Chapter 2
Literature Survey on Stochastic Wave Models

This chapter aims at providing a comprehensive, up-to-date review of statistical models proposed for modeling long-term variability in extreme waves and sea states as well as a review of alternative approaches from other areas of application. A review of wave climate projections is also included. Efforts have been made to include all relevant and important work to make this literature survey as complete as possible, which has resulted in a rather voluminous list of references at the end of the chapter. Notwithstanding, due to the enormous amount of literature in this field some important works might inevitably have been omitted. This is unintended and it should be noted that important contributions to the discussion herein might exist of which I have not been aware. Nevertheless, it is believed that this literature study contains a fair review of relevant literature and as such that it gives a good indication of state-of-the art within the field and may serve as a basis for further research on stochastic modeling of extreme waves and sea states. A brief review of available wave data sources is also presented in this chapter. The literature survey presented in this chapter is based on [196].

2.1 Wave Data and Data Sources

As in all statistical modeling, a crucial prerequisite for any sensible modeling and reliable analysis is the availability of statistical data. For example, if models describing the spatio-temporal variability of extreme waves are to be developed, wave data with sufficient spatio-temporal resolution is needed. Furthermore, the lack of adequate coverage in the data will restrict the scope of the statistical models that can be used.

Wave data can be obtained from buoys, laser measurements, satellite images, shipborne wave recorders, or be generated by numerical wave models. Of these, buoy measurements are most reliable, but the spatial coverage is limited. For regions where buoy data are not available, satellite data may be an alternative for estimation of wave heights [117, 152], and there are different satellites that collect such data.

E. Vanem, *Bayesian Hierarchical Space-Time Models with Application to Significant Wave Height*, Ocean Engineering & Oceanography 2,
DOI: 10.1007/978-3-642-30253-4_2,

Examples of satellite missions are the European Remote Sensing Satellites (ERS-1 and ERS-2), the Topex/Poseidon mission and Jason-1 and -2 missions. Some of the data from these satellite missions are available from various online sources on the Internet.

Wave parameters derived from satellite altimeter data were demonstrated to be in reasonable agreement with buoy measurements by the end of last century [102]. More recently, further validation of wave heights measured from altimeters have been performed, and the agreement with buoy data is generally good [63, 156]. However, corrections due to biases may be required, and both negative and positive biases for the significant wave height have been reported, indicating that corrections are region-dependent [136]. Sea state parameters such as significant wave height derived from synthetic aperture radar images taken from satellites were addressed in [123].

Ship observations are another source of wave data which covers areas where buoy wave measurements are not available. The Voluntary Observing Ship (VOS) scheme has been in operation for almost 150 years and has a large set of voluntary collected data. However, due to the fact that ships tend to avoid extreme weather whenever possible, extreme wave events are likely to be under-represented in ship observations and hence such data are not ideally suited to model extreme wave events [57, 150].

Recently, a novel wave acquisition stereo system (WASS) based on a variational image sensor and video observational technology in order to reconstruct the 4D dynamics of ocean waves was developed [68]. The spatial and temporal data provided by this system would be rich in statistical content compared to buoy data, but the availability of such data are presumably still limited. Data quality and validation may also be an open issue.

In general, measurements of wave parameters are more scarce than meteorological data such as wind and pressure fields which are collected more systematically and covering a wider area. An alternative is therefore to use output from wave models that uses meteorological data as input rather than to use wave data that are measured directly.

Wave models are normally used for forecast or hindcast of sea states [90]. Forecasts typically predict sea states up to 3–5 days ahead. Hindcast modeling can be used to calibrate the models after precise meteorological measurements have been collected. It can also be used as a basis for design but it is stressed that quality control is necessary and possible errors and biases should be identified and corrected [24]. In general, it is acknowledged that wave buoys are regarded as highly accurate instruments, and it is stated in, e.g., [22] that both the systematic and random error of significant wave height measurements by buoys are negligible. However, when calibrating hindcast data against observations, the data will still be subject to epistemic uncertainty due to the way the calibration is carried out and high values of significant wave height will normally be more affected by uncertainties, as discussed in [24].

Currently, data are available from various reanalysis projects [38]. For example, 40 year of meteorological data are available from the NCEP/NCAR reanalysis project [111] that could be used to run wave models [52, 187]. A more recent reanalysis project, ERA-40 [193], was carried out by the European Center for Medium-Range Weather Forecasts (ECMWF) and covers a 45-year period from 1957 to 2002. The data contain six-hourly fields of global wave parameters such as significant wave height, mean wave direction and mean wave period as well as mean sea level pressure and wind fields and other meteorological parameters. A large part of this reanalysis data are freely available for download from their website for research purposes.[1]

It has been reported that the ERA-40 dataset contains some inhomogeneities in time and that it underestimates high wave heights [185], but corrected datasets for the significant wave height have been produced [36]. Hence, a new 45-year global six-hourly dataset of significant wave height has been created, and the corrected data show clear improvements compared to the original data. In [39] it is stated that this dataset can be obtained freely from the authors for scientific purposes.

2.2 Review of Statistical Models for Extreme Waves

In order to model long-term trends in the intensity and frequency of occurrence of extreme wave events or extreme sea states due to climate change, appropriate models must be used. There are numerous stochastic wave models proposed in the literature, but most of these are developed for other purposes than predicting such long-term trends. Models used for wave forecasting, for example in operational simulation of safety of ships and offshore structures typically have a short-term perspective, and cannot be used to investigate long-term trends. Also, many wave models assume stationary or cyclic time series, which would not be the case if climate change is a reality.

There are different approaches to estimating the extreme wave heights at a certain location based on available wave data, and some of the most widely used are the initial distribution method, the annual maxima method, the peak-over-threshold method, and the MEan Number of Up-crossings (MENU) method. The initial distribution method uses data (measured or calculated) of all wave heights and the extreme wave height of a certain return period is estimated as the quantile h_p of the wave height distribution $F(h)$ with probability p. The annual maxima approach uses only the annual (or block) maxima and the extreme wave height will have one of the three limit distributions referred to as the family of the Generalized Extreme Value distribution. The peak-over-threshold approach uses data with wave heights greater than a certain threshold, and thus allows for increased number of samples compared to the annual maxima approach. Waves exceeding this threshold would then be modeled according to the Generalized Pareto distribution. However, the peaks-over-threshold method has demonstrated a clear dependence on the threshold and is therefore not very reliable. The MENU method determines the return period of an extreme wave of a certain wave height by requiring that the expected or mean number of up-crossings of this wave height will be one for that time interval.

Another approach useful in extreme event modeling is the use of quantile functions, an alternative way of defining a probability distribution [78]. The quantile func-

[1] Data available from url: http://data-portal.ecmwf.int/data/d/era40_daily/

tion, Q, is a function of the cumulative probability of a distribution and is simply the inverse of the cumulative density function: $Q(p) = F^{-1}(p)$ and $F(x) = Q^{-1}(x)$. This function can then be used in frequency analysis to find useful estimates of the quantiles of relevant return periods T of extreme events in the upper tail of the frequency distribution, $Q_T = Q(1 - 1/T)$.

Yet another approach for estimating the maxima of a stationary process is to model the number of extreme events, defined as the number of times the process crosses a fixed level u in the upward direction, as a Poisson process (a counting process $\{N(t), t \geq 0\}$ with $N(0) = 0$, independent increments and with number of events in a time interval of length t Poisson distributed with mean λt is said to be a Poisson process with rate λ) and apply the Rice formula to compute the intensity of the extreme events (see e.g., [164]).

In the following, a brief review of some wave models proposed in the literature will be given. This includes a brief description of some short-term and stationary wave models as well as a more comprehensive review of proposed approaches to modeling long-term trends due to global climatic changes. An introduction to stochastic analysis of ocean waves can be found in [149] and [191], albeit the latter with a particular emphasis on freak or rogue waves.

2.2.1 Short-Term Stochastic Wave Models

Waves are generated from wind actions and wave predictions are often based on knowledge of the generating wind and wind-wave relationships. Most wave models for operational wave forecasting are based on the energy balance equation; there is a general consensus that this describes the fundamental principle for wave predictions, and significant progress has been made in recent decades [106]. Currently, the third-generation wave model WAM is one of the most widely used models for wave forecasting [82, 115] computing the wave spectrum from physical first principles. Other widely used wave models are Wave Watch and SWAN, and there exist a number of other models as well [84]. However, wave generation is basically an uncertain and random process which makes it difficult to model deterministically, and in [19, 58] approaches using neural networks were proposed as an alternative to deterministic wave forecasting models.

There are a number of short-term, statistical wave models for modeling of individual waves and for predicting and forecasting sea states in the not too distant future. Most of the models for individual waves are based on Gaussian approaches, but other types of stochastic wave models have also been proposed to account for observed asymmetries (e.g., adding random correction terms to a Gaussian model [129] or based on Lagrangian models [2, 125]). Asymptotic models for the distribution of maxima for Gaussian processes for a certain period of time exist, and under certain assumptions, the maximum values are asymptotically distributed according to the Gumbel distribution. However, as noted in [165], care should be taken when using this approximation for the modeling of maxima of wave crests. A similar concern was expressed in [48], albeit not related to waves.

A comparison between significant wave height data predicted by a numerical wave prediction model and corresponding satellite measurements was recently published in [73]. A novel approach utilizing information geometry was used to quantify bias, and one important finding was that the wave prediction model seemed to overestimate the significant wave height slightly but consistently. They also reported significant spatial variation in the distribution of the significant wave height data.

Given the short-term perspective of these types of models, they cannot be used to describe long-term trends due to climate change, nor to formulate design criteria for ships and offshore structures, even though they are important for maritime safety during operation. Improved weather and wave forecasts will of course improve safety at sea, but the main interest in the present study is on long-term trends in ocean wave climate, and the effect this will have on maritime safety and on the design of marine structures. Therefore, short-term wave models will not be considered in great detail herein.

2.2.1.1 Significant Wave Height as a Function of Wind Speed

The significant wave height for a fully developed sea, sometimes referred to as the equilibrium sea approximation, given a fixed wind speed has been modeled as a function of the wind speed in different ways, for example as $H_S \propto U^{5/2}$ or $H_S \propto U^2$ [112]. This makes it possible to make short-term predictions of the significant wave height under the assumptions of a constant wind speed and assuming unlimited fetch and duration. For developing sea conditions, with limited fetch or limited wind duration, the significant wave height as a function of wind speed, U (m/s), and respectively fetch X (km) and duration D (h) has been modeled in different ways, for example as $H_S \sim X^{1/2}U$ and $H_S \propto D^{5/7}U^{9/7}$ [151].

However, it is observed that the equilibrium wind sea approximation is seldom valid, and an alternative model for predicting the significant wave height for wind waves, H_S from the wind speed U_{10} at a reference height of 10 m was proposed in [7], using a different, yet simple parametrization. 18 years of hourly data of significant wave height and winds speed for 12 different buoys were used in order to estimate the model which can be written on the following form:

$$H_S = C(D)I(U_{10} \leq 4\,\mathrm{m/s}) + \left[a(D)U_{10}^2 + b(D)\right] I(U_{10} > 4\,\mathrm{m/s}) \tag{2.1}$$

D denotes the water depth and C, a and b are depth-dependent parameters. Based on comparison with measurements it was concluded that this model is reliable for wind speeds up to at least $U_{10} = 25\,\mathrm{m/s}$.

