

Chapter 2

Kinematics of Micropolar Continuum

In this chapter we briefly recall general kinematical relations for a micropolar continuum. For a comprehensive approach, we refer the reader to [1, 2]. The symbolic (direct) tensor notation follows the one by [3, 4], see also Appendix A.

The description of motion of a particle of a micropolar continuum (medium) is based on the assumption that every particle of the micropolar body has six degrees of freedom, see [1, 2]. This is similar to the description of a rigid body in classical mechanics. Three of the degrees of freedom are translational as in classic elasticity, and other three degrees are *orientational* or *rotational*.

In the actual configuration χ at instant t , the position of a particle of micropolar continuum is given by the position vector \mathbf{r} . The particle orientation is defined by an orthonormal trihedron \mathbf{d}_k ($k = 1, 2, 3$) whose vectors are called *directors*. The two vector fields \mathbf{r} and \mathbf{d}_k define the translational and rotational motions of a particle.

To describe the medium relative deformation, we use some fixed position of the body that may be taken at $t = 0$ or another fixed instant; we call this position the *reference configuration* κ . Here the state of particle is defined by the position vector \mathbf{R} , whereas its orientation by directors \mathbf{D}_k (cf. Fig. 2.1). Let us note that as the reference configuration can be chosen not only the real state but also any one.

The motion of a micropolar continuum can be described by the following vectorial fields

$$\mathbf{r} = \mathbf{r}(\mathbf{R}, t), \quad \mathbf{d}_k = \mathbf{d}_k(\mathbf{R}, t). \quad (2.1)$$

In the process of deformation the trihedron \mathbf{d}_k stays orthonormal, $\mathbf{d}_k \cdot \mathbf{d}_m = \delta_{km}$. The change of the directors can be described by an orthogonal tensor that is

$$\mathbf{H} = \mathbf{d}_k \otimes \mathbf{D}_k.$$

\mathbf{H} is called the *microrotation tensor*. So \mathbf{r} describes the position of the particle of the continuum at time t , whereas \mathbf{H} defines its orientation. The orientation of

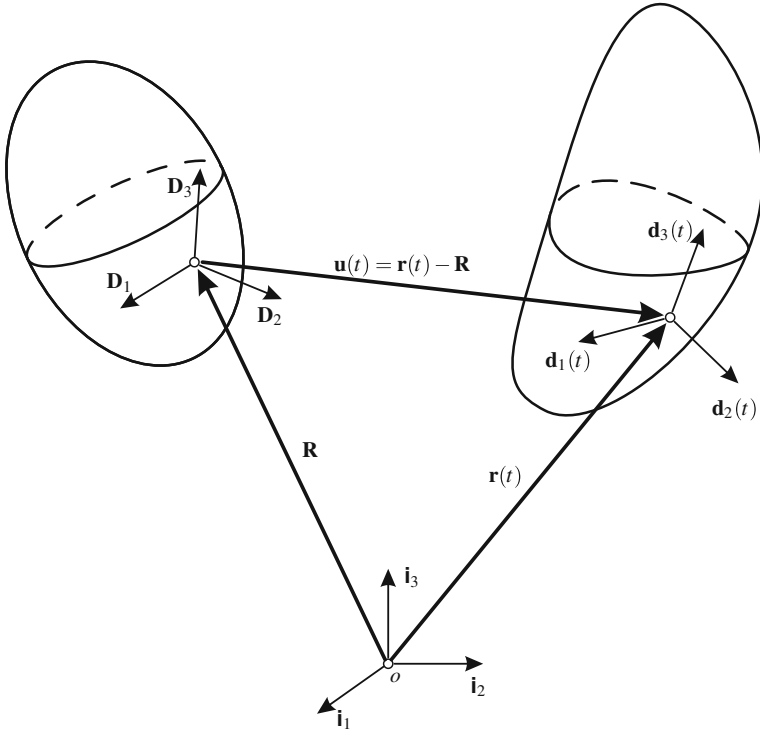


Fig. 2.1 Deformation of a micropolar body (reference and actual configurations)

\mathbf{D}_k and \mathbf{d}_k can be selected the same, so \mathbf{H} is proper orthogonal. Hence, the micropolar continuum deformation can be described by the following relations

$$\mathbf{r} = \mathbf{r}(\mathbf{R}, t), \quad \mathbf{H} = \mathbf{H}(\mathbf{R}, t). \quad (2.2)$$

The linear velocity is given by the relation

$$\mathbf{v} = \dot{\mathbf{r}}. \quad (2.3)$$

For brevity, we use the notation $\dot{(\dots)} \equiv \frac{d}{dt}(\dots)$, where $\frac{d}{dt}$ denotes the material derivative with respect to t . As in classical mechanics, see (B.9), the angular velocity vector, called *microgyration vector*, is given by

$$\boldsymbol{\omega} = -\frac{1}{2} \left(\mathbf{H}^T \cdot \dot{\mathbf{H}} \right)_{\times}, \quad (2.4)$$

where the dot denotes the dot (inner) product and $(\dots)^T$ -transposed. The symbol $(\dots)_{\times}$ stands for the vector invariant of a second-order tensor (cf. (A.4)). In particular,

for a dyad $\mathbf{a} \otimes \mathbf{b}$ we have $(\mathbf{a} \otimes \mathbf{b})_{\times} = \mathbf{a} \times \mathbf{b}$, where \times is the vector (cross) product. Relation (2.4) means that $\boldsymbol{\omega}$ is the axial vector associated with the skew-symmetric tensor $\mathbf{H}^T \cdot \dot{\mathbf{H}}$.

References

1. A.C. Eringen, C.B. Kafadar, Polar field theories. in *Continuum Physics*, vol. IV, ed. by A.C. Eringen (Academic Press, New York, 1976), pp. 1–75
2. A.C. Eringen, *Microcontinuum Field Theory. I. Foundations and Solids* (Springer, New York, 1999)
3. L.P. Lebedev, M.J. Cloud, V.A. Eremeyev, *Tensor Analysis with Applications in Mechanics* (World Scientific, New Jersey, 2010)
4. A.I. Lurie, *Theory of Elasticity* (Springer, Berlin, 2005)

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