutions of system (2),

$$u_1 = u_2 = \eta \operatorname{sech}(\eta\tau) \exp\left(\left(\frac{1}{2}\eta^2 + \kappa\right)z\right) \quad (3)$$

(here  $\eta$  is an arbitrary amplitude of the soliton), replacing them by nontrivial asymmetric two-component soliton modes. The point of the onset of the symmetry-breaking instability of the symmetric solitons with the increase of the soliton's peak power,  $\eta^2$ , was found in an exact analytical form, as  $\eta_{\text{crit}}^2 = 4/3$ , in Ref. [8]. The resulting transition to asymmetric solitons was first predicted, in an approximate analytical form, by means of the variational approximation, in Refs. [9] and [10]. Afterward, it was found that, on the contrary to the supercritical bifurcation of the CW states in system (1) with the Kerr nonlinearity, the symmetry-breaking bifurcation of the solitons in system (2) is subcritical [11, 12].

An independent line of the studies of the SSB originated from the consideration of models of atomic BECs trapped in potential landscapes of the DWP type. The scaled form of the corresponding GPE for the single-particle wave function,  $\psi(x, t)$ , is

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2}\frac{\partial^2\psi}{\partial x^2} + \sigma|\psi|^2\psi + U(x)\psi, \quad (4)$$

where  $\sigma = +1$  and  $-1$  for the repulsive and attractive collision-induced nonlinearity, respectively, and the DWP can be taken, e.g., as

$$U(x) = U_0(x^2 - a^2)^2, \quad (5)$$

where  $U_0$  and  $a^2$  are positive constants. It is relevant to mention that the connection between the equation in the form of PDE (4) and a simpler ODE system (1) (with  $z$  replaced by time  $t$ ) can be established by means of the *tight-binding approximation* [13], replacing  $\psi(x, t)$  by a linear superposition of two stationary wave functions,  $\phi$ , corresponding to states trapped separately in the two potential wells, with centers at points  $x = \pm a$ :

$$\psi(x, t) = u_1(t)\phi(x - a) + u_2(t)\phi(x + a). \quad (6)$$

The analysis of the SSB in BEC and quantum models based on Eq. (4) and similar models was initiated in Refs. [14] and [15]. Most typically, the BEC nonlinearity is self-repulsive, which, as mentioned above, gives rise to the spontaneous breaking of the antisymmetry of the odd states, with  $\psi(-x) = -\psi(x)$ . Further, the GPE can

be extended by adding an extra spatial coordinate, on which the DWP potential does not depend, i.e., one arrives at a two-dimensional GPE with a *double-trough potential*. In such a setting, the self-attractive nonlinearity (which, although being less typical in BEC, is possible too) gives rise to matter-wave solitons, which may self-trap in the free direction [16]. Accordingly, symmetric solitons are possible in the double-trough potential, which are replaced, via the subcritical bifurcation, by stable asymmetric solitons, provided that the number of particles in the BEC (which determines the effective strength of the intrinsic nonlinearity) exceeds a certain critical number [17].

The above discussion was dealing with static symmetric and asymmetric modes supported by various nonlinear systems. The consideration of dynamical regimes, most typically in the form of oscillations of the norm of the wave function between two wells of the DWP structure, i.e., as a matter of fact, between the two mutually degenerate asymmetric states existing above the critical values of the effective strength of the nonlinearity, has been developed too. Following the analogy with well-known Josephson oscillations of the wave function of superconducting electrons in tunnel junctions, formed by bulk superconductors separated by a narrow dielectric layer [18] (note that topological solitons, in the form of quanta of trapped magnetic flux, are well-known in long Josephson junctions of this type [19]), the possibility of similar oscillations in *bosonic Josephson junctions* was predicted [20]. The simplest model of the Josephson oscillations in bosonic systems is based on the dynamical version of Eq. (1), which was derived from the full GPE by means of the tight-binding approximation relying upon expansion (6).

As is the case with many other general topics, especially those in the area of nonlinear science, the variety of theoretically predicted results concerning the SSB phenomenology by far exceeds the number of experimental works. Nevertheless, some experimental manifestations of the SSB have been observed in a clear form. In particular, the self-trapping of a macroscopically asymmetric state of the atomic condensate of  $^{87}\text{Rb}$  atoms with repulsive interactions between them, loaded into the DWP, as well as Josephson oscillations in that setting, were reported in Ref. [21]. On the other hand, the SSB of laser beams coupled into an effective transverse DWP created in the SF photorefractive medium has been explicitly demonstrated in Ref. [22]. Still another result of an experimentally observed SSB effect in nonlinear optics is the spontaneously established asymmetric regime of operation of a symmetric pair of coupled lasers [23]. More recently, symmetry breaking was experimentally demonstrated in a plasmonic coupler [24], although in the latter case the effect was not spontaneous, being induced by a structural element of the system.