It is out of scope of the present literature survey to review all models for predicting wave heights from wind speed or other meteorological data. Such models are an integral part of the various wave models available for wave forecasting, but cannot be used directly to model long-term variations in wave height. However, given adequate long-term wind forecasts, such relationships between wind speed and wave height may be exploited in simulating long-term wave data for long-term predictions of wave climate.

2.2.2 Stationary Models

A thorough survey of stochastic models for wind and sea state time series is presented in [142]. Only time series at the scale of the sea state have been considered without modeling events at the scale of individual waves, and only at given geographical points. One section of [142] is discussing how to model non-stationarity such as trends in time series and seasonal components, but for the main part of the paper it is assumed that the studied processes are stationary. The models have been classified in three groups: Models based on Gaussian approximations, other non-parametric models and other parametric models. In the following, the main characteristics for these different types of wave models are highlighted.

Although ocean wave time series cannot normally be assumed to be Gaussian, it may be possible to transform these time series into time series with Gaussian marginal distributions when they have a continuous state space [142]. The transformed time series can then be simulated by using existing techniques to simulate Gaussian processes. If $\{Y_t\}$ is a stationary process in $\mathbf{R}^d$, assume that there exists a transformation $f : \mathbf{R}^d \to \mathbf{R}^d$ and a stationary Gaussian process $\{X_t\}$ so that $Y_t = f(X_t)$. Such a procedure consists of determining the transformation function f, generation of realizations of the process $\{X_t\}$ and then transforming the generated samples of $\{X_t\}$ into samples of $\{Y_t\}$ using f. A number of such models for the significant wave height have been proposed in the literature (e.g., [54, 198] for the univariate time series for significant wave height, H_s, [85, 144] for the bivariate time series for significant wave height and mean wave period, (H_s, T) and [57] for the trivariate time series for significant wave height, mean wave period and mean wave direction, (H_s, T, Θ_m)). However, it is noted that the duration statistics of transformed Gaussian processes have been demonstrated not to fit too well with data, even though the occurrence probability is correctly modeled [107].

Multimodal wave models for combined seas (e.g., with wind-sea and swell components) have also been discussed in the literature (see e.g., [66, 189, 190]), but these are generally not required to describe severe sea states where extremes occur [23].

A few non-parametric methods for simulating wave parameters have been proposed, as reported in [142]. One may for example assume that the observed time series are Markov chains and use non-parametric methods such as nearest-neighbor resampling to estimate transition kernels. In [36], a non-parametric regression method was proposed to correct outputs of meteorological models. A continuous space, discrete time Markov model for the trivariate time-series of wind speed, significant wave height and spectral peak period was presented in [143]. However, one major drawback of non-parametric methods is the lack of descriptive and predictive power.

An approach based on copulas for multivariate modeling of oceanographic variables, accounting for dependencies between the variables, was proposed in [197] and applied to the joint bivariate description of extreme wave heights and wave periods.

Parametric models for wave time series include various linear autoregressive models, nonlinear retrogressive models, finite state space Markov chain models and circular time series models. A modified Weibull model was proposed in [145] for modeling of significant and maximum wave height. For short-term modeling of wave parameters, different approaches of artificial neural networks (see e.g., [11, 58, 133, 134]) and data mining techniques [130, 131] have successfully been applied. A nonlinear threshold autoregressive model for the significant wave height was proposed in [174].

2.2.3 Non-Stationary Models

Many statistical models for extreme waves assume the stationarity of extreme values, but there are some non-stationary models proposed in the literature. In the following, some non-stationary models for extreme waves that are known and previously presented in the literature will be reviewed. A review of classical methods for asymptotic extreme value analysis used in extreme wave predictions is presented in [178].

2.2.3.1 Microscopic Models

A number of statistical models have been presented in the literature where the focus has been to use sophisticated statistical methods to estimate extreme values at certain specific geographical points (e.g., based on data measurements at that location). This approach is natural, given the limited spatial resolution of available wave data, and aims at exploiting available data measurements at certain locations to the maximum, i.e., to obtain as good predictions as possible for locations where wave data are available. In the following, some of these will be briefly reviewed, even though it is noted that the aim of this study is to extend the scope and broaden the perspective of the statistical models to also include the spatial dimension.

A method for calculating return periods of various levels from long-term non-stationary time series data of significant wave height based on a new definition of the return period is presented in [182, 183]. This definition is based on the mean number of upcrossings of a particular level and was first introduced in the context of prediction of sea-level extremes in [140]. In [179] and [89], new de-clustering methods and filtering techniques are proposed in order to apply the r-largest-order statistics for long-term predictions of significant wave height. A new de-clustering method was also suggested in [177] for applying the peaks-over-threshold method for H_s time series. An approach using stochastic differential equations for clarifying the relationship between long-term time-series data and its probability density functions in order to extrapolate long-term predictions from shorter historical data is proposed in [141]. Two approaches for estimating long-term extreme waves are discussed in [94] (i.e., an initial distribution approach and a Peak Over Threshold (POT) approach for storm events) and issues related to sampling variability, model fitting and threshold selection (for the POT analysis) are addressed.

Duration statistics of long time series of significant wave height H_s (i.e., the duration of sea states with different intensities) where analyzed in [181] using a bottom–up segmentation algorithm. This analysis makes use of the increasing or decreasing intensity of successive sea state conditions, and subdivide long-term H_s time series into subsequent series of monotonically increasing or decreasing intensities. This would correspond to developing and decaying sea states, and the segmentation algorithm should ensure that a meaningful subdivision of the long-term time series is obtained. A sensitivity analysis of this approach, investigating the effect of the maximum allowed error on the segmentation of the H_s time series is reported in [180].

Return periods of storms with an extreme wave above a certain threshold are found based on an equivalent triangular storm model in [9]. This approach is extended to find return periods analytically for storms with two or more waves exceeding the threshold in [8, 10]. The basic idea behind the equivalent triangular storm model is that it, for a fixed location, associates a triangle to each actual storm and represents a significant wave height time series by means of a sequence of triangular storms. The triangle height is the maximum significant wave height during the actual storm and the triangle base is such that the maximum expected wave height in the actual storm equals the maximum expected wave height in the triangular storm model [25]. The equivalent power storm model was presented in [67] as a generalization of the equivalent triangular storm model to predict return periods for waves above a certain threshold. It is noted that the equivalent triangular storm is firmly based on what has become known as the Borgman Integral [27], which gives the distribution function for the largest wave, $F_m(h) = P(H_m \leq h)$ as follows, with H_m denoting the largest wave height, $a^2(t)$ time varying Rayleigh parameter and $T(t)$ typical wave period at time t:

$$F_m(h) = e^{\int \log[1-e^{-h^2/a^2(t)}] \frac{\mathrm{d}t}{T(t)}} \tag{2.2}$$

A non-stationary stochastic model for long-term time series of significant wave height is presented in [12], where the time series is modeled by decomposing de-trended time series to a periodic mean value and a residual time series multiplied with a periodic standard deviation: $X(\tau) = \overline{X}_{\mathrm{trend}}(\tau) + \mu(\tau) + \sigma(\tau)W(\tau)$. It was then showed that $W(\tau)$ could be considered stationary. Short-term and long-term wave characteristics of ocean waves were combined in order to develop nested, stochastic models for the distribution of maximum wave heights in [155]. Different time scales were introduced, i.e., fast time and slow time, and a stochastic process was modeled in the fast time where the state variables were modeled as a stochastic process in the slow time.

The seasonal effects on return values of significant wave height were investigated in [139], where a time-dependent generalized extreme value model was used for monthly maxima of significant wave height. Non-stationarity representing annual and semiannual cycles is introduced in the model via the location, scale, and shape parameters and the inclusion of seasonal variabilities is found to reduce the residuals of the fitted model substantially. Hence, the model provides a way of quantitatively examining the long-term seasonal distribution of significant wave height.

Various other models for the long-term distribution of significant wave height have been suggested (e.g., using the Beta and Gamma models [69], using the Annual Maxima and Peak Over Threshold methods [88] using nonlinear threshold models [174], using time-dependent Peak Over Threshold models for the intensity combined with a Poisson model for frequency [137, 138], employing different autoregressive models [86, 87], and using a transformation of the data and a Gaussian model for the transformed data [70]). Short- and long-term statistics were combined in [116] in order to establish distributions of maximum wave heights and corresponding periods. Some considerations of bias and uncertainty in methods of extreme value analysis were discussed in [77], leading to some recommended approaches for such analyses and applied on a set of wave data.

More recently, an interesting approach to long-term predictions of significant wave height, combining Bayesian inference methodology, extreme value techniques and Markov chain Monte Carlo (MCMC) procedures is presented in [175]. The benefits of using a Bayesian approach compared to a traditional likelihood-based approach is that prior knowledge about parameter values θ can be used together with observed data x to update a posterior distribution $\pi(\theta|x)$. Simulations of this posterior distribution can be obtained by constructing a Markov chain whose invariant distribution, or target distribution, is proportional to the posterior distribution by employing the Metropolis-Hastings algorithm (see [161]). This Bayesian approach was used to analyze a dataset of significant wave height collected in the northern North Sea.

Another Bayesian approach to estimating posterior distributions of return periods for extreme waves is proposed in [65]. Here, the occurrence of extreme events is modeled as a Poisson-process with extreme wave heights distributed according to a generalized Pareto distribution.

2.2.3.2 Combining Long- and Short-Term Wave Height Statistics

The Borgman Integral (Eq. 2.2) is a fundamental tool for combining the long-term distribution of significant wave height with short-term distributions for the individual wave heights [27]. This is often desired for estimating the maximum wave or crest height occurring in a long return interval. A similar method was proposed by [15]. It is noted that the particular expression of the Borgman Integral as presented in Eq. 2.2 is based on the assumption of a Rayleigh distribution for the individual wave height. A more general form would be, letting $P(h|H_s)$ denote the short-term distribution of the individual wave height conditioned on the sea state,

$$F_m(h) = e^{\int \log[P(h|H_s)] \frac{dt}{T(t)}} \tag{2.3}$$

Long time series of individual wave heights are typically not available and calculations must therefore be based on time series of sea state parameters such as the significant wave height. Hence, the problem of modeling the maximum wave height in a long time interval comprises three aspects: modeling of long-term sea state parameters (e.g., significant wave height), short-term modeling of individual wave heights conditioned on the sea state and combining the two distributions. This can be done by first fitting a short-term distribution and then apply the Borgman Integral to this distribution. Integration of short-term second order models over time series of measured sea states was performed by [118]. A recent study concerned with finding the most accurate method for combining long- and short-term wave statistics was reported in [71].

2.2.3.3 Spatio-Temporal Models for Extreme Waves

The spatial and temporal variability of ocean wave fields is complex, and the fields will generally be inhomogeneous in space and non-stationary in time, with strong temporal and spatial variation [108]. Different models have been proposed in the literature for modeling these variabilities and for analysing and synthesizing spatio-temporal wave data.

