
2.1 A Digest of the Development of Income Taxation

An income tax in the modern sense requires a monetary economy. In barter economies part of the crops, commonly the tithe, had to be ceded to the landlord or to the authorities. Hence, it is no wonder that the first income tax was introduced in the then most industrialized country, viz. the United Kingdom. In December 1798 William Pitt the Younger introduced an income tax to finance the Napoleonic Wars. This tax started with a rate of 0.83 percent for incomes in excess of £60 and increased to 10 percent for incomes exceeding £200; hence, Pitt's income tax was progressive. In the United States, an income tax was introduced in 1861 to finance the Civil War. Its rate was 3 percent for incomes exceeding \$800. In 1862 it was made progressive with rates of 3 percent for incomes between \$800 and \$10,000, and 5 percent for income exceeding \$10,000. In 1864 a third bracket was introduced and the rates were increased to 5 percent, 10 percent, and 15 percent. After some forerunner taxes, several German states adopted income taxes (Bremen in 1848, Prussia in 1851, Hesse in 1869, Saxony in 1874). Prussia's Minister of Finance Miquel introduced a modern progressive income tax in Prussia in 1891. An income tax for Germany as a whole (Reichseinkommensteuer) was introduced in 1920 by Minister of Finance Erzberger. In Austria, a comprehensive tax system including an income tax (Personalsteuergesetz largely elaborated by Böhm-Bawerk) was enacted in 1896 and came in force in 1898, replacing a provisional arrangement enacted in 1848. Although the histories of the income taxes in the various countries were multifaceted (the earliest income taxes were suspended after the wars, then again introduced, again suspended, etc.), one can appraise the nineteenth century as the age of the income tax. Eventually the income tax became considered the "Queen of Taxes."

All proponents of the income tax agreed that it should be *equitable*. However, equitableness was understood in different ways. The two most prominent concepts of equitableness can be ascribed to such venerable sources as the Bible and Aristotle: they considered an equal or a proportional burden sharing as equitable. According to the Bible (Exodus, Chap. 30, Verses 13–15), God considered a poll tax as equitable.

God required each Israelite to contribute half a shekel per year for maintaining the tent of revelation (precursor of the temple), and states explicitly that the rich should not pay more and the poor should not pay less than half a shekel (see also Exodus, Chap. 38, Verse 26).¹ As coins were unknown at that time, half a shekel was given a weight in silver; it amounted to 5.7 grams. Aristotle (Ethics, Book V) remarked “What is just . . . is what is proportional, and what is unjust is what violates the proportion.”

However, it is not immediate how to justify income tax progression in view of these two principles of equitableness: equal versus proportional split of the tax burden. The loophole for reconciliation of tax progression with the two principles of equitableness consisted in the adoption of the principles of equal absolute and proportional sacrifice in terms of equal decreases in the utility of income.² This approach was pioneered by Mill (1848),³ Sidgwick (1883),⁴ Cohen-Stuart (1889), Edgeworth (1897), Cassel (1901), Dalton (1922/1954), and Pigou (1928).

Upon having established the ethical justification of tax progression, several quarters set on to devise progressive tax schedules. These were at first the legislators in the various countries or, more precisely, the jurists who prepared the tax laws. As the jurists had no training in mathematics, they devised rather clumsy tax schedules in three varieties: increasing absolute tax amounts, average tax rates, or marginal tax rates for consecutive tax brackets.⁵ The first method defines absolute tax amounts for the income tax brackets which increase by more than the taxable income in relative terms. This approach has the drawback of internal regression as the average tax rate decreases within each income tax bracket. The second method defines average tax rates for the income tax brackets which increase for consecutive brackets. This approach has the drawback of jumps in the tax burden at the beginning of each bracket such that the additional tax exceeds the additional income for some interval. The third method defines increasing marginal tax rates for consecutive brackets. This method yields a continuous tax schedule. This approach was therefore widely employed, although it has the drawback of discontinuous marginal tax rates. Most of the contemporary tax schedules follow indeed the third method.⁶

¹The New Jerusalem Bible interprets this as a sign that the poor and the rich are equal before God.

²We skip equal marginal sacrifice because it leads to total progression for identical utility functions.

³Mill (1848, p. 804) remarked: “Equality of taxation . . . as a maxim of politics, means equality of sacrifice.” Indeed, Blum and Kalven (1953, p. 49, Footnote 125) attribute the sacrifice principles to Bentham, however, without any quotation.