## Survey of Chapters in the Present Volume

Studies of SSB effects, self-trapping, and Josephson oscillations in very diverse nonlinear systems have been subjects of a great number of publications, chiefly theoretical ones. These general topics have seen a great deal of development in

many directions. Despite the obvious imbalance between the theoretical and experimental works, the topics call for a comprehensive review article, or even a book, which, as a matter of fact, is still missing. The present volume partly compensates this omission in the literature, offering a collection of 28 chapters which cover many (although definitely not all) aspects of the general themes named in the title of the volume, as well as related topics (even if symmetry breaking, self-trapping, or Josephson junctions are not mentioned in titles of particular chapters, some of these research items are considered in all of them). Most of the chapters are written as semi-review articles, giving an adequate presentation of the respective topics, and also offering references for further reading.

The chapters are briefly surveyed below under rubrics corresponding to different types of physics which are considered in them. It is worthy to note that, quite naturally, not all important branches of the field are covered by particular chapters. In particular, the SSB occurs too in a class of systems with symmetric *pseudopotentials* (rather than usual potential structures), which are induced by appropriate spatial modulations of the local nonlinearity strength. Models of this type were comprehensively reviewed in Ref. [25]. A more specific topic is the SSB of discrete solitons in parallel-coupled dual-core nonlinear lattices [26].

## ***Nonlinear Optics and Plasmonics***

This topical section is the largest one in the volume, including 14 chapters, see the list following below. It is relevant to stress that one of the chapters, “[Spontaneous Formation and Switching of Optical Patterns in Semiconductor Microcavities](#)”, by J. Scheuer and M. Orenstein, includes a vast experimental material. Two chapters, viz., “[Defect Modes, Fano Resonances and Embedded States in Magnetic Metamaterials](#)”, by M. I. Molina, and “[Sub-Wavelength Plasmonic Solitons in 1D and 2D Arrays of Coupled Metallic Nanowires](#)”, by F. Ye, D. Mihalache, and N. C. Panoiu, deal not with optics proper, but rather with plasmonics and metamaterials, which are new directions in studies of the propagation of electromagnetic fields in artificially built media. It is relevant to note that chapters “[Frequency and Phase Locking of Laser Cavity Solitons](#)”, by T. Ackemann, Y. Noblet, P. V. Paulau, C. McIntyre, P. Colet, W. J. Firth, and G.-L. Oppo, “[Guided Modes and Symmetry Breaking Supported by Localized Gain](#)”, by Y. V. Kartashov, V. V. Konotop, V. A. Vysloukh, and D. A. Zezyulin, and “[Pattern Formation Under a Localized Gain](#)”, by A. A. Nepomnyashchy, are dealing with dissipative models of nonlinear optics, while chapter “[Spatial Solitons in Parity-Time-Symmetric Photonic Lattices: Recent Theoretical Results](#)”, by Y.-J. He and B. A. Malomed, presents a short review of solitons in  $\mathcal{PT}$ -symmetric nonlinear models, which are intermediate between conservative systems and usual dissipative ones.

- (1) Nonlinear Dynamics of Bloch wave packets in honeycomb lattices, by M. J. Ablowitz and Y. Zhu.