There has been significantly more focus on the temporal variability compared to the spatial variability of wave fields, but the spatial behavior (i.e., the spatial interdependence and radius of influence of a set of spatially distributed stations) of significant wave height is investigated in [5]. The methodology is based on the concept of trigonometric point cumulative semivariograms, consisting of cumulative broken lines where the angle between two successive lines connecting two station records is a measure of the regional dependence, ranging from 0 (complete independence) to 1 (complete dependence). Another approach for predicting the maximum wave height over a spatial area was proposed in [68], based on 4D video data of sea states acquired through a wave acquisition stereo system (WASS) and using Euler Characteristics' theory. A regional frequency analysis of extreme wave heights, analyzing peaks-over-threshold wave data from 9 locations along the Dutch North Sea coast was reported in [195]. The different locations could be considered as a homogeneous region and it was shown that the Generalized Pareto Distribution is an optimal regional probability distribution for the extreme wave heights for the region. Notable differences were found for the regional quantile estimates compared to the at-site quantile estimates, indicating that it would be better to rely on the regional estimates in decision making.

Models for stochastic simulation of the annual [28] and synoptic [29] variability of inhomogeneous met-ocean fields were proposed as expansions of the field $\zeta(r, t)$ in terms of periodical empirical orthogonal functions in [28, 29]:

$$\zeta(\mathbf{r}, t) = m(\mathbf{r}, t) + \sum_k a_k(t)\phi_{kt}(\mathbf{r}, t) + \varepsilon(\mathbf{r}, t) \tag{2.4}$$

where $m(\mathbf{r}, t)$ are the mathematical expectations, $\phi_{kt}(\mathbf{r}, t)$ are the spatio-temporal basis functions, $\varepsilon(k, t)$ is inhomogeneous white noise and $a_k(t)$ are the coefficients. $\mathbf{r}$ denotes the geographical coordinates and t time. The results of simulating these models are a set of simulated met-ocean fields $\zeta(r, t)$ in a discrete set of grid points and at discrete times. They could then be used to investigate the field extremes and rare events in terms of both spatial and temporal extremes, and wave data from the Barents Sea region have been used to test the models with reasonable agreement. The stochastic models for annual variability can be regarded as field generalizations of periodically correlated stochastic processes. The model for synoptic variability uses a Lagrangian approach and the temporal sequence of storm centers is modeled as a finite-state Markov chain with the storm extensions and field properties as spatio-temporal impulses.

Recently, spatio-temporal statistical models for the significant wave height have been reported that describes the variability of significant wave height over large areas by stochastic fields [17, 18]. This is based on constructing a homogeneous model valid for a small region and then extending this to a non-homogeneous model valid for large areas. Global wave measurements from satellites have been used for model fitting, providing wave data of spatial variability, but limited physical knowledge about the wave phenomena have been incorporated into the models. The resulting models can then be used to estimate the probability of a maximum significant wave height to exceed a certain level or to estimate the distribution of the (spatial) length of a storm [16]. However, the temporal validity of this model is limited to the order of hours [17], and therefore it does not seem suited for studying long-term trends and the effects of climate change.

The study reported in [40] used two approaches to model the extremes of non-stationary time series, i.e., the non-homogeneous Poisson process and a non-stationary generalized extreme value model. The non-homogeneous Poisson process was used to model extreme values of the significant wave height, obtained from the 40-year ECMWF reanalysis (ERA-40) [193] and compared to estimates obtained using a non-stationary generalized extreme value model (NS-GEV). The parameter of the Poisson distribution in this model was on the form $\lambda = \int\int \lambda(t, x)\mathrm{d}t\mathrm{d}x$, where :

$$\lambda(t, x) = \frac{1}{\sigma(t)}\left[1 + \xi(t)\frac{x - \mu(t)}{\sigma(t)}\right]_{+}^{-(\frac{1}{\xi(t)})-1} \tag{2.5}$$

From projections of the sea level pressure under three different forcing scenarios ([26, 146]), projections of the parameters in the non-homogeneous Poisson process are made up to the end of the twenty-first century. Trends in these parameters are then determined, projections of return value estimates of H_S are projected and their uncertainties are assessed.

2.3 Relevant Statistical Models from Other Areas of Application

Extreme value analysis has a wide area of applications aside from ocean waves, in particular in various environmental sciences where events are also associated with spatio-temporal variations, and it is believed that some lessons can be learned by examining different statistical models for other types of extreme events.

An interesting discussion on the use of asymptotic models for the description of the variation of extremes is available in [48], within the context of extreme rainfall modeling. It is concerned with the lack of ability of such models to predict extreme, catastrophic events leading to inadequate designs and lack of preparedness for such rare events. One of the reasons for this, according to [48] is models that do not take the uncertainties in both model and predictions adequately into account. For example, it is argued that even in cases where data support the reduction of the generalized extreme value model to a Gumbel model, this should not be done without an appraisal of the uncertainty this decision introduces and as a general advice it is suggested to use the generalized extreme value model rather than Gumbel reduction. Furthermore, the preference for Bayesian analysis over the classical likelihood analysis is emphasized, even if using diffuse priors.

In this section, a review of relevant time- and space-dependent statistical models from other areas of application is presented. Further work will then focus on how these approaches can be used for statistical modeling of extreme waves and sea states.

2.3.1 Bayesian Hierarchical Space-Time Models

Modeling of wave data in space and time is an alternative to the common approach of extreme value analysis based on a point process representation, provided that adequate space-time wave data can be obtained. [209] proposes a hierarchical Bayesian space-time model as an alternative to traditional space-time statistical models and applies it on an atmospheric data set of monthly maximum temperatures. Such models generally consist of three stages often referred to as the data stage, the process stage, and the parameter stage.

Similar models have also been used for modeling tropical ocean surface winds [210], North Atlantic sea surface temperatures [124], concentrations of PM_{10} pollution[2] [46], ozone levels [167], and earthquake data [147]. A brief overview of hierarchical approaches applied to environmental processes is presented in [208]. More recently, various hierarchical Bayesian space-time models for extreme precipitation events were proposed in [168]. Bayesian hierarchical space-time models are treated in the recent book by Cressie and Wikle [53].

[2] PM_{10} is the fraction of aerosol particles with aerodynamic diameter less than 10 μm

The modeling of earthquake data for earthquake prediction, using a Bayesian hierarchical space-time approach in [147] considered a spatial resolution of $0.5 \times 0.5°$ (about 50×50 km) and a temporal resolution of 4 months. The observations are, for each time period, the magnitude of the largest earthquakes (by the Richter scale) observed within each grid. The model is implemented within a Markov chain Monte Carlo framework using Gibbs sampling and additional Metropolis-Hastings steps. Four different model alternatives were suggested in a hierarchical structure, one main model and three levels of simplified models, nested within the model at the higher level. The model contains a large number of parameters, and all prior parameter distributions are considered independent. A Markov chain Monte Carlo approach using the Gibbs sampler and also an additional Metropolis-Hastings step for some of the parameters, was adopted for generating independent samples from the posterior distributions in order to arrive at posterior estimates and predictions. A brief introduction to Markov chain Monte Carlo methods, including the Gibbs sampler and the Metropolis-Hastings algorithm, are given in appendix A. For a more detailed treatment, reference is made to [161] or similar textbooks.

2.3.2 Continuous Space Models

Although wave data are generally only available at certain specific locations, extreme waves should in principle be considered as a continuous process in space and time rather than a spatially discrete process. Considering the continuous space modeling of a process' extremes, this would require the specification of a continuous space model for the marginal behavior of the extremes of the process and a continuous space specification of the dependence structure of the extremes. Hence, a generalization of the dependence structure of multivariate extremes to the infinite dimensional case is needed, and one such generalization is provided by the theory of max-stable processes [93]. By definition, a stochastic process $\{Y_t\}$ is called a max-stable process if the following property holds:

If $\{Y_t^{(i)}\}_{t\in T}$, $i = 1, \ldots, r$, are independent copies of the process then the process $\{\max_{i\le r} Y_t^{(i)}\}_{t\in T}$ has the same distribution as $\{rY_t^{(1)}\}_{t\in T}$.

In the following, a procedure for using the theory of max-stable processes for modeling data which are collected on a grid of points in space are reviewed. This approach is considered as an infinite dimensional extension of multivariate extreme value theory and has the advantage that it can be used to aggregate the process over the whole region and for interpolation to anywhere within the whole region. Models based on the resulting family of multivariate extreme value distributions are suitable for a large number of grid points.

In [50, 51] a class of max-stable process models for regional modeling of extreme storms was specified which can be estimated using all relevant extreme data and which is consistent with the multivariate extreme structure of the data. The essence of this approach is to describe the process of storms by the following components:

i. A phase space S of storm types so that the storm type is independent of their size
ii. An index space T for the region, conveniently referred to as the region itself
iii. A measure $\nu(ds)$ on S describing the relative frequency of storm types
iv. A function $f(s, t)$ interpreted as the proportion of a storm of type s observed at t.

With x_j interpreted as the size of the jth storm, s_j the type of the jth storm, if $\{(x_i, s_i); i = 1, \ldots\}$ are taken to be the points of a Poisson process on $(0, \infty) \times S$ with intensity $\mu(\mathrm{d}x, \mathrm{d}s) = x^{-2}\mathrm{d}x\,\nu(\mathrm{d}s)$ and letting $f(x, s)$ be a positive function on $S \times T$, then the process

$$Z_t = \max_i\{X_i f(Si, t)\} \tag{2.6}$$

is a max-stable process for $t \in T$.

For statistical modeling of extreme storms as such a max-stable process, it was assumed that the spatial variability of storms could be described adequately by variability within a subset of data sites T_1 [50]. Then, a multivariate extreme value model is fitted to the data for this subset and the model is extended smoothly as a max-stable process through suitable functions $f(\cdot, \cdot)$ on the basis of information from the remaining data sites. Such a model was fitted for rainfall data collected from 11 sites, and in spite of some interesting qualitatively observations, the quality of fit of the model was rather poor.

In [33] a somewhat different approach of using max-stable processes for the modeling of spatial extreme rainfall is proposed based on random fields. Whereas [50, 51] indicate how to analytically calculate quantiles of areal rainfall, in [33], the 100-year quantile of the total rainfall over an area in Holland is found by simulating synthetic daily rainfall fields using their estimated model. An extended Gaussian max-stable model for spatial extreme rainfall was also presented in [176], where Bayesian techniques are used in order to incorporate information other than data into the model, i.e., by using informative priors for the marginal site parameters and non-informative priors for parameters relating to the dependence structure of the process. The extended model is estimated using a pairwise likelihood within the Bayesian analysis and Markov chain Monte Carlo techniques were used to simulate from the posterior distributions, using a Gibbs sampler with a Metropolis step. Max-stable processes have also been applied to, e.g., modeling of extreme wind speeds [49].

2.3.3 Process Convolution Models

Several models for spatio-temporal processes based on process convolution have been proposed in the literature (e.g., [41, 42, 99, 169]). The main idea is to convolve independent processes to construct a dependent process by some convolution kernel. This kernel may evolve over space and time thus specifying models with non-stationary dependence structures.

The model proposed in [99] is motivated by estimation of the mean temperature field in the North Atlantic Ocean based on 80 year of temperature data for a region.

