

Contents

1	Introduction	1
1.1	What are Green's Functions?	2
1.2	The Importance of Green's Functions for FE-Analysis	3
1.3	A 1-D Problem	4
1.3.1	The Analytical Solution	4
1.3.2	Green's Function	4
1.3.3	Finite Elements	6
1.3.4	Finite Elements and Green's Function	7
1.4	Entanglement.	9
1.4.1	Functionals	12
1.4.2	Proof	16
1.5	Goal Oriented Refinement.	18
1.6	Model Adaptivity.	19
1.6.1	Local & Global	26
1.7	How to Calculate Influence Functions with Finite Elements	27
	References	34
2	Basic Concepts.	35
2.1	Elements of Functional Analysis	36
2.1.1	Notation	37
2.1.2	Vector Spaces and Scalar Product	38
2.1.3	Linear Functionals	39
2.1.4	Projection	41
2.1.5	Variational Problems	42
2.1.6	Equivalent Norms	43
2.1.7	Galerkin Method	46
2.1.8	Sobolev Spaces	46
2.2	Green's Identities.	49
2.2.1	Gauss' Theorem.	49
2.2.2	The Laplace Operator.	49

2.2.3	Linear Self-Adjoint Operators	53
2.3	Duality	54
2.3.1	Linear Algebra.	55
2.3.2	Vectors and Linear Functionals	57
2.4	Influence Functions	58
2.4.1	Influence Function for $u(x)$	59
2.4.2	Influence Function for $u'(x)$	61
2.4.3	Weak Influence Function for $u(x)$	63
2.4.4	A Sequence that Converges to G_1	67
2.4.5	Elevators and Escalators	69
2.4.6	Influence Functions in Higher Dimensions	71
2.4.7	Weak Influence Functions.	73
2.4.8	Non-Zero Boundary Values.	74
2.4.9	Average Values of Stresses	75
2.5	Properties of Green's Functions	79
2.5.1	Modern Approach	82
2.5.2	Maxwell	82
2.5.3	Modes of Decay	83
2.5.4	Dipoles and Monopoles	85
2.5.5	Multipole Expansion	87
2.5.6	Infinite Energy.	91
2.5.7	Genealogy of Influence Functions	92
2.6	Sobolev's Embedding Theorem	93
2.7	Fundamental Solutions	95
2.7.1	Influence Function	96
2.8	Ill-Posed Problems	97
2.9	Nonlinear Problems	98
2.9.1	Lagrange Multiplier	99
2.9.2	Lagrange Multiplier and Linear Algebra.	100
2.9.3	Nonlinear Functionals.	101
2.9.4	Nonlinear Problems	103
2.10	Mixed Problems.	106
	References	107
3	Finite Elements and Green's Functions.	109
3.1	Poisson Equation	110
3.2	The FE-Load Case p_h	112
3.2.1	Distributions	114
3.2.2	Notation	114
3.3	Extensions.	118
3.3.1	Betti's Theorem: Extended	118
3.3.2	Tottenham's Equation.	119
3.3.3	Maxwell's Theorem: Extended	122