⁴Sidgwick (1883) remarked: “The obvious equitable principle . . . is that equal sacrifice should be imposed on all.”

⁵See Voigt (1912, pp. 1–23), Bräuer (1927, pp. 24–62), Kipke (1931, pp. 11–40), Pollak (1980, p. 249–53).

⁶For an early sharp criticism of tax schedules as prepared by jurists see Voigt (1912, pp. 23–42). Voigt (1912, p. 34) remarked sarcastically: “Will man Belege sammeln für die Irrationalität menschlicher Einrichtungen, so wird einem kaum ein Gebiet des Lebens deren mehr bieten als die Steuergesetzgebung.” [“If one wants to amass evidence of the irrationality of human institutions, hardly any sphere of human life will feature more instances than tax legislation.”]

The other method of devising tax schedules was taken by some mathematically trained economists who proposed mathematical formulae as possible candidates for progressive tax schedules. Among others, we have to refer to Cassel (1901), Voigt (1912), Timpe (1934, p. 99–112), Folliet (1947), and (with particular emphasis on trigonometric functions) Seidl et al. (1970). These authors proposed a variety of functional shapes of tax schedules.

The justification of progressive taxation and the availability of the machinery for progressive taxation as well as its widespread use in most countries gave impetus to the study of economic effects of progressive taxation. Respective work started with Seligman (1908). Other important contributors were Vickrey (1947), Blum and Kalven (1953), Schmidt (1960), Haller (1970), Lambert (1985a), and—extremely critical—Hayek (1952, 1956).

The identification of the main ingredients of tax progression provoked the demand for measuring the incidence and the intensity of tax progression as well as progression comparisons of different tax schedules. Note that the early attempts of measuring tax progression were confined to tax schedules only. The profession became aware of the role of the income distribution for tax progression only after Dalton (1920) and other scholars had initiated research on income distributions and after data on income distributions became available. In the next section we will provide a concise overview of measures of tax progression restricted to tax schedules only. These measures are commonly called *local* measures of tax progression.

2.2 Measuring Progression of Tax Schedules

For our analyses we introduce the following notation: Y denotes gross (pre-tax) income, $[Y_*, Y_{**}]$, $Y_* > 0$, denotes the support of the income distribution, $T : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, $0 \leq T(Y) < Y$, denotes the income tax schedule, $T(Y)/Y$ denotes the average income tax schedule, $dT(Y)/dY \geq 0$ denotes the marginal income tax schedule. $\Delta(Y) := dT(Y)/dY - T(Y)/Y$ denotes the difference between the marginal and the average tax schedule, $\varepsilon(Y) := (dT(Y)/dY)/(T(Y)/Y)$ denotes the tax elasticity which measures *liability progression*, and $\eta(Y) := (d[Y - T(Y)]/dY)/([Y - T(Y)]/Y)$ denotes the residual income elasticity which measures *residual income progression*. Verbally expressed, the tax elasticity is the ratio of marginal and average tax rates, and the residual income elasticity is the ratio of the marginal and the average retention rates. For the sake of mathematical convenience we assume that all tax schedules are continuously differentiable.

Obviously we have (see also Börs and Genser 1977, p. 416):

$$\frac{d}{dY} \frac{T(Y)}{Y} = \frac{1}{Y} \Delta(Y),^7 \quad (2.1)$$

⁷This progression measure was proposed by Slitor (1948) and Musgrave and Thin (1948, p. 498, Footnote 2).

$$\varepsilon(Y) = 1 + \frac{Y}{T(Y)} \Delta(Y), \quad (2.2)$$

$$\eta(Y) = 1 - \frac{Y}{Y - T(Y)} \Delta(Y), \quad (2.3)$$

$$\varepsilon(Y) = \frac{Y}{T(Y)} + \eta(Y) \left(1 - \frac{Y}{T(Y)}\right), \quad (2.4)$$

$$\varepsilon(Y) = 1 + \frac{Y^2}{T(Y)} \frac{d}{dY} \frac{T(Y)}{Y}, \quad (2.5)$$

$$\eta(Y) = \frac{1 - \frac{T(Y)}{Y} \varepsilon(Y)}{1 - \frac{T(Y)}{Y}}, \quad (2.6)$$

$$\eta(Y) = 1 - \frac{Y^2}{Y - T(Y)} \frac{d}{dY} \frac{T(Y)}{Y}. \quad (2.7)$$

The traditional notion of a progressive [proportional, regressive] tax schedule implies an increasing [constant, decreasing] average tax schedule.⁸ From (2.5) and (2.7) it follows that the elasticity of average tax rates equals $\varepsilon(Y) - 1$ and that the elasticity of average retention rates equals $\eta(Y) - 1$.⁹ A positive [negative] elasticity of average tax rates [elasticity of average retention rates] means that the average tax rates [average retention rates] increase [decrease] relatively in consequence of relative increases in taxable income. This implies the pattern depicted in Table 2.1.