First, the temperature field $y(s, t)$ is modeled as a process over space s and time t as the sum of two processes

$$y(s, t) = z(s, t) + \varepsilon(s, t) \tag{2.7}$$

where $z(s, t)$ is a smooth Gaussian process and $\varepsilon(s, t)$ is an independent error process. The smooth process $z(s, t)$ is constructed to model the data by taking the convolution of a three-dimensional lattice process. Given a grid process $x = (x_1, \ldots, x_m)$ with space-time coordinates $(\omega_1, \tau_1), \ldots, (\omega_m, \tau_m)$, the smooth field is expressed as

$$z(s, t) = \sum_{j=1}^{m} K_s(s - \omega_j, t - \tau_j) \cdot x_j \tag{2.8}$$

where the properties of the convolution kernel determine the smoothness of z. A separable kernel were used (i.e., a product of a kernel that smooths over space and one that smooths over time): $K_s(\Delta s, \Delta t) = C_s(\Delta s) \cdot R(\Delta t)$. Inference on the resulting model was made using a Bayesian approach and simulating the posterior distribution of the mean temperature field over space and time using Markov chain Monte Carlo methods.

Following a similar approach, but using non-separable, discrete convolution kernels, regional temperature measurements were modeled in space and time in [169]. Two alternative sets of models were suggested. The first was to convolute spatial Gaussian processes with a kernel providing temporal dependencies and the second was to convolute autoregressive models with a kernel providing spatial interactions. In other words, the data could either be considered as a number of time series at each location (temporal convolution model) or as a number of realizations of spatial processes observed at some locations (spatial convolution model).

A dynamic process convolution model extends the discrete process convolution approach by defining the underlying process x to be a time-dependent process that is spatially smoothed by a smoothing kernel at each time-step [41, 42]. Such models have been used in air quality assessment (e.g., in bivariate modeling of levels of particulate matter $PM_{2.5}$ and PM_{10} in [42] and for multivariate modeling of the concentration of five pollutants in [41]). A continuous version in space and time is considered in [31], where a model is formulated in discrete time and continuous space and a limit argument is applied to obtain continuous time as well. A general approach using cross-convolution of covariance functions for modeling of multivariate geostatistical data were proposed in [132]. All of the convolution models discussed above used Bayesian approaches and Markov chain Monte Carlo methods for model specification.

Finally, it is noted that some limitations to the convolution model approach are reported in [99] and [42]. One is the impact of prior assumptions on the posterior distributions. Furthermore, it is stated that it would be preferable to allow the data to determine the kernels, which could depend on space and time, rather than specifying it apriori. In addition, the model for particulate matter is not able to handle extreme observations very well and permits non-sensible predictions.

2.3.4 Non-Stationary Covariance Models

Many spatio-temporal models assume separability in space and time so that that the space-time covariance function can be represented as the product of two models: one as a function of space and the other as a function of time. However, the rationale for using a separable model is often convenience rather than the ability of such models to describe the data well, and the assumption is often unrealistic. Other simplifying assumptions often employed are stationarity (e.g., second order stationarity which means that the mean function is assumed constant and the space-time covariance function is assumed to depend only on the directional distance between measurement sites) and isotropy (i.e., that the covariance function is dependent only on the length of the separation and not on its direction). An example of a spatio-temporal covariance model where the assumptions of stationarity and separability is relaxed is presented in [32], applied to tropospheric ozone data.

Due to the increased availability of satellite measurements of many geophysical processes, global data are increasingly available. Such data often show strong non-stationarity in the covariance structure. For example, processes may be approximately stationary with respect to longitude but with highly dependent covariance structures with respect to latitude. In order to capture the non-stationarity in such global data, with a spherical spatial domain, a class of parametric covariance models is proposed in [110]. These assume that processes are axially symmetric, i.e., that they are invariant to rotations about the earth's axis and hence stationary with respect to longitude.

Assuming a homogeneous, zero-mean process Z_0, a zero-mean non-stationary process Z may be defined by applying differential operators with respect to latitude and longitude, letting L and l denote latitude and longitude, respectively [109],

$$Z(L, l) = \left\{ A(L)\frac{\partial}{\partial L} + B(L)\frac{\partial}{\partial l} \right\} Z_0(L, l) + C Z_0(L, l) \tag{2.9}$$

Now, A and B denote non-random functions depending on latitude (and may also in principle depend on longitude, but this would break the axial symmetry). A non-negative constant C corresponds to including homogeneous models for the case $A(L) = B(L) = 0$. In order to apply this model to real applications, the A and B functions need to be estimated, and it is suggested to use linear combinations of Legendre polynomials [110].

The covariance model is applied to global column ozone level data and it is shown that the strong non-stationarity with respect to latitudes as well as the local variation of the process can be well captured with only a modest number of parameters. Thus, it may be a promising candidate for modeling spatially dependent data on a sphere. Furthermore, an extension to spatio-temporal processes would be obtained by introducing a similar differential operator with respect to time in addition to the ones with respect to latitude and longitude. Then, such models should be able to capture spatial-temporal non-stationary behavior and to create flexible space-time interactions such as space-time asymmetry. Reviews of various methods and recent developments for the construction of spatio-temporal covariance models are presented in [113] and [127].

2.3.5 Coregionalization Models

A multivariate spatial process is a natural modeling choice for multivariate, spatially collected data. When the interest is in modeling and predicting such joint processes it will be important to account for the spatial correlation as well as the correlation among the different variables. If this is modeled using a Gaussian process, the main challenge is the specification of an adequate cross-covariance function [173], which can be developed through linear models of coregionalization (LMC). The linear model of coregionalization is reviewed in [76] where the notion of spatially varying LMC is proposed in order to enhance the usefulness by providing a class of multivariate non-stationary processes.

Traditionally, linear models of coregionalization have been used to reduce dimensions, approximating a multivariate process through a lower dimensional representation. However, it may also be used in multivariate process construction, i.e., obtaining dependent multivariate processes by linear transformation of independent processes. A general multivariate spatial model could be on the form

$$\mathbf{Y}(s) = \boldsymbol{\mu}(s) + \mathbf{v}(s) + \boldsymbol{\varepsilon}(s) \tag{2.10}$$

where $\boldsymbol{\varepsilon}(s)$ is a white noise vector (i.e., $\boldsymbol{\varepsilon}(s) \sim N(0, \mathbf{D})$ where $\mathbf{D}$ is a diagonal matrix with $(D_{jj}) = \tau_j^2$), $\mathbf{v}(s)$ arises from a linear model of coregionalization from independent spatial processes $\mathbf{w}(s) = (w_1(s), \ldots w_p(s))$: $\mathbf{v}(s) = \mathbf{A}\,\mathbf{w}(s)$ and where $\boldsymbol{\mu}(s)$ may be assumed to arise linearly in the covariates, i.e., $\boldsymbol{\mu}_j(s) = \mathbf{X}_j^T(s)\beta_j$ where each component may have its own set of covariates $\mathbf{X}_j$ and its own coefficient vector β_j. If ignoring the term $\boldsymbol{\mu}(s)$ and the $w_j(s)$ processes are assumed to have mean 0, variance 1 and a stationary correlation function $\rho_j(h)$, then $E(\mathbf{Y}(s)) = 0$ and the cross-covariance matrix associated with $\mathbf{Y}(s)$ becomes

$$\Sigma_{\mathbf{Y}(s),\mathbf{Y}(s')} \equiv C(s - s') = \sum_{j=1}^{p} \rho_j(s - s')\mathbf{T}_j, \qquad \mathbf{T}_j = \mathbf{a}_j\mathbf{a}_j^T \tag{2.11}$$

with $\mathbf{a}_j$ the jth column of $\mathbf{A}$. Priors on the model parameters $\boldsymbol{\theta}$ consisting of $\left\{\beta_j\right\}$, $\left\{\tau_j^2\right\}$, $\mathbf{T}$ and ρ_j, $j = 1, \ldots, p$ would then complete the model specification in a Bayesian setting, obtaining the posterior distribution of the model parameters

$$\pi(\boldsymbol{\theta}|\mathbf{Y}) \propto f(\mathbf{Y}|\left\{\beta_j\right\}, \mathbf{D}, \left\{\rho_j\right\}, \mathbf{T})\pi(\boldsymbol{\theta}) \tag{2.12}$$

The extension to a spatially varying linear model of coregionalization is obtained by letting $\mathbf{A}$ be spatially dependent, i.e., replacing $\mathbf{A}$ with $\mathbf{A}(s)$ in $\mathbf{v}(s) = \mathbf{A}(s)\mathbf{w}(s)$ [76]. $\mathbf{v}(s)$ will then no longer be a stationary process. Further extensions to spatio-temporal versions of the model, modeling $\mathbf{v}(s,t) = \mathbf{A}(s,t)\mathbf{w}(s,t)$, where the components of $\mathbf{w}(s,t)$ are independent spatio-temporal processes may also be feasible, but this was not further investigated.

A stationary Bayesian linear coregionalization model for multivariate air pollutant data was presented in [173] and [76] presents a commercial real estate example of a spatially varying model. Rather than taking the Bayesian approach, an Expectation-Maximization (EM) algorithm for the maximum-likelihood estimation of the parameters in a linear coregionalization model is developed in [215], and applied on a spatial model of soil properties.

2.3.6 Generalized Extreme Value Models

The Generalized Extreme Value distribution is a cornerstone of extreme value modeling, and in [101] non-stationary, location-dependent processes are studied using the GEV distribution where the parameters are allowed to vary in space and time. The modeling is based on a hierarchical structure assuming an underlying spatial model. Parameter changes over time (i.e., for the location, scale, and shape parameters) are modeled by use of Dynamic Linear Models (DLM) [207] which is a very general class of time series models. Now, the trends are not constrained to have a specific parametric form and the significance of short-term changes can be assessed together with the long-term changes. It is also possible to estimate how the effects of covariates change over time. An extension of this model to include changes in space as well as in time is made using a process convolution approach in defining a Dynamic Linear Model on the parameters.

Several approaches for estimation of parameters and quantiles of the GEV distribution have been applied, such as maximum likelihood estimation, L-moments estimation, Probability Weighted Moments estimation and the method of moments. Recently, an alternative to these, employing a full Bayesian GEV estimation method which contains a semi-Bayesian framework of generalized maximum likelihood estimators and considers the shape, location, and scale parameters as random variables was developed [214]. However, these approaches do not consider non-stationarity.

A generalized Probability Weighted Moments (PWM) method was suggested in [159] to model temporal covariates and provide accurate estimation of return levels from maxima of non-stationary random sequences modeled by a GEV distribution. This is a generalization of the PWM method that has proved to be efficient in estimating the parameters of the GEV distribution for iid processes and is an alternative to Maximum Likelihood Estimation (MLE) for cases when the iid assumption is violated (e.g., in non-stationary cases). The approach is illustrated by applying it on time series of annual maxima of CO_2 concentrations and seasonal maxima of accumulated daily precipitations.

An alternative to GEV models could be to use threshold models [20]. For example, various statistical methods for exploring the properties of extreme events in large grid point datasets were presented in [47], and a flexible generalized Pareto model that is able to account for spatial and temporal variation in the distribution of excesses was outlined. The generalized Pareto distribution parameters may incorporate the dependence of the extreme values and different explanatory variables related to spatial and temporal changes such as climate change. The methods were illustrated using mean surface temperatures of the Northern Hemisphere.