3.4	Proxies	124
3.4.1	$G_h = G$ on \mathcal{V}_h^*	125
3.4.2	$\delta_h = \delta$ on \mathcal{V}_h	126
3.4.3	$J_h(u) = J(u)$ on \mathcal{V}_h	129
3.4.4	Summary	133
3.5	Dirac Energy	135
3.6	Generalized Green's Functions	139
3.6.1	Arbitrary Deltas	140
3.7	Influence Functions for Integral Values	144
3.7.1	Nodal Forces	145
3.8	Weak Influence Functions	147
3.9	Weak Influence Functions Have More Choices	151
3.10	Nodal Form of Influence Functions	153
3.10.1	Numerical Effort	155
3.10.2	Sensitivity Plots	155
3.10.3	What is $j^T u$?	158
3.11	Nodal Values of Green's Functions	158
3.11.1	Finite Differences and Finite Elements	159
3.11.2	Influence Function for $p(x)$	159
3.11.3	The Foot Print of p_h	162
3.12	The Inverse Stiffness Matrix	163
3.12.1	Examples	165
3.13	Condition of a Stiffness Matrix	167
3.13.1	The Triple Product	170
3.14	Interpolation	171
3.14.1	The Nodal Vector u_I	173
3.15	Infinite Stresses	178
3.15.1	Archimedes' Lever	179
3.15.2	Continuous Beam	182
3.15.3	Cantilever Plate	183
3.15.4	Summary	185
3.16	Why Do Singularities Matter?	185
3.17	Nature Makes No Jumps: Finite Elements Do	186
3.18	Influence Functions for Support Reactions	188
3.18.1	Global Equilibrium	190
3.18.2	A Paradox?	192
3.19	The Path the Load Takes	192
3.20	The Path the Influence Function takes	195
3.21	Mixed Problems	197
3.21.1	Tottenham's Equation for Mixed Problems	202
3.22	Condensation of a Stiffness Matrix	203
3.23	p -Method	205
	References	208

4	The Discretization Error	209
4.1	Asymptotic Error Analysis	210
4.2	Goal-Oriented Refinement	213
4.3	Comparison	214
4.4	Primal and Dual Error	216
4.5	An Analysis of the Goal-Oriented Error Estimator	217
4.6	The Algebra of the Residuals	219
4.7	Goal-Oriented Refinement for Nonlinear Problems	220
4.7.1	Estimates	221
4.7.2	Nonlinear Functionals	224
4.7.3	Implementation	224
4.8	Drift	227
4.9	Combination of Modeling and Discretization Error	229
4.10	Pollution	230
4.11	Gauss Points and Green's Functions	237
	References	239
5	Modeling Error	241
5.1	Linear Algebra	242
5.2	Summary	244
5.2.1	Determining u_c	244
5.2.2	Determining Effects	245
5.3	Woodbury-Sherman-Morrison Formula	248
5.3.1	One Entry on the Diagonal Changes, $k_{ii} + \Delta k$	248
5.3.2	The Inverse of the Updated Stiffness Matrix K_c	249
5.4	Direct Formulations	251
5.5	Force Method	251
5.5.1	Notation	253
5.5.2	Changes on the Diagonal	254
5.5.3	The Inverse	255
5.6	Example	256
5.6.1	What It Means	257
5.6.2	The Inverse of ΔK_e	259
5.6.3	Example	262
5.6.4	Collapse	265
5.7	Functionals	266
5.7.1	The Gradient of a Functional	267
5.8	Weak Formulations and the d -Term	268
5.8.1	Linear Problems	269
5.8.2	The Error in Functionals	271
5.9	The Basic Idea	278
5.9.1	Continuous and Discrete Case	278
5.9.2	Long & Strong and Short & Weak	280
5.9.3	Estimates	281

5.10	The Approximation $u_c \approx u$	281
5.11	Linearization	283
5.12	Engineering Sensitivity Analysis	286
5.12.1	Focus on a Point	286
5.12.2	Focus on an Element	295
5.12.3	Beams	301
5.12.4	Poisson Problem	303
5.12.5	Kirchhoff Plates	305
5.12.6	Analysis	306
5.13	Equations for the Unknown Stresses on Ω_e	307
5.13.1	Computational Aspects	309
5.14	Adjoint Method of Sensitivity Analysis	313
5.15	Linear Versus Nonlinear	315
	References	317
6	Appendix	319
6.1	Nonlinear Elasticity	319
6.1.1	Linearization	321
6.2	Software	322
	Reference	323
	Index	325



<http://www.springer.com/978-3-642-29522-5>

Green's Functions and Finite Elements

Hartmann, F.

2013, XIV, 330 p., Hardcover

ISBN: 978-3-642-29522-5