Table 2.1 shows us that the categorization of tax schedules is consistent. This means that if a particular tax schedule is categorized as progressive according to one measure, it is also categorized as progressive according to the other measures.

For comparisons of two tax schedules $T^1(Y)$ and $T^2(Y)$ defined on the same support of the income distributions we have three options. First, $T^1(Y)$ is considered to be more progressive than $T^2(Y)$ at Y , if $T^1(Y)/T^2(Y)$ is increasing at Y (that is, the tax increases more under T^1 than under T^2), second, if $(Y - T^1(Y))/(Y - T^2(Y))$ is decreasing at Y (that is the net income decreases more under T^1 than under T^2), and, third, if $T^1(Y)/Y - T^2(Y)/Y$ is increasing at Y (that is, the average tax rate is higher and increases more for T^1 than for T^2). Simple calculations show that

⁸One can also consider the second derivative of the average tax rate. If the first derivative is positive and the second derivative is positive [zero, negative], then the tax schedule is called accelerated [linear, delayed] progressive. Note that every progressive tax schedule has to be delayed progressive eventually because otherwise the average tax rate would exceed the 100 percent mark (see Pollak 1980, pp. 245–49). The German tax schedule is in its middle part linear progressive (see Fig. 5.13). For more local progression measures see Pfähler and Lambert (1991/92, pp. 289–90).

⁹From (2.6) follows $\eta(Y) - 1 = -T(Y)/(Y - T(Y)) \times [\varepsilon(Y) - 1]$.

Table 2.1 Tax progression/proportionality/regression

Tax schedule	$\frac{d}{dY} \frac{T(Y)}{Y}$	$\Delta(Y)$	$\varepsilon(Y)$	$\eta(Y)$	$\varepsilon(y) - 1$	$\eta(Y) - 1$
Progressive	> 0	> 0	> 1	< 1	> 0	< 0
Proportional	$= 0$	$= 0$	$= 1$	$= 1$	$= 0$	$= 0$
Regressive	< 0	< 0	< 1	> 1	< 0	> 0

$$\frac{d}{dY} \frac{T^1(Y)}{T^2(Y)} > 0 \Leftrightarrow \varepsilon^1(Y) > \varepsilon^2(Y) , \quad (2.8)$$

$$\frac{d}{dY} \frac{Y - T^1(Y)}{Y - T^2(Y)} < 0 \Leftrightarrow \eta^1(Y) < \eta^2(Y) , \quad (2.9)$$

$$\frac{d}{dY} \left[\frac{T^1(Y)}{Y} - \frac{T^2(Y)}{Y} \right] > 0 \Leftrightarrow \Delta^1(Y) > \Delta^2(Y) . \quad (2.10)$$

However, this does not imply that these measures are equivalent with respect to progression comparisons. Note that a change in $\eta(\cdot)$ or $\varepsilon(\cdot)$ will usually be accompanied by a change in $T(Y)$, and, as (2.4) and (2.6) show, the progression as measured by $\eta(\cdot)$ or $\varepsilon(\cdot)$ needs not indicate a change in progression according to the respective other measure.¹⁰ As concerns the average tax schedule, (2.5) shows that $\varepsilon(Y)$ increases if $d[T(Y)/Y]/dY$ increases and $T(Y)$ decreases. Equation (2.7) shows that $\eta(Y)$ decreases if $d[T(Y)/Y]/dY$ increases and $T(Y)$ increases.