- (2) Light self-localization and power-dependent steering in anisotropic dielectrics: Spatial solitons in uniaxial nematic liquid crystals, by A. Alberucci and G. Assanto.
- (3) Frequency and phase locking of laser cavity solitons, by T. Ackemann, Y. Noblet, P. V. Paulau, C. McIntyre, P. Colet, W. J. Firth, and G.-L. Oppo.
- (4) Light-induced breaking of symmetry in photonic crystal waveguides with nonlinear defects as a key for all-optical switching circuits, by E. Bulgakov, A. Sadreev, and K. N. Pichugin.
- (5) Spatial solitons in parity-time-symmetric photonic lattices: Recent theoretical results, by Y.-J. He and B. A. Malomed.
- (6) Spontaneous symmetry breaking of pinned modes in nonlinear gratings with an embedded pair of defects, by I. V. Kabakova, I. Uddin, J. Jeyaratnam, C. M. de Sterke, and B. A. Malomed.
- (7) Guided modes and symmetry breaking supported by localized gain, by Y. V. Kartashov, V. V. Konotop, V. A. Vysloukh, and D. A. Zezyulin.
- (8) Analytical solitary wave solutions of a nonlinear Kronig–Penney model for photonic structures consisting of linear and nonlinear layers, by Y. Kominis and K. Hizanidis.
- (9) Trapping polarization of light in nonlinear optical fibers: An ideal Raman polarizer, by V. V. Kozlov, J. Nuño, J. D. Ania-Castañón, and S. Wabnitz.
- (10) Studies of existence and stability of circularly polarized few-cycle solitons beyond the slowly varying envelope approximation, by H. Leblond, D. Mihalache, and H. Triki.
- (11) Defect modes, Fano resonances, and embedded states in magnetic metamaterials, by M. I. Molina.
- (12) Pattern formation under a localized gain, by A. A. Nepomnyashchy.
- (13) Spontaneous formation and switching of optical patterns in semiconductor microcavities, by J. Scheuer and M. Orenstein.
- (14) Sub-wavelength plasmonic solitons in 1D and 2D arrays of coupled metallic nanowires, by F. Ye, D. Mihalache, and N. C. Panoiu.

## ***Bose–Einstein Condensates***

The second largest topical section, which includes nine chapters, is dealing with BEC and related subjects, such as bosonic Josephson junctions. In addition to atomic quantum gases, condensates of quasiparticles (polaritons) are considered too, in chapter “[Symmetry-breaking effects for polariton condensates in double-well potentials](#)”, by A. S. Rodrigues, P. G. Kevrekidis, J. Cuevas, R. Carretero-Gonzalez, and D. J. Frantzeskakis. It is relevant to mention that chapter “[Classical dynamics of a two-species Bose-Einstein condensate in the presence of nonlinear maser processes](#)”, by B. M. Rodriguez-Lara and R.-K. Lee, is dealing with a situation which combines problems in the fields of both BEC and optics.

- (1) From coherent modes to turbulence and granulation of trapped gases, by V. S. Bagnato and V. I. Yukalov.
- (2) Bright solitary matter waves: Formation, stability, and interactions, by T. P. Billam, A. L. Marchant, S. L. Cornish, S. A. Gardiner, and N. G. Parker.
- (3) Temporal quantum fluctuations in the fringe-visibility of atom interferometers with interacting Bose–Einstein condensate, by D. Cohen and A. Vardi.
- (4) Temperature effects on the quantum coherence of bosonic Josephson junctions, by B. Juliá-Díaz, J. Martorell, and A. Polls.
- (5) Multiple fluxon analogues and dark solitons in linearly coupled Bose–Einstein condensates, by M. I. Qadir, H. Susanto, and P. C. Matthews.
- (6) Symmetry-breaking effects for polariton condensates in double-well potentials, by A. S. Rodrigues, P. G. Kevrekidis, J. Cuevas, R. Carretero-González, and D. J. Frantzeskakis.
- (7) Classical dynamics of a two-species Bose–Einstein condensate in the presence of nonlinear maser processes, by B. M. Rodríguez-Lara and R.-K. Lee.
- (8) Existence, stability, and nonlinear dynamics of vortices and vortex clusters in anisotropic Bose–Einstein condensates, by J. Stockhofe, P. G. Kevrekidis, and P. Schmelcher.
- (9) Josephson tunneling of excited states in a double-well potential, by H. Susanto and J. Cuevas.

### ***General Models of Nonlinear Symmetric Systems***

Two chapters deal with general aspects of the symmetry breaking in models of nonlinear systems, which are based on discrete and continual equations of the NLSE type. One chapter is dealing with dissipative dynamics, and the other one reports results for a conservative model:

- (1) Solitons in a parametrically driven damped discrete nonlinear Schrödinger equation, by M. Syafwan, H. Susanto, and S. M. Cox.
- (2) Conditions and stability analysis for saddle-node bifurcations of solitary waves in generalized nonlinear Schrödinger equations, by J. Yang.

### ***Josephson Junctions in Superconductivity***

One chapter addresses the area in which the Josephson junctions had been first predicted and created, namely, traditional low-temperature superconductivity:



- (1) Escape time of Josephson junctions for signal detection, by P. Addesso, G. Filatrella, and V. Pierro.

## ***Micromechanical Systems***

A separate chapter is dealing with a physical setting different from those considered in other chapters, namely, dynamics of nonlinear micromechanical elements:

- (1) Symmetry breaking criteria in electrostatically loaded bistable curved/pre-buckled micro beams, by L. Medina, R. Gilat and S. Krylov.

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