Chapter 2
Hammerstein Systems

Abstract A discrete-time cascade of a static nonlinearity followed by a linear dynamics, i.e. the Hammerstein system, is presented. Its equivalence (from the proposed identification algorithms point of view) to some static nonlinear system with the dynamics acting as the source of an additive noise is pointed out. The ample classes of admissible memoryless nonlinear and linear dynamic elements are defined, and the assumptions concerning the input and noise signals are imposed. Selected examples of other block-oriented systems which can be described by the equivalent static system input–output equation are shown. Possible applications to high power amplifier or transmission line modeling are proposed.

We consider a discrete-time Hammerstein system (see Fig. 2.1), that is, a cascade of a nonlinear static (memoryless) block followed by a linear dynamics.

The leitmotif of the book and the main goal of the presented algorithms is to recover the nonlinear characteristics, $m(u)$, of the static part from the pairs of the system input and output measurements $\{(u_k, y_k)\}$, $k = 1, 2, \ldots$. The system is described by the input–output equation

$$y_k = \sum_{i=0}^{\infty} \lambda_i m(u_{k-i}) + z_k \tag{2.1}$$

We assume that the interconnecting signal $v_k = m(u_k)$, between the static and the dynamic part, is not available and that the system output is disturbed by an additive noise, z_k. The following proposition founds a basis for the identification algorithms considered in the book (cf. [52, 55, 57, Chap. 2]).

Proposition 2.1. *The Hammerstein system in Fig. 2.1a is equivalent to the nonlinear memoryless element in Fig. 2.1b, with the input–output equation; cf. (2.1):*

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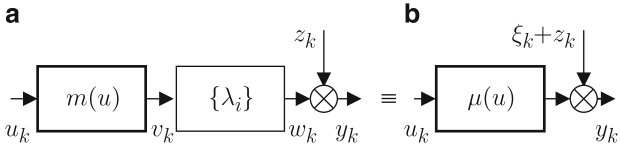

Fig. 2.1 (**a**) A generic Hammerstein system. (**b**) An equivalent static system seen from input–output viewpoint

$$
\begin{aligned}
y_k &= \lambda_d m(u_{k-d}) + \sum_{\substack{i=0 \\ i \neq d}}^{\infty} \lambda_i m(u_{k-i}) + z_k \\
&= \mu(u_{k-d}) + \xi_k + z_k, \qquad (2.2)
\end{aligned}
$$

in which the nonlinear system characteristics *(the* system nonlinearity*), $\mu(u) = \lambda_d m(u) + b_d$, where $\lambda_d \neq 0$ and $b_d = Em(u_1) \sum_{i=0, i \neq d}^{\infty} \lambda_i$,*[1] *is observed in the presence of the external noise z_k and the (zero-mean) system noise, $\xi_k = \sum_{i=0, i \neq d}^{\infty} \lambda_i m(u_{k-i}) - b_d$.*

Our a priori knowledge about the system characteristics and the signals is nonparametric, and we assume that:

1. The input, u_k, is a stationary white noise signal with a probability density function, $f(u)$ being Lipschitz or piecewise-Lipschitz, and strictly positive in the standardized identification interval $[0, 1]$.
2. The nonlinearity $m(u)$ is either a Lipschitz or a piecewise-Lipschitz function in that interval.
3. The dynamic part is linear and asymptotically stable and has the impulse response, $\{\lambda_i\}, i = 0, 1, \ldots$, which is of a finite or an infinite length. We assume that there is no delay in the system, i.e., $\lambda_0 \neq 0$.
4. The external noise, z_k, is any zero-mean second-order stationary signal, white or correlated.

Assumptions 1–4—being of the nonparametric nature—express rather poor prior information about the target system before the identification experiment and impose rather weak restriction on the system characteristics and on the identification conditions. In particular, the input signal can have virtually any bounded and compactly supported probability density function, viz. uniform, triangular, piecewise-constant distribution or Gauss, Cauchy, or Laplace one (truncated to the identification interval).

[1]Note that the multiplicative constant factor λ_d depends only on the system impulse response, while the additive second one, b_d, also on the probability density function of the input signal (i.e., on the identification conditions).