For local measures of tax progression we can also make use of Kakwani's (1977b) idea to model progression as a deviation from proportionality. Consider a proportional tax schedule with rate π . Then for a tax schedule $T(Y)$ we can determine an income threshold Y_0 such that $T(Y_0) = \pi Y_0$. $T(Y)$ is defined to be progressive with respect to π if (cf. Pfähler and Lambert 1991/92, p. 302)

$$\frac{T(Y) - \pi Y}{\pi Y} \begin{cases} > 0 & \forall Y > Y_0 \\ \leq 0 & \text{else} , \end{cases}$$

or

$$\frac{[Y - T(Y)] - (1 - \pi)Y}{(1 - \pi)Y} \begin{cases} < 0 & \forall Y > Y_0 \\ \geq 0 & \text{else} . \end{cases}$$

Alternatively, we can define progression as (cf. Pfähler and Lambert 1991/92, p. 303)

¹⁰For numerical examples of former German tax reforms see Seidl and Kaletha (1987) and Seidl and Traub (1997).

$$\frac{T(Y) - T(Y_0)}{T(Y_0)} \left\{ \begin{array}{ll} > \frac{Y - Y_0}{Y_0} & \forall Y > Y_0 \\ \leq \frac{Y - Y_0}{Y_0} & \forall Y \in (0, Y_0] \end{array} \right. ,$$

or

$$\frac{[Y - T(Y)] - [Y_0 - T(Y_0)]}{Y_0 - T(Y_0)} \left\{ \begin{array}{ll} < \frac{Y - Y_0}{Y_0} & \forall Y > Y_0 \\ \geq \frac{Y - Y_0}{Y_0} & \forall Y \in (0, Y_0] \end{array} \right. ,$$

i.e., the relative increase in tax [net income] exceeds [is less than] the relative increase in income for incomes above the threshold, and vice versa for incomes below the threshold.

Note that these comparisons concern arbitrary proportional tax schedules, which can be used as a kind of calibration device for progression measurement. In applications the mean tax rate τ replaces π . However, this would require to explicitly take into account the income distribution, crossing the border line to global measures of tax progression, which are dealt with in the next chapter.

2.3 Equal Sacrifice Principles

The equal sacrifice principles establish a connection between income and utility. Hence, the next step is to look for the properties which the utility functions of income should possess to produce progressive tax schedules according to the equal sacrifice principles. Formally stated, the principle of equal absolute sacrifice as endorsed by Mill (1848) reads as

$$U(Y) - U[Y - T(Y)] := K, \quad K \geq 0, \quad \forall Y \in [Y_*, Y_{**}]. \quad (2.11)$$

The principle of equal proportional sacrifice as endorsed by Cohen-Stuart (1889) reads as

$$\frac{U(Y) - U[Y - T(Y)]}{U(Y)} = 1 - \frac{U[Y - T(Y)]}{U(Y)} := k, \quad 0 \leq k \leq 1 \quad \forall Y \in [Y_*, Y_{**}]. \quad (2.12)$$

Note that both equal sacrifice principles are interrelated. Applying an exponential transformation to (2.11) gives us

$$e^{U(Y) - U[Y - T(Y)]} = e^K \Rightarrow -\frac{e^{U[Y - T(Y)]}}{e^{U(Y)}} = -e^{-K}.$$

Adding $1 = e^{U(Y)}/e^{U(Y)}$ on both sides yields

$$\frac{e^{U(Y)} - e^{U[Y-T(Y)]}}{e^{U(Y)}} = 1 - e^{-K} = 1 - \exp(-K) . \quad (2.13)$$

Write $k := 1 - \exp(-K)$; hence, we have $0 < k \leq 1$, as $0 < \exp(-K) \leq 1$ for $K \geq 0$. But (2.13) is just equal proportional sacrifice as applied to the exponentially transformed utility function $U(\cdot)$, viz. $\exp[U(\cdot)]$.

Conversely, starting from equal proportional sacrifice,

$$1 - \frac{U[Y - T(Y)]}{U(Y)} = k, \quad 0 < k < 1 ,$$

and applying a logarithmic transformation gives us

$$\ln \frac{U[Y - T(Y)]}{U(Y)} = \ln(1 - k) ,$$

and, furthermore,

$$\ln U(Y) - \ln U[Y - T(Y)] = -\ln(1 - k) > 0 . \quad (2.14)$$

Writing $K := -\ln(1 - k)$ demonstrates that $K > 0$, as $0 < k < 1$. But this is just equal absolute sacrifice as applied to the logarithmically transformed utility function $U(\cdot)$, viz. $\ln U(\cdot)$.

This means that we need only investigate one equal sacrifice principle. The results for the other can be attained by applying the respective transformation.