A generalized PWM method was introduced in [59] in order to estimate the parameters of the generalized Pareto distribution (GPD) from finite length time series. A Bayesian framework for analysis of extremes in a non-stationary context was proposed in [158] with a case study on peak-over-threshold data. Several probabilistic models, including stationary, step-wise changing and linear trend models, and different extreme value distributions were considered allowing modeling uncertainty to be taken into account.

An alternative to the standard approach of modeling non-stationarity in threshold models (i.e., retaining a constant threshold and letting the parameters of the GPD be functions of some covariates) is proposed in [64]. This involves preprocessing; attempting to model the non-stationarity in the entire data set and then removing this non-stationarity from the data. If this preprocessing is successful, the extremes of the preprocessed data will have most, if not all, of its non-stationarity removed and a simple extreme value analysis of the preprocessed data can be employed. It is argued that this approach provides improved description of the non-stationarity of the extremes, clearer interpretation, easier threshold selection and reduced threshold sensitivity. The approach was also found to be superior to approaches with continuous varying thresholds.

A brief introduction to traditional approaches to extreme value modeling is given in appendix B.

2.3.7 Optimality Models

One type of statistical models that has recently been applied in evolutionary sciences is optimality models. These assume the evolution of some biological trait toward an optimal state dictated by the environmental conditions. Due to a randomly changing environment, the optimal state is assumed to change over time, and the species are assumed to be adapting to this changing optimality with a certain phylogenetic inertia. One choice of process models for analysing such an adaptation-inertia problem is the Ornstein-Uhlenbeck process, as suggested in, e.g., [95, 157], represented by the stochastic differential equation

$$\mathrm{d}y = -\alpha(y - \theta)\mathrm{d}t + \sigma_y \mathrm{d}W_y \tag{2.13}$$

Here, $\mathrm{d}y$ is the change in some random variable y over a time step $\mathrm{d}t$, α is a parameter measuring the rate of adaptation toward the optimum θ, $\mathrm{d}W_y$ is a random noise process and σ_y is the standard deviation of the random changes. Thus, evolution according to this model has two components: One is a deterministic pull toward the primary optimum and the other is a stochastic change without direction.

A layered process is introduced for modeling adaptation to a randomly changing optimum, assuming that the optimum at any point on the phylogeny (that is, the history of organismal lineages as they change through time) is a function of a randomly changing predictor variable x. Thus, the model is extended to the coupled stochastic differential equations below where the predictor indirectly influences the trait through its influence of the optimum.

$$\mathrm{d}y = -\alpha(y - \theta(x))\mathrm{d}t + \sigma_y \mathrm{d}W_y \tag{2.14}$$

$$\mathrm{d}x = \sigma_x \mathrm{d}W_x \tag{2.15}$$

Additional layers of hidden processes may also be modeled in this way, where each layer is responding to changes in the layer beneath. The model may also be extended in that the predictor variable itself may be modeled as an Ornstein-Uhlenbeck process, tracing some optimum. The Ornstein-Uhlenbeck process has also been proposed for modeling of drought and flood risks [192] and survival data [1] and has been widely used in financial modeling [14, 21, 184].

It could be worthwhile to investigate whether an analogy to this approach would be appropriate for the development of extreme waves, i.e., whether the distribution of extreme sea states are trying to adapt to a changing mean state due to the changing environment. For example, will there be a certain average wave climate given the changing environmental conditions such as the level of CO_2 concentration in the atmosphere, global temperatures, greenhouse gas emissions etc? In other words, it could be investigated whether the distribution of extreme waves in a changing environment could be adequately modeled using layered Ornstein-Uhlenbeck processes in some way.

2.3.8 Bayesian Maximum Entropy Models

Bayesian maximum entropy (BME) models have been used to model spatio-temporal random fields. For example, in [45], this approach was used for developing a systematic epidemic forecasting methodology used to study the space-time risk patterns of influenza mortality in California during wintertime. Influenza mortality rates were represented as spatio-temporal random fields and the Bayesian maximum entropy method was used to map the rates in space and time and thus generate predictions. Bayesian maximum entropy models have also been used for space-time mapping of soil salinity [61], urban climate [122] and the contamination pattern from the Chernobyl fallout [172] and for modeling geographic distributions of species [154].

In short, the principle of maximum entropy states that the probability distribution best representing the current state of knowledge, which may be incomplete, is the one with the largest entropy. If some testable information about a probability distribution function is given, then, considering all trial probability distributions that encode this information, the probability distribution that maximizes the information entropy is the true probability distribution with respect to the testable information. This principle is applicable to problems of inference with a well-defined hypothesis space and incomplete data without noise and the Bayesian maximum entropy method can be used to predict the value of a spatio-temporal random field at an unsampled point in space-time based on precise (hard) and imprecise (soft) data.

The BME method applied to influenza mortality risk [45] consists of three stages with different knowledge bases at each stage: the general knowledge base (core knowledge), the specificatory knowledge base (case-specific knowledge) and the integration knowledge base (union of the general and specificatory knowledge bases). The influenza risk is represented as a spatio-temporal random field $X(\mathbf{p})$ defined at each space-time point $\mathbf{p} = (s, t)$. The influenza modeling approach then follows the three BME stages:

a. A probability density function, $f_g(\mathbf{x}_{\text{map}})$ is constructed on basis of the general knowledge base, where the vector $\mathbf{x}_{\text{map}}$ denotes a possible realization of the random field associated with the point vector $\mathbf{p}_{\text{map}}$. The $\mathbf{x}_{\text{map}}$ generally includes hard data $\mathbf{x}_{\text{hard}} = (x_1, \ldots, x_h)$ at points $\mathbf{p}_{\text{hard}} = (\mathbf{p}_1, \ldots, \mathbf{p}_{m_h})$, soft data $\mathbf{x}_{\text{soft}} = (x_{m_h+1}, \ldots, x_m)$ at points $\mathbf{p}_{\text{soft}} = (\mathbf{p}_{m_h+1}, \ldots, \mathbf{p}_m)$ and the unknown estimates x_k at points $\mathbf{p}_k$.
b. At the specificatory stage, the specificatory knowledge base considers hard data and soft data.
c. At the integration stage, the general and specificatory knowledge bases are combined in a total knowledge base to give the integration pdf $f_\kappa(x_\kappa)$ at each mapping point $\mathbf{p}_k$ using the operational Bayesian formula,

$$f_\kappa(x_\kappa) = A^{-1} \int_D f_g(x_{\text{map}}) d\,\Xi_S(x_{\text{soft}}) \tag{2.16}$$

where A is a normalizing constant and Ξ_S and D denote an integration operator and the range determined by the specificatory knowledge base respectively.

From the integration probability density function above, mortality estimates can be derived across space and time and an estimate of the mode is obtained by maximizing $f_\kappa(x_\kappa)$.

2.3.9 Stochastic Diffusion Models

A continuous time parameter stochastic process is referred to as a diffusion process if it possesses the Markov property and its sample paths $X(t)$ are continuous functions of time t. Many physical and other phenomena can be reasonably modeled by diffusion processes. Diffusion processes may be characterized by two infinitesimal parameters describing the mean and the variance of the infinitesimal displacements, defined as the following limits: Let the increment of the process accrued over a time interval h be $\Delta_h X(t) = X(t+h) - X(t)$, then the infinitesimal parameters of the process are:

$$\mu(x,t) = \lim_{h\to 0} E\left[\Delta_h X(t)|X(t) = x\right] \tag{2.17}$$

$$\sigma^2(x,t) = \lim_{h\to 0} E\left[\{\Delta_h(X(t)\}^2\,|X(t) = x\right] \tag{2.18}$$

$\mu(x,t)$ is sometimes referred to as the drift parameter, infinitesimal mean or the expected infinitesimal displacement and $\sigma^2(x,t)$ is called the diffusion parameter or the infinitesimal variance and these are generally continuous functions in x and t. Alternative characterizations of diffusion processes exist, e.g., based on stochastic differential equations.

A methodology for analyzing secular trends in the time evolution of certain variables, modeling the variables by non-homogenous stochastic diffusion processes with time-continuous trend functions is proposed in [92]. The methodology was applied to the evolution of CO_2 emissions in Spain with the Spanish GDP as an exogenous factor affecting the trend component and hence introducing non-homogeneity. The trend can be analyzed by means of statistical fit of the trend functions of the stochastic diffusion model to the observed data, and the models were also found appropriate for medium-term forecasts.

Stochastic diffusion models have been applied to other temporal or spatial problems as well, such as modeling of tumor growth [4], ion channel gating [194], financial volatility [188] and scaling behavior of precipitation statistics [119].

2.3.10 Regional Frequency Analysis

A method commonly used in hydrology, referred to as Regional Frequency analysis, utilizes data from several similar sites in order to estimate event frequencies, typically extreme events, at a particular site. The main idea is that data from neighboring or other sites where the frequency of the event to be investigated is similar provide additional information and hence yield more accurate predictions than data from the particular site alone. This approach can also be used to interpolate to ungauged sites where there are no data, based on data from similar sites.

The main idea is that, given data from N similar sites so that one may assume the sites to form a homogeneous region, i.e., that the frequency distributions of the different sites are identical apart from a site-specific scaling factor, the quantile function of the frequency distribution at a site i can be modeled by this scaling factor and a regional quantile function common to every site, referred to as the regional growth curve:

$$Q_i(F) = \mu_i q(F), \qquad i = 1, \ldots, N \tag{2.19}$$

In the equation above, $Q_i(F)$ denotes the site-specific quantile function at site i, μ_i denotes the site-specific scaling factor, often referred to as the index flood, and $q(F)$ is the regional growth curve. F is the cumulative distribution function of the frequency distribution of the quantity of interest (e.g., significant wave height).

A typical regional frequency analysis will consist of the following four steps:

i Screening of the data: Eliminating gross errors and inconsistencies and checking whether the data are homogeneous over time
ii Identification of homogeneous regions: Assign the sites to regions whose frequency distributions are similar
iii Choice of regional frequency distribution
iv Estimating the frequency distribution: Estimating the distribution at each site to give a regional average.

A thorough description of the regional frequency analysis approach is given in [100], together with an outline of regional model estimators based on L-moments, a widely used approach for estimation in regional frequency analysis. Estimation based on Bayesian Markov chain Monte Carlo methods in regional frequency analysis was proposed in [75]. Regional frequency analysis is widely used in hydrology and there is abundant literature on applications to extreme rainfall [72, 148, 213] and flooding [121, 166]. Regional frequency analysis has also been applied in ocean engineering problems such as modeling of significant wave heights [128, 195] and the height of wave crests [105].

2.4 Selecting a Modeling Approach

In the preceding sections of this chapter, a number of different modeling approaches have been reviewed, which may all be appropriate for modeling long-term trends in extreme wave climate. However, the approach based on Bayesian hierarchical space-time models are believed to be superior and offer several benefits compared to the other approaches that has been reviewed. The hierarchical Bayesian approach to modeling data and processes with different scales of spatial and temporal variability consists of different stages (e.g., the data stage, the process stage, and the parameter stage) and such models are generally very flexible and intuitive to work with, as outlined in [209].