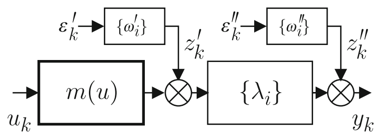

Fig. 2.2 The Hammerstein system with both the interconnecting signal v_k and the system output w_k disturbed by the external noise signals

The target nonlinearity, $m(u)$, can have isolated (jump) discontinuities. The number of jumps is unknown but finite. This assumption is satisfied by, e.g. piecewise-constant, or piecewise-Lipschitz, or, in particular, by piecewise-polynomial functions.

The next assumption, about the dynamic part, admits any discrete-time linear stable systems, that is, the systems with an absolutely summable impulse response, $\sum_{i=0}^{\infty} |\lambda_i| < \infty$. The system can thus have a finite or infinite impulse response. The response can have damped oscillations as shown in the following example:

Example 2.1. Let the dynamic system transfer function, $K_\lambda(z)$, possess only simple poles and no pole at the origin (i.e., there is no delay in the system). With ζ_r, $r = 1, \ldots, p$, denoting real and $(\eta_r, \bar{\eta}_r)$, $r = 1, \ldots, q$, denoting pairs of complex poles of $K_\lambda(z)$, the impulse response is of the well-known form

$$\lambda_n = K_\lambda(0)\,\delta_n + \sum_{r=1}^{p} \alpha_r \zeta_r^n + 2\sum_{r=1}^{q} |\beta_r|\,|\eta_r|^n \cos(n\omega_r + \varphi_r)$$

where $\alpha_r = \lim_{z\to\zeta_r}(z-\zeta_r)\,K_\lambda(z)/\zeta_r$, $\beta_r = \lim_{z\to\eta_r}(z-\eta_r)\,K_\lambda(z)/\eta_r$, and $\varphi_r = \arg\beta_r$ (δ_n is the Kronecker's delta function). In particular, if there exist complex poles $(\eta_r, \bar{\eta}_r)$ or some real poles, ζ_r, are negative, then the impulse response, $\{\lambda_n\}$, includes oscillating components (in our—stable—system, all $|\zeta_r|, |\eta_r| < 1$, i.e., all poles are located within the unit circle, and the oscillations are damped).

Note that the "no-delay" assumption, $\lambda_0 \neq 0$, is made for the clarity of exposition. If $\lambda_0 = 0$, i.e., in the presence of a delay in the system, one can take any other $\lambda_d \neq 0$ and, in the following algorithms, use pairs $\{(u_k, y_{k+d})\}$ instead of $\{(u_k, y_k)\}$; cf. for comparison [57, Chap. 2.2].

As it concerns the external noise, z_k, it can—in general—be correlated and can act on both the input and the output of the Hammerstein system dynamics (cf. Figs. 2.2 and 2.5 in Example 2.5).

Example 2.2 (Multiple noise sources). Let the noise signals $\{\varepsilon_k'\}$ and $\{\varepsilon_k''\}$ in Fig. 2.2 be zero-mean, finite variance *i.i.d.* processes. The equivalent external noise is $z_k = z_k' + z_k''$ with $z_k' = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \lambda_j \omega_i' \varepsilon_{k-(i+j)}'$ and $z_k'' = \sum_{i=0}^{\infty} \omega_i'' \varepsilon_{k-i}''$. Assumption 4 holds provided that the noise filters, $\{\omega_j'\}$ and $\{\omega_j''\}$, are stable.

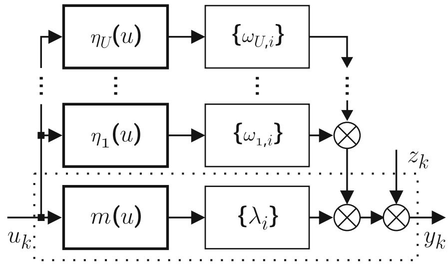

Fig. 2.3 The Uryson system

2.1 Other Systems

Several block-oriented structures can be represented in an equivalent Hammerstein system-like form, and subsequently, their nonlinearities can be recovered using the algorithms designed for the (canonical) Hammerstein systems; cf. [57, 71, 72, 110, Chap. 12]. These systems can be, in turn, used to model any phenomenon having an input nonlinearity followed by a linear dynamics of arbitrary structure, and below, several illustrative examples of such systems and circuits are demonstrated.