Next, let us combine the equal absolute sacrifice principle with the types of tax schedules. We focus on progressive taxation; the other two types are immediate. Differentiating (2.11) with respect to income yields

$$U'(Y) = [1 - T'(Y)]U'[Y - T(Y)]$$

and, hence,

$$T'(Y) = 1 - \frac{U'(Y)}{U'[Y - T(Y)]} .$$

Substituting $T'(Y)$ into $\eta(Y)$ gives us for progressive tax schedules, i.e., $\eta(Y) < 1$:

$$\frac{Y U'(Y)}{[Y - T(Y)]U'[Y - T(Y)]} < 1 ,$$

which implies

$$Y U'(Y) < [Y - T(Y)]U'[Y - T(Y)] , \text{ where } T(Y) > 0 ,$$

which shows that $YU'(Y)$ is a decreasing function for progressive tax schedules. Differentiating $YU'(Y)$ and setting the derivative negative gives us after re-arrangement

$$-\frac{U''(Y)}{U'(Y)}Y > 1. \quad (2.15)$$

This result demonstrates that a tax schedule derived under the equal sacrifice principle is progressive if the absolute value of the elasticity of the marginal utility used to derive this tax schedule exceeds one. The respective conditions for proportional or regressive tax schedules are gained by replacing the $>$ -sign in (2.15) by a $=$ -sign or a $<$ -sign.

Suppose that $-U''(Y)/U'(Y) \times Y > 1$ had come about by a logarithmic transformation of the utility function $V(\cdot)$, i.e., $U(\cdot) = \ln V(\cdot)$. This means that $V(\cdot)$ describes equal proportional sacrifice. Inserting $U'(Y) = V'(Y)/V(Y)$ and $U''(Y) = \{V''(Y)V(Y) - [V'(Y)]^2\}/[V(Y)]^2$ into (2.15) gives us

$$-\frac{V''(Y)}{V'(Y)}Y + \frac{V'(Y)}{V(Y)}Y > 1. \quad (2.16)$$

Equation (2.16) demonstrates that a tax schedule under equal proportional sacrifice is progressive if the sum of the absolute value of the elasticity of the marginal utility and of the elasticity of utility used to derive this tax schedule exceeds one. For proportional and regressive tax schedules replace $>$ by $=$ or $<$.

Hence, (2.15) and (2.16) demonstrate that the concavity property of a utility function of income does not suffice to establish progression of the resulting tax schedule. Eligible utility functions have to be characterized by additional elasticity properties depending on the sacrifice principle applied.

Recall that both equal sacrifice principles spring from the same root, viz. equal absolute sacrifice. The equal sacrifice principles were originally formulated ad hoc. In view of their venerable old history it is bewildering that they were axiomatized only rather late by Young (1988). Young (1988, pp. 324–5) postulated four axioms (given in our formulation):

1. CONSISTENCY: The tax of each subject should only depend on his or her own taxable income.
2. STRICT MONOTONICITY: Everyone's taxes increase when the total tax burden increases.
3. COMPOSITION: The method used to raise a given amount of tax revenue must also be used to raise any increment in tax revenue.
4. STRICT ORDER PRESERVATION:¹¹ If two individuals would be equally well off (have the same utility level) in the absence of taxation, they should also

¹¹This axiom dates back to the Feldstein (1976, p. 83) formulation of horizontal and vertical equity (Feldstein did not use the term vertical equity outright, but his requirement comes up to vertical equity). We applied Feldstein's formulations.

be equally well off if there is a tax (horizontal equity). More generally, the introduction of a tax should not alter the ordering of individuals by utility level (vertical equity).

Theorem 1. (Young 1988, pp. 326–30) *A tax schedule satisfies axioms 1–4 if and only if it is an equal sacrifice method.*

If the tax schedule is, in addition, also required to be inflation-proof, i.e., scale invariant, then equal absolute sacrifice implies that only the following utility functions are admissible (see Young 1988, pp. 331–2):

$$U(Y) = a + b \ln Y, \text{ or } U(Y) = a - bY^\gamma, \text{ where } a > 0, b > 0, \gamma < 0. \quad (2.17)$$

The exponential transformation of these utility functions yields the respective utility functions for equal proportional sacrifice

$$U(Y) = \tilde{a}Y^b, \text{ or } U(Y) = \tilde{a}e^{-bY^\gamma}, \tilde{a} = e^a > 0, b > 0, \gamma < 0. \quad (2.18)$$