Some of the advantages of a hierarchical approach are the flexibility such models allow. One may build up the models in a modular, hierarchical manner where different aspects of the model can be treated separately. Extensions to the model may easily be incorporated, if necessary, and the different modules may be updated individually as knowledge and insight increase. Knowledge about physical aspects may be incorporated in the models, as illustrated by the earthquake model in [147] and such models are very flexible with regard to how they are built up. Furthermore, hierarchical models, incorporating knowledge about the physical phenomena they represent, perform better with regard to interpretation of results.

One crucial assumption applied in the model of earthquakes [147] is the Markovian assumption (i.e., that the spatial process or field in one location is only dependent on its nearest neighbours). Although this assumption needs to be challenged on a case by case basis, it is presumably a reasonable assumption also for ocean waves. Hence, it may be reasonable to model ocean waves as a random Markov field along the lines of [147].

There are also benefits from utilizing a Bayesian approach, related to the fact that knowledge about the physical process and its characteristics may be exploited by way of the prior distributions. This is clearly an advantage in modeling of physical phenomena where such knowledge are available, as is the case of ocean waves. Furthermore, by adopting a Bayesian approach the uncertainty in model parameters is taken into account. Hence, of all the modeling approaches reviewed herein, Bayesian hierarchical space-time models are believed to be the most promising candidate for further developments in long-term time-dependent stochastic modeling of extreme waves and it is suggested that further research and model development are focused in this direction. Indeed, in the following chapters of this book, various Bayesian hierarchical space-time models for significant wave height will be presented, and it is argued that they generally perform well also for modeling of oceanic sea states.

2.5 Wave Climate Projections

2.5.1 Climate Change

The IPCC's fourth assessment report states that "*Warming of the climate system is unequivocal, as is now evident from observations of increases in global average air and ocean temperatures, widespread melting of snow and ice and rising global average sea level*" [103]. It predicts further global warming and that many changes in the global climate are very likely to be larger during the twenty-first century than what has been observed during the twentieth century. Furthermore, the frequency and intensity of extreme events are expected to change as the global climate changes, some of which have already been observed. A more recent up-to-date overview on climate change research [160] has as one of its key messages that recent observations indicate that the climate change may be more severe and occur earlier than the fourth assessment report predicts. Even though a more recent IPCC report is somewhat more moderate [104], there are reasons to believe that the future climate may be more severe than past and present climates.

However, in spite of climate change being a global phenomenon, regional variability is large, and it has been observed that for example the Arctic has warmed at double rate compared to the rest of the world in recent decades [3]. Regional differences are also presumed to be predominant in future changes of the climate, but overall, the globe is expected to warm and the intensity and frequency of extreme climatic events are likely to increase.

Hence, one important question for the stakeholders involved in maritime transport is to what extent the observed and projected global warming will influence the wave climate on short and long term and what impact this will have on the safety of maritime transportation. In the following, a review of some projections of wave climate within the context of this global warming will be presented as well as analyses of previous and current trends.

2.5.2 Current Trends in the Wave Climate

Evidence for a statistical significant increasing trend in mean wave height in the North Atlantic was observed more than 30 years ago [13, 44]. Since then, there are a number of studies reported in the literature which try to identify and assess previous and current trends in extreme wave climate, most with a focus on the North Atlantic, by different hindcast and reanalysis techniques combined with statistical analyses (see e.g., [12, 111, 193]). Some of these will be briefly outlined in the following.

Seasonal trends in extremes of significant wave height were assessed in [200, 201] for the North Atlantic and the North Pacific by simulating a 40-year global wave hindcast. For both oceans, no statistically significant changes were observed for the past century, but significant changes were found in some regions and for some seasons for the past four decades. Most notably an increase was found for the winter season in the North Atlantic, matched by a decrease in the subtropical North Atlantic and a significant increase in the North Pacific for the winter and spring seasons. Extensive validation of a 40 year global wave hindcast against available wave observations (from buoys, platforms, ships and satellites) has shown generally good agreement over the entire frequency distribution for such reanalyzed data [52].

A previous study, somewhat limited in scope with regards to the period and area covered compared to the assessments in [200, 201], reported a similar increasing trend in significant wave height at several north-east Atlantic locations since 1960, as well as a decrease south of 40°N [120]. Similar patterns were also suggested in [186]. An increase in frequency of extreme events in the last four decades were reported for the North Sea in [206], although no significant changes were found with regard to intensity and duration. Also, an analysis of wave hindcast for 1955–1994 reported in [83] suggests an increasing trend in both the North Sea and the Norwegian Sea, but with decreasing trends in other regions. A global wave climate trend analysis was reported in [39] where an ERA-40 dataset, corrected for inhomogeneities, was used and significant increasing trends for mean, 90- and 99-percentiles were found for a large part of the global oceans. An intercomparison of significant wave height data derived from different reanalyses was presented in [38], and in spite of differences in the data quality and scope, it was reported that most of the long-term characteristics such as trends and variability, were equally present in all datasets.

Hence, most studies from the turn of the century generally agree that the wave climate of the North Atlantic became rougher in the past decades of the twentieth century. These general conclusions have been supported by analysis of microseismological data [81], by significant wave height data from ship observations [91] and by satellite altimetry data [43, 212]. Buoy measurements have also suggested an increase in wave height for the western Atlantic Ocean, but only for the summer hurricane season [114]. More recent studies observe the continuation of this increasing trend into the twenty-first century [34, 60], although there are still uncertainties as to whether, or to what extent, the trend can be ascribed to global warming [205, 211]. Increasing trends have also been found in other oceans than the Atlantic [39, 79, 163, 171].

However, it is noted that opposite trends have been reported for different regions, some studies reporting decreasing trends for particular regions [62, 135] and for different seasons (e.g., decreasing trends for the months February was reported in contrast to increasing trends in January for the Baltic Sea in [162]) so care should always be exercised when extrapolating conclusions arrived at from one location to another or from one season to another. Notwithstanding, there are evidence for a general overall trend of rougher wave climate in the North Atlantic as well as in various other ocean areas.

2.5.3 Projections of Future Trends in the Wave Climate

In light of the observed increasing trends in recent extreme wave climate in many areas of the world, a much relevant question is whether, or to what extent, this trend will prevail in the coming decades and how the future wave climate is expected to develop. In the following, a review of some attempts to make projections of future trends in the wave climate will be presented, with an emphasis on the trends for extreme waves.

The modeling approach outlined in [40], modeling extreme waves as non-homogeneous Poisson processes (NPP), utilizing the statistical relationship between wave height and sea level pressure, and compared to a non-stationary generalized extreme value model (NS-GEV), is already discussed briefly in previous sections. Previous works focusing on the relationship between wave height and sea level pressure include [199, 202, 203]. One interesting finding is that the regression model best describing the 20-year significant wave height time series toward 2099 is quadratic in time, in contrast to the the present climate where the trends are linear [37].

The changes of the projected extreme wave climate toward 2099 arrived at from the NPP model were found to be dependent on season and location and the spatial patterns were very similar for the different scenarios that were investigated. However, the magnitude of the estimated changes as well as the time evolution of the projections (i.e.,how fast the changes will occur) were scenario dependent. Comparing the projected times series of significant wave height return values obtained from the NPP and the NS-GEV models, it was found that they are highly correlated and hence compatible in some sense, but significant differences in means and variances were found, mainly in tropical areas. The seasonal projections of the 20-year significant wave height toward 2099, under different forcing scenarios and using a NPP model with parameters estimated from present climate data [40], predict significant changes in SWH_{20} in different regions of the world. The rate of future changes depends on the scenario, but under all scenarios considered, significant positive trends are predicted in the North Pacific. This is in agreement with the projections made in [170], which predict increases in significant wave height by up to 0.4 m over a wide area of the western North Pacific.

Projections of future wind, wave, and storm surge climate in the North Atlantic based on regional wave models are reported in [55, 56], where a future climate for the period 2030–2050 is compared to a control climate for the period 1980–2000. The initial study did not identify statistically significant changes in wave height [55], but when the study was revisited with more recent IPCC scenarios, the following statistically significant changes in extreme significant wave height were found: Significant increases west of the British Isles and in the eastern North Sea and in the Skagerrak and significant decreases west of 30°W [56].

Projections of extreme wave heights for the Northwest Pacific Ocean toward the year 2032 were presented in [163], based on various approaches, i.e., time-dependent Generalized Pareto-Poisson model with time-dependent event rate, Generalized Extreme Value-model with time-dependent trends in the location and scale parameters of the GEV distribution, and based on non-stationary r-largest extreme value analysis. The projected 100-year return level from the different modeling approaches showed a robust trend but with significant spread for the year 2032. Projections further into the future would yield still greater spread in the model projections. It is therefore cautioned against using projected extreme values in actual engineering problems and such projections should be considered as uncertain until the underlying causes of the long-term trends are better understood

In order to extend the confidence and coverage of future wave climate projections a proposal for a coordinated effort toward global wave climate projections was made in [96], suggesting a shift in focus from regional projections. Such a global study was recently published in [97], where projected changes in the global wave climate based on projections from an ensemble of previous studies, obtained by different models, were presented. The study reports an agreed projected decrease in annual mean significant wave height over 25.8 % of the global oceans and a projected increase over 7.1 %, predominantly in the Southern Ocean. However, a general caveat when using multimodel mean projections is that there might be a tendency of underestimating projected trends from individual models. Significant trends from different models might cancel each other out and become insignificant in a multimodel average and it might also be difficult to interpret the multimodel results. It is also noted in [97] that the study methodology is the dominating source of uncertainty. Notwithstanding, it is noted that results presented in [97] indicate a projected decrease in the annual mean significant wave height for a notable part of the global oceans.

Other wave climate projections based on dynamical downscaling of projections from global atmosphere-ocean climate models are reported for different regions of the world in [6, 35, 98, 126, 153], the details of which will not be covered herein.

Various downscaling methods for estimation of statistics for significant wave height were investigated in [204], evaluated against the ERA-40 wave data. Statistical downscaling approaches, typically based on the observed statistical relationship between atmospheric variables and wave height were deemed better than dynamical methods, which typically involves using atmospheric variables to drive ocean wave models. Furthermore, different atmospheric covariates were analysed in nested regression models (i.e., sea level pressure anomalies, sea level pressure gradients and anomalies of seasonal mean squared surface wind speeds) and analysis of the various models suggests that it is sufficient to use the wind-based predictor alone since this model performs very similar to the full model. Projections made from the different approaches show similar patterns for both seasonal means and extremes. In wintertime, increases in the eastern and western subtropical North Atlantic and decreases most other places were the predominant pattern whereas autumn projections were characterized by increases in the mid-latitudes and eastern subtropical North Atlantic and decreases in some other areas.

Dynamic and statistical downscaling techniques were also investigated in [74], and a combination of dynamical and statistical approaches was proposed as a faster, less computational-intensive alternative to purely dynamical methods for downscaling of medium-scale wave data, a conclusion supported by [30]. The method demonstrated reasonable agreement with observed wave conditions for simulations of an near-shore area around Helgoland.