Example 2.3 (Uryson system). The Uryson system is an example of a multichannel nonlinear system. Its input–output equation has the following form (see Fig. 2.3 and cf., e.g., [38]):

$$\begin{aligned} y_k &= \sum_{i=0}^{\infty} \lambda_i m(u_{k-i}) + \sum_{u=1}^{U} \sum_{i=0}^{\infty} \omega_{u,i} \eta_u(u_{k-i}) + z_k \\ &= \mu(u) + \xi_k + z_k, \end{aligned}$$

where the system nonlinearity is given by the formula

$$\mu(u) = \lambda_0 m(u) + \sum_{u=1}^{U} \omega_{u,0} \eta_u(u) + b_0$$

with $b_0 = \sum_{i=1}^{\infty} \lambda_i Em(u_{k-i}) + \sum_{u=1}^{U} \sum_{i=1}^{\infty} \omega_{u,i} E\eta_u(u_{k-i})$, i.e., the system nonlinearity $\mu(u)$ is now a weighted sum of all nonlinearities from the system's branches (with unknown and system dependent weights). Observe however that the single nonlinearity $m(u)$ can still be separated from other nonlinearities $\eta_u(u)$, $u = 1, \ldots, U$, when the dynamics in their channels have nonzero delays (cf. Example 2.4). Moreover, when all the channel nonlinearities $\eta_u(u)$ are active (nonzero) in input signal ranges nonoverlapping with the active input range of $m(x)$, i.e., if it holds that $\operatorname{supp} \mu(x) \cap \operatorname{supp} \eta_u(x) = \emptyset$ for all $u = 1, \ldots, U$, then again, the $m(x)$ is separated from other nonlinearities in its activity region.

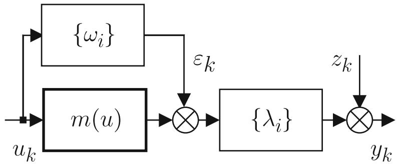

Fig. 2.4 The Hammerstein system with a parasitic dynamics

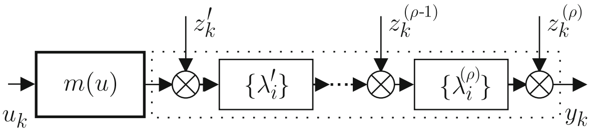

Fig. 2.5 The nonlinear transmission line modeled as a Hammerstein system

Example 2.4 (Parasitic parallel dynamics). A nonlinear system with a parallel nuisance (parasitic, lumped) dynamics (Fig. 2.4) has the following Hammerstein system representation:

$$y_k = \sum_{i=0}^{\infty} \lambda_i \left[m\left(u_{k-i}\right) + \sum_{i=0}^{\infty} \omega_i u_{k-i} \right] + z_k = \mu\left(u_k\right) + \xi_k + z_k,$$

where

$$\mu\left(u\right) = \lambda_0 \left[m\left(u\right) + \omega_0 u\right] + b_0,$$

with $b_0 = b_0' + b_0''$, and $b_0' = \sum_{i=1}^{\infty} [\lambda_i Em\left(u_{k-i}\right) + \lambda_0 \omega_i Eu_{k-i}]$, $b_0'' = \sum_{i=1}^{\infty} \sum_{j=0}^{\infty} \lambda_i \omega_j Eu_{k-i-j}$. Such system noise has a bit more complicated structure, $\xi_k = \xi_k' + \xi_k''$, where $\xi_k' = \sum_{i=1}^{\infty} [\lambda_i m\left(u_{k-i}\right) + \lambda_0 \omega_i u_{k-i}] - b_0'$ and $\xi_k'' = \sum_{i=1}^{\infty} \lambda_i \sum_{j=0}^{\infty} \omega_j u_{k-(i+j)} - b_0''$. Note that if there is a delay in the parasitic channel (and e.g. $\omega_0 = 0$), then we get the "memoryless system" relation

$$\mu\left(u\right) = \lambda_0 m\left(u\right) + b_0,$$

with $b_0'' = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \lambda_i \omega_j Eu_{k-i-j}$, and $\xi_k'' = \sum_{i=1}^{\infty} \lambda_i \sum_{j=1}^{\infty} \omega_j u_{k-(i+j)} - b_0''$.