The utility functions resulting from Young's analysis are virtually the same as the ones resulting from a seminal paper by Luce (1959) on the psychophysical laws. Luce combined continua of the stimulus and sensation variables for three types of scales, viz. ratio scales, interval scales, and ordinal scales, which gave him nine cases. When we focus on the case in which a ratio scale is the independent (stimulus) continuum (to neutralize inflation), and the dependent (sensation) continuum is an interval scale (to mimic cardinal utility), and if dimensional constants are absent, then a psychophysical function (here being a proxy for the utility of income) can only be a logarithmic or a power function, the first representing the Weber–Fechner law of poikilitic measurement (see Weber 1834 and Fechner 1860), the second Stevens' theory of magnitude measurement (see Stevens 1975).

Note that the equal sacrifice principles can also be derived from an optimization model:¹²

$$\min_{T(Y)} \left\{ \max_{Y \in [Y_*, Y_{**}]} \{U(Y) - U(Y - T(Y))\} \right\} \text{ subject to } \int_{Y_*}^{Y_{**}} T(Y) f(Y) dY \geq R, \quad (2.19)$$

where $f(Y)$ denotes the density function of the income distribution and R denotes the revenue requirement. The solution of (2.19) by means of the calculus of variations yields a tax schedule which embodies equal absolute sacrifice.

Now we dispose of the necessary ingredients to look at the tax schedules resulting from the above utility functions considering the equal sacrifice principles. It is easily seen that the equal absolute sacrifice applied to logarithmic utility yields a proportional tax schedule with tax rate $(1 - e^{-K})$; the power utility function yields

¹²See Seidl and Schmidt (1988, p. 57), following a suggestion of Wolfram Richter. We indicate only equal absolute sacrifice; equal proportional sacrifice follows immediately.

$$T(Y) = Y - \left[Y^\gamma - \frac{K}{b} \right]^{\frac{1}{\gamma}}. \quad (2.20)$$

Note that the very same tax schedules result if the equal proportional sacrifice is applied to the utility functions shown in (2.18).

Applying the equal proportional sacrifice to the logarithmic utility function of (2.17) yields

$$T(Y) = \left[1 - \frac{Y_0}{Y^b} \right] Y, \quad (2.21)$$

where Y_0 denotes the basic allowance of the tax schedule.

Equation (2.21) has remarkable properties. First, its residual income elasticity is $(1 - b)$, a constant. It is easily checked that (2.21) is progressive with respect to all other measures in Table 2.1. Second, if all gross incomes change by $\lambda > 0$, all net incomes change by $\lambda(1 - b)$.¹³ The problem with this tax schedule is that $\lim_{Y \rightarrow \infty} T(Y)/Y = 1$, that is, the average tax rate converges to 100 percent as taxable income increases indefinitely. Fortunately, the convergence of the average tax rate is rather slow, so this tax schedule is a serious candidate for becoming an actual tax schedule if its average tax rate is bounded from above so that it becomes proportional beyond a certain income threshold.

The tax schedules of the shapes (2.20) and (2.21) are consistent with the equal sacrifice principles. Scores of other progressive tax schedules were suggested, many of them ad hoc.¹⁴ An alternative approach by Pfingsten (1985, 1986, 1987) derived tax schedules by sets of axioms, viz. scale invariance, monotonicity, and μ -compatibility. The latter means that the level of tax progression does not change if gross income inequality does not change. Amongst other things, he showed that for scale invariant inequality concepts tax progression can be expressed in terms of a strictly increasing function of one minus the residual income elasticity.

To recap, this chapter describes local measures of tax progression. Interestingly enough, we will encounter them again in Sect. 4.1, where we adapt these concepts to international and intertemporal comparisons of tax progression.

¹³This means that if all incomes change by $\lambda > 0$, then not only the Lorenz curve of the gross incomes remains the same, but also the Lorenz curve of the net incomes remains the same under this tax schedule. For this reason, Genser (1980) called this tax schedule *Lorenz-equitable*. This tax schedule has a long history. It was, e.g., proposed by Voigt (1912, p. 55) and Vickrey (1947, p. 376). Similar shapes were suggested by Cohen-Stuart (1889), Dalton (1922/1954, p. 68), and Cassel (1901).

¹⁴For an account see Seidl et al. (1970).

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