Preface

This book is about finite elements and Green's functions, two seemingly very different topics, one representing modern numerical analysis and the other representing the old way of doing it, slowly converging infinite sums and strange looking integrals—none of which will appear in this book—and limited in its scope to linear problems. But the opposite is true, finite elements and Green's functions have very many things in common, may we just mention the fact that the columns of the inverse stiffness matrix are the discrete Green's functions of the nodal values. To a young engineer who is easily fascinated by all the powerful tools and machinery he has available today it may come as a surprise that the finite element method basically can be seen as a Green's function method and that when it is applied to nonlinear problems many of these features shine through as the success of goal-oriented adaptive refinement proves conclusively. So in some sense a relic of the old past is the driving force in the computer programs we use today.

The Green's functions are so to speak the *physical basis functions*, the "true" basis functions of a problem and finite element analysis is all about approximating these function with nodal basis functions. These discrete Green's functions form the machinery behind the finite element code, they produce the output the engineer sees on the screen.

In 1999 the author attended an IUTAM-symposium on boundary elements in Cracow, Poland. At lunch Wolfgang Wendland mentioned that Fehmi Çirak in Stuttgart had applied "Rannacher's method" to an engineering problem in shell theory [1], that Çirak had traveled for one month twice a week to Heidelberg for private lessons by Rannacher on goal-oriented adaptive refinement with Green's functions.

Later Ekkehard Ramm, Çiraks thesis adviser, discovered that the key equation had already been published by Hugh Tottenham in Southampton as early as 1970 [2], though hardly anyone noticed this at that time or seemingly understood the central message this equation contained—otherwise we could have had goal-oriented adaptive refinement as early as 1970.

As an aside, probably there are also other precursors because in mathematics Green's functions are the tool of choice when it comes to pointwise estimates [3]. The intention of this (unproven) remark is not to diminish the pioneering insight of Hugh Tottenham.

The conference in Cracow was the first time the author heard about goal-oriented refinement. After returning home and reading Çirak's thesis it suddenly dawned on him that Green's functions were not just a tool for pointwise estimates but that they were the gist of the matter: the finite element method itself was a Green's function method—from the start.

Seemingly, there is no need to draw circles around the point loads, to soften their impact, to avoid the infinite energy associated with most Green's functions. The algorithm itself does not care with the consequence that the machinery in an finite element program, so to speak, consists of kernel functions which are the solutions of ill-posed problems. One is reminded of the success of the X-FEM which also does everything "wrong".

A first tentative paper was published [4], and in a later book on Structural Analysis with Finite Elements we and our co-author Casimir Katz chartered the new found territory in all directions [5]. Suddenly we could explain things from a new perspective, point at possible difficulties due to the complex nature of the influence functions. With each day the two authors learnt something new. Now after more than 10 years have elapsed since the conference in Cracow the current author has the feeling that it is time to treat the subject more systematically, to summarize the main points, and to trace out the main features. This is why we have written this book.

It is the attempt of an engineer to come to terms with the subject of Green's functions and finite elements. We are well aware that a more elaborate exposition of this topic would have been desirable but we hope that what the book misses in rigor and exactness is made up by an ample provision of engineering examples and applications.

Kassel, Germany, March 2012 Friedel Hartmann

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