Example 2.5 (Transmission line). A transmission line with an input nonlinearity can be modeled as the Hammerstein system, see Fig. 2.5. The Assumptions 3–4 are clearly fulfilled if all noises, $z_k^{(r)}$, $r = 0, \ldots, \rho$ are zero-mean second-order stationary processes and all the elementary components of the transmission line $\{\lambda_i^{(r)}\}$, $r = 1, \ldots, \rho$, are linear and asymptotically stable dynamics; cf. Example 2.2.

Example 2.6 (Doherty amplifier). The Doherty amplifier is a nonlinear circuit used in radio amplifiers and has recently been applied in many microwave devices (e.g., in OFDMA-based wireless transmitters; see [20,77,88,114]). It can be seen as a system with the input nonlinearity composed of two parallel nonlinear static subsystems

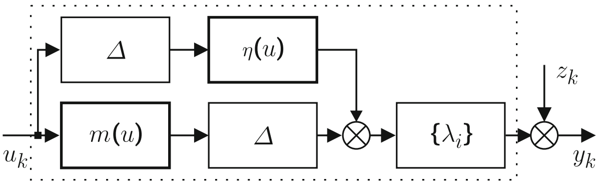

Fig. 2.6 The Doherty amplifier (Δ denotes a pure (unit) delay subsystem, i.e., $\Delta\left[m\left(u_k\right)\right] = m\left(u_{k-1}\right)$ and $\eta\left(\Delta\left[u_k\right]\right) = \eta\left(u_{k-1}\right)$)

$m(u)$ and $\eta(u)$, followed and preceded by the pure-delay elements Δ, respectively, see Fig. 2.6. Such a model has thus an equivalent, Hammerstein-like, input–output equation:

$$y_k = \sum_{i=0}^{\infty} \lambda_i \left\{\Delta\left[m\left(u_{k-i}\right)\right] + \eta\left(\Delta\left[u_{k-i}\right]\right)\right\} + z_k$$

$$= \sum_{i=1}^{\infty} \lambda_{i-1}\left[m\left(u_{k-i}\right) + \eta\left(u_{k-i}\right)\right] + z_k = \mu\left(u_{k-1}\right) + \xi_k + z_k,$$

where the system nonlinearity is a (weighted) sum of both nonlinearities

$$\mu(u) = \lambda_0\left[m(u) + \eta(u)\right] + b_0,$$

with $b_0 = \sum_{i=2}^{\infty} \lambda_{i-1} E\left[m\left(u_1\right) + \eta\left(u_1\right)\right]$, and $\xi_k = \sum_{i=2}^{\infty} \lambda_{i-1}[[m\left(u_{k-i}\right) + \eta\left(u_{k-i}\right) - E\left[m\left(u_1\right) + \eta\left(u_1\right)\right]]$.

2.2 Notes

The assumption about the input signal independence, while often met in the literature (see, e.g., the classical lectures by Wiener [153] and by Lee and Schetzen [93]), can clearly be pointed out as a limitation in those applications where the input signal is neither white nor can be controlled; cf. [123] and [97]. Still, in modern transmission systems, one can find the *i.i.d.* input signals being generated by the stream encoding/compressing transmitters (since a well-compressed datastream is, *in principle*, a white (and, furthermore, often of uniform distribution) process; cf. e.g. [143]).

In case when stochastic dependence of the input signal cannot be neglected, one should consider the *Wiener system* model, in which (in the simplest case) a single input nonlinearity is preceded by a linear dynamics or a *sandwich* structure, where the Wiener and the Hammerstein systems are connected in a cascade; see Fig. 2.7.

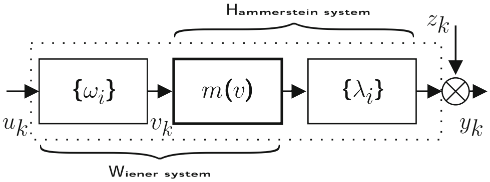

Fig. 2.7 The sandwich system being a cascade of the Wiener $(\{\omega\}, m(v))$ and Hammerstein systems $(m(v), \{\lambda_i\})$

Nevertheless, it should be noted that nonparametric identification of these systems and, in particular, recovery of their nonlinearities remains a challenging problem (see the very recent results by Greblicki e.g. [48–50, 57, Chaps. 9 and 14] and [51], Pawlak et al. [111], Mzyk [105], and cf. Giri and Bai [42]).

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