The uncertainty of the impact of climate change on future extreme wave conditions in the North Sea was investigated in [80] by running the WAM wave model [82] over an ensemble of four different climate change realizations for the 30-year period 2071–2100. Wind field data sets were obtained by simulation outputs from two global circulation models for two emission scenarios, and compared to two control scenarios. The study revealed that there are large uncertainties in the magnitude and the spatial patterns of the climate change signals, and results indicate that the uncertainties due to different climate models are larger, by a factor exceeding five in some regions, than the uncertainties related to the different scenarios. Notwithstanding these uncertainties, it was general agreement between the simulations in that extreme wave heights will increase in large parts of the North Sea and that the future frequency of severe sea states will increase due to global warming.

2.6 Summary and Conclusions

This chapter has presented a comprehensive review of the literature concerning probabilistic modeling of ocean waves and sea states. It has addressed the importance of available wave data in order to develop sensible probabilistic models, and although buoy measurements are generally regarded as most reliable, the spatial coverage of such data may be inadequate for spatial models. Alternatives exist in satellite data and in reanalysis data obtained from wave models forced by various meteorological parameters. In particular, data from the ERA-40 reanalysis project that are freely available for scientific purposes have been identified.

Numerous statistical models for extreme waves have been reported in the literature, and some of these have been presented herein. Many of these either have short-term scope or are microscopic in the sense that they focus on a particular location where wave data have been available. That is, the spatial variability is not covered by many of these models. The long-term trends and time-dependencies due to observed and projected climate change are also poorly incorporated in some of these models. There have been some attempts to develop spatio-temporal models for extreme waves, and these have been discussed herein. However, due to the modest number of attempts to establish spatio-temporal models for extreme waves, a glance at models proposed in other areas of application has also been reported. Hence, a review of some models used in earthquake modeling, storm modeling, temperature modeling, and air pollution modeling has been presented. It is suggested that similar approaches might be appropriate for spatio-temporal modeling of extreme waves and further work should focus on developing such models.

In particular, the framework of Bayesian hierarchical space-time models has been identified as a promising candidate for further development of long-term stochastic models of extreme wave climate. It is believed that such a framework offers significant improvements in the statistical understanding and modeling of extreme waves and may be used in modeling and projecting long-term trends due to climate change.

Following the review of different stochastic models, a review of projections of future wave climate has been presented. Most of these predict changes in the global wave climate toward the end of the century, but the changes are very region-dependent and also highly dependent on the methodologies and models that have been applied. However, the overall message is that, at least for some parts of the Northern Atlantic, the wave climate will tend to become rougher during the present century. This means that historic wave data may no longer be adequate as basis for design of ships and offshore structures or for use in risk assessment and that new knowledge about the time-dependence and long-term trends of extreme wave climate is of crucial importance. However, different studies of wave climate projections disagree and this just serves to illustrate the complexity and the high degree of uncertainty that persist related to future projections of regional and global wave climates. On the other hand, this should motivate further research into the statistical relationships and development of improved spatio-temporal models for extreme waves.

References

1. Aalen, O.O., Gjessing, H.K.: Survival models based on the Ornstein-Uhlenbeck process. Lifetime Data Anal. **10**, 407–423 (2004)
2. Aberg, S., Lindgren, G.: Height distribution of stochastic Lagrange ocean waves. Probab. Eng. Mech. **23**, 359–363 (2008)
3. ACIA: Arctic Climate Impact Assessment. Cambridge University Press, Cambridge (2005)
4. Albano, G., Giorno, V.: A stochastic model in tumor growth. J. Theor. Biol. **242**, 329–336 (2006)
5. Altunkaynak, A.: Significant wave height prediction by using a spatial model. Ocean Eng. **32**, 924–936 (2005)
6. Andrade, C., Pires, H., Taborda, R., Freitas, M.: Projecting future changes in wave climate and coastal response in Portugal by the end of the 21st century. J. Coast. Res. SI **50**, 253–257 (2007)
7. Andreas, E.L., Wang, S.: Predicting significant wave height off the northeast coast of the United States. Ocean Eng. **34**, 1328–1335 (2007)
8. Arena, F., Barbaro, G., Romolo, A.: Return periods of a sea storm with at least two waves higher than a fixed threshold. In: Proceedings of the 28th International Conference on Offshore Mechanics and Arctic Engineering (OMAE 2009). American Society of Mechanical Engineers (ASME) (2009)
9. Arena, F., Pavone, D.: Return period of nonlinear high wave crests. J. Geophys. Res. **111**, C08, 004 (2006)
10. Arena, F., Pavone, D.: A generalized approach for long-term modelling of extreme crest-to-trough wave heights. Ocean Model. **26**, 217–225 (2009)
11. Arena, F., Puca, S.: The reconstruction of significant wave height time series by using a neural network approach. J. Offshore Mech. Arct. Eng. **126**, 213–219 (2004)
12. Athanassoulis, G., Stefanakos, C.N.: A nonstationary stochastic model for long-term time series of significant wave height. J. Geophys. Res. **100**(C8), 16, 149–16, 162 (1995)
13. Bacon, S., Carter, D.: Wave climate changes in the North Atlantic and North Sea. Int. J. Climatol. **11**, 545–558 (1991)
14. Barndorff-Nielsen, O.E., Shephard, N.: Non-gaussian Ornstein-Uhlenbeck-based models and some of their uses in financial economics. J. R. Stat. Soc. B **63**, 167–241 (2001)
15. Battjes, J.A.: Long-term wave height distribution at seven stations around the british isles. Ocean Dyn. **25**, 179–189 (1972)
16. Baxevani, A., Borgel, C., Rychlik, I.: Spatial models for the variability of the significant wave height on the world oceans. In: Proceedings of the 17th International Offshore and Polar Engineering Conference (ISOPE 2007). The International Society of Offshore and Polar Engineering (ISOPE) (2007)
17. Baxevani, A., Caires, S., Rychlik, I.: Spatio-temporal statistical modelling of significant wave height. Environmetrics **20**, 14–31 (2009)
18. Baxevani, A., Rychlik, I., Wilson, R.J.: A new method for modelling the space variability of significant wave height. Extremes **8**, 267–294 (2005)
19. Bazargan, H., Bahai, H., Aryana, F.: Simulation of the mean zero-up-crossing wave period using artificial neural networks trained with a simulated annealing algorithm. J. Mar. Sci. Technol. **12**(1), 22–33 (2007)
20. Behrens, C.N., Lopes, H.F., Gamerman, D.: Bayesian analysis of extreme events with threshold estimation. Stat. Modelling **4**, 227–244 (2004)
21. Benth, F.E., Kallsen, J., Meyer-Brandis, T.: A non-gaussian Ornstein-Uhlenbeck process for electricity spot price modeling and derivatives pricing. Appl. Math. Finance **14**, 153–169 (2007)
22. Bitner-Gregersen, E.M., Hagen, Ø.: Uncertainties in data for the offshore environment. Struct. Saf. **7**, 11–34 (1990)

23. Bitner-Gregersen, E.M., Toffoli, A.: Uncertainties of wind sea and swell prediction from the Torsethaugen spectrum. In: Proceedings of the 28th International Conference on Offshore Mechanics and Arctic Engineering (OMAE 2009). American Society of Mechanical Engineers (ASME) (2009)
24. Bitner-Gregersen, E.M., de Valk, C.: Quality control issues in estimating wave climate from hindcast and satellite data. In: Proceedings of the 27th International Conference on Offshore Mechanics and Arctic Engineering (OMAE 2008). American Society of Mechanical Engineers (ASME) (2008)
25. Bocotti, P.: Wave mechanics for ocean engineering. Elsevier Oceanography Series. Elsevier Science B.V., New York (2000)
26. Boer, G.J., Flato, G., Reader, M.C., Ramsden, D.: A transient climate change simulation with greenhouse gas and aerosol forcing: experimental design and comparison with the instrumental record for the twentieth century. Clim. Dyn. **16**, 405–425 (2000)
27. Borgman, L.E.: Probabilities for highest wave in hurricane. J. Waterw. Harb. Coast. Eng. Div. ASCE **99**, 185–207 (1973)
28. Boukhanovsky, A.V., Krogstad, H.E., Lopatoukhin, L.J., Rozhkov, V.A.: Stochastic simulation of inhomogeneous metocean fields. parti: Annual variability. In: P. Sloot, D. Abramson, A. Bogdanov, J. Dongarra, A. Zomaya, Y. Gorbachev (eds.) Computational science—ICCS 2003, Lecture notes in computer science, vol. 2658, pp. 213–222. Springer, Heidelberg (2003)
29. Boukhanovsky, A.V., Krogstad, H.E., Lopatoukhin, L.J., Rozhkov, V.A., Athanassoulis, G.A., Stephanakos, C.N.: Stochastic simulation of inhomogeneous metocean fields. partii: Synoptic variability and rare events. In: P. Sloot, D. Abramson, A. Bogdanov, J. Dongarra, A. Zomaya, Y. Gorbachev (eds.) Computational science—ICCS 2003, Lecture notes in computer science, vol. 2658, pp. 223–233. Springer, Heidelberg (2003)
30. Breivik, Ø., Gusdal, Y., Furevik, B.R., Aarnes, O.J., Reistad, M.: Nearshore wave forecasting and hindcasting by dynamical and statistical downscaling. J. Mar. Syst. **78**, 235–243 (2009)
31. Brown, P.E., Karesen, K.F., Roberts, G.O., Tonellato, S.: Blur-generated non-separable space-time models. J. R. Stat. Soc. Ser. B **62**, 847–860 (2000)
32. Bruno, F., Guttorp, P., Sampson, P.D., Cocchi, D.: A simple non-seperable, non-stationary spatiotemporal model for ozone. Environ. Ecol. Stat. **16**, 515–529 (2009)
33. Buishand, T., de Haan, L., Zhou, C.: On spatial extremes: with application to a rainfall problem. Ann. Appl. Stat. **2**, 624–642 (2008)
34. Caires, S., Groeneweg, J., Sterl, A.: Changes in the North Sea extreme waves. In: Preprints of 9th International Workshop on Wave Hindcasting and Forecasting (2006)
35. Caires, S., Groeneweg, J., Sterl, A.: Past and future changes in the North Sea extreme waves. In: Proceedings of the 31st International Conference on Coastal Engineering (ICCE 2008) (2008)
36. Caires, S., Sterl, A.: A new nonparametric method to correct model data: application to significant wave height from ERA-40 re-analysis. J. Atmospheric Ocean. Technol. **22**, 443–459 (2005)
37. Caires, S., Sterl, A., Bidlot, J.R., Graham, N., Swail, V.: Climatological assessment of reanalysis ocean data. In: Preprints of 7th International Workshop on Wave Hindcasting and Forecasting (2002)
38. Caires, S., Sterl, A., Bidlot, J.R., Graham, N., Swail, V.: Intercomparison of different wind-wave reanalyses. J. Clim. **17**, 1893–1913 (2004)
39. Caires, S., Swail, V.: Global wave climate trend and variability analysis. In: Preprints of 8th International Workshop on Wave Hindcasting and Forecasting (2004)
40. Caires, S., Swail, V.R., Wang, X.L.: Projection and analysis of extreme wave climate. J. Clim. **19**, 5581–5605 (2006)
41. Calder, C.A.: Dynamic factor process convolution models for multivariate space-time data with application to air quality assessment. Environ. Ecol. Stat. **14**, 229–247 (2007)
42. Calder, C.A.: A dynamic process convolution approach to modeling ambient particulate matter concentrations. Environmetrics **19**, 39–48 (2008)

43. Carter, D.: Variability and trends in the wave climate of the North Atlantic: a review. In: Proceedings of the 9th International Offshore and Polar Engineering conference (ISOPE 1999). The International Society of Offshore and Polar Engineering (ISOPE) (1999)
44. Carter, D., Draper, L.: Has the north-east Atlantic become rougher? Nature **332**, 494 (1988)
45. Choi, K.M., Yu, H.L., Wilson, M.L.: Spatiotemporal statistical analysis of influenza mortality risk in the State of California during the period 1997–2001. Stoch. Env. Res. Risk Assess. **22**, S15–S25 (2009)
46. Cocchi, D., Greco, F., Trivisano, C.: Hierarchical space-time modelling of PM_{10} pollution. Atmos. Environ. **41**, 532–542 (2007)
47. Coelho, C., Ferro, C., Stephenson, D., Steinskog, D.: Methods for exploring spatial and temporal variability of extreme events in climate data. J. Clim. **21**, 2072–2092 (2008)
48. Coles, S., Pericchi, L.R., Sisson, S.: A fully probabilistic approach to extreme rainfall modeling. J. Hydrol. **273**, 35–50 (2003)
49. Coles, S., Walshaw, D.: Directional modelling of extreme wind speeds. J. R. Stat. Soc. **43**, 139–157 (1994)
50. Coles, S.G.: Regional modelling of extreme storms via max-stable processes. J. R. Stat. Soc. B **55**, 797–816 (1993)
51. Coles, S.G., Tawn, J.A.: Modelling extremes of the areal rainfall process. J. R. Stat. Soc. B **58**, 329–347 (1996)
52. Cox, A.T., Swail, V.R.: A global wave hindcast over the period 1958–1997: validation and climate assessment. J. Geophys. Res. **106**, 2313–2329 (2001)
53. Cressie, N.: Wikle, C.K.: Statistics for spatio-temporal Data. Wiley, Hoboken (2011)
54. Cunha, C., Guedes Soares, C.: On the choice of data transformation for modelling time series of significant wave height. Ocean Eng. **26**, 489–506 (1999)
55. Debernard, J., Sætra, Ø., Røed, L.P.: Future wind, wave and storm surge climate in the northern North Atlantic. Clim. Res. **23**, 39–49 (2002)
56. Debernard, J.B., Røed, L.P.: Future wind, wave and storm surge climate in the Northern Seas: a revisit. Tellus **60A**, 427–438 (2008)
57. DelBalzo, D.R., Schultz, J.R., Earle, M.D.: Stochastic time-series simulation of wave parameters using ship observations. Ocean Eng. **30**, 1417–1432 (2003)
58. Deo, M., Jha, A., Chaphekar, A., Ravikant, K.: Neural networks for wave forecasting. Ocean Eng. **28**, 889–898 (2001)
59. Diebolt, J., Guillou, A., Rached, I.: Approximation of the distribution of excesses through a generalized probability-weighted moments method. J. Stat. Plan. Inference **137**, 841–857 (2007)
60. Dodet, G., Bertin, X., Tabora, R.: Wave climate variability in the North-East Atlantic Ocean over the last six decades. Ocean Model. **31**, 120–131 (2010)
61. Douanik, A., van Meirvenne, M., Tóth, T., Serre, M.: Space-time mapping of soil salinity using probabilistic bayesian maximum entropy. Stoch. Env. Res. Risk Assess. **18**, 219–227 (2004)
62. Dupuis, H., Michel, D., Sottolichio, A.: Wave climate evolution in the Bay of Biscay over two decades. J. Mar. Syst. **63**, 105–114 (2006)
63. Durrant, T.H., Greenslade, D.J., Simmonds, I.: Validation of Jason-1 and Envisat remotely sensed wave heights. J. Atmospheric Ocean. Technol. **26**, 123–134 (2009)
64. Eastoe, E.F., Tawn, J.A.: Modelling non-stationary extremes with application to surface level ozone. J. R. Stat. Soc. Ser. C (Appl. Stat.) **58**, 25–45 (2009)
65. Egozcue, J., Pawlowsky-Glahn, V., Ortego, M.: Wave-height hazard analysis in eastern coast of Spain - Bayesian approach using generalized Pareto distribution. Adv. Geosci. **2**, 25–30 (2005)
66. Ewans, K., Bitner-Gregersen, E., Guedes Soares, C.: Estimation of wind-sea and swell components in a bimodal sea state. J. Offshore Mech. Arct. Eng. **128**(4), 265–270 (2006)
67. Fedele, F., Arena, F.: The equivalent power storm model for long-term predictions of extreme wave events. In: Proceedings of the 28th International Conference on Offshore Mechanics and Arctic Engineering (OMAE 2009). American Society of Mechanical Engineers (ASME) (2009)

68. Fedele, F., Sampath, P., Benetazzo, A., Forristall, G., Gallego, G., Yezzi, A., Tayfun, M., Cavaleri, L., Sclavo, M., Bastianini, M.: Beyond waves & spectra: Euler characteristics of ocean sea states. In: Proceedings of the 28th International Conference on Offshore Mechanics and Arctic Engineering (OMAE 2009). American Society of Mechanical Engineers (ASME) (2009)
69. Ferreira, J., Guedes Soares, C.: Modelling the long-term distribution of significant wave height with the Beta and Gamma models. Ocean Eng. **26**, 713–725 (1999)
70. Ferreira, J., Guedes Soares, C.: Modelling distributions of significant wave height. Coast. Eng. **40**, 361–374 (2000)
71. Forristall, G.Z.: How should we combine long and short term wave height distributions? In: Proceedings of the 27th International Conference on Offshore Mechanics and Arctic Engineering (OMAE 2008). American Society of Mechanical Engineers (ASME) (2008)
72. Fowler, H., Kilsby, C.: A regional frequency analysis of United Kingdom extreme rainfall from 1960 to 2000. Int. J. Clim. **23**, 1313–1334 (2003)
73. Galanis, G., Chu, P.C., Kallos, G., Kuo, Y.H., Dodson, C.T.J.: Wave height characteristics in the north Atlantic ocean: a new approach based on statistical and geometrical techniques. Stoch. Env. Res. Risk Assess. **26**, 83–103 (2012)
74. Gaslikova, L., Weisse, R.: Estimating near-shore wave statistics from regional hindcasts using downscaling techniques. Ocean Dyn. **56**, 26–35 (2006)
75. Gaume, E., Gaál, L., Viglione, A., Szolgay, J., Kohnová, S., Blöschl, G.: Bayesian MCMC approach to regional flood frequency analysis involving extraordinary flood events an ungauged sites. J. Hydrol. **394**, 101–117 (2010)
76. Gelfand, A.E., Schmidt, A.M., Benerjee, S., Sirmans, C.: Nonstationary multivariate process modeling through spatially varying coregionalization. Test **13**, 263–312 (2004)
77. Gibson, R., Forristall, G.Z., Owrid, P., Grant, C., Smyth, R., Hagen, O., Leggett, I.: Bias and uncertainty in the estimation of extreme wave heights and crests. In: Proceedings of the 28th International Conference on Offshore Mechanics and Arctic Engineering (OMAE 2009). American Society of Mechanical Engineers (ASME) (2009)
78. Gilchrist, W.G.: Statistical modelling with quantile functions. Chapman & Hall/CRC, Boca Raton (2000)
79. Gower, J.: Temperature, wind and wave climatologies, and trends from marine meteorological buoys in the northeast Pacific. J. Clim. **15**, 3709–3718 (2002)
80. Grabemann, I., Weisse, R.: Climate change impact on extreme wave conditions in the North Sea: an ensemble study. Ocean Dyn. **58**, 199–212 (2008)
81. Grevemeyer, I., Herber, R., Essen, H.H.: Microseismological evidence for a changing wave climate in the northeast Atlantic Ocean. Nature **408**, 349–352 (2000)
82. Group, T.W.: The WAM model—a third generation ocean wave prediction model. J. Phys. Oceanogr. **18**, 1775–1810 (1988)
83. Group, T.W.: Changing waves and storms in the Northeast Atlantic. Bull. Am. Meteorol. Soc. **79**, 741–760 (1998)
84. Group, T.W., Cavaleri, L., Alves, J.H., Ardhuin, F., Babanin, A., Banner, M., Belibassakis, K., Benoit, M., Donelan, M., Groeneweg, J., Herbers, T., Hwang, P., Janssen, P., Janssen, T., Lavrenov, I., Magne, R., Monbaliu, J., Onorato, M., Polnikov, V., Resio, D., Rogers, W., Sheremet, A., McKee Smith, J., Tolman, H., van Vledder, G., Wolf, J., Young, I.: Wave modelling—the state of the art. Prog. Oceanogr. **75**, 603–674 (2007)
85. Guedes Soares, C., Cunha, C.: Bivariate autoregressive models for the time series of significant wave height and mean period. Coast. Eng. **40**, 297–311 (2000)
86. Guedes Soares, C., Ferreira, A.: Representatin of non-stationary time series of significant wave height with autoregressive models. Probab. Eng. Mech. **11**, 139–148 (1996)
87. Guedes Soares, C., Ferreira, A., Cunha, C.: Linear models of the time series of significant wave height on the Southwest Coast of Portugal. Coast. Eng. **29**, 149–167 (1996)
88. Guedes Soares, C., Scotto, M.: Modelling uncertainty in long-term predictions of significant wave height. Ocean Eng. **28**, 329–342 (2001)

89. Guedes Soares, C., Scotto, M.: Application of the r largest-order statistics for long-term predictions of significant wave height. Coast. Eng. **51**, 387–394 (2004)
90. Guedes Soares, C., Weisse, R., Carretero, J.C., Alvarez, E.: A 40 years hindcast of wind, sea level and waves in European waters. In: Proceedings of the 21st International Conference on Offshore Mechanics and Arctic Engineering (OMAE 2002). American Society of Mechanical Engineers (ASME) (2002)
91. Gulev, S.K., Hasse, L.: Changes of wind waves in the North Atlantic over the last 30 years. Int. J. Climatol. **19**, 1091–1117 (1999)
92. Gutiérrez, R., Gutiérrez-Sánchez, R., Nafidi, A.: Trend analysis using nonhomogeneous stochastic diffusion processes. Emission of CO2; Kyoto protocol in Spain. Stoch. Env. Res. Risk Assess. **22**, 57–66 (2008)
93. de Haan, L.: A spectral representation for max-stable processes. Ann. Probab. **12**, 1194–1204 (1984)
94. Hagen, Ø.: Estimation of long term extreme waves from storm statistics and initial distribution approach. In: Proceedings of the 28th International Conference on Offshore Mechanics and Arctic Engineering (OMAE 2009). American Society of Mechanical Engineers (ASME) (2009)
95. Hansen, T.F., Pienaar, J., Orzack, S.H.: A comparative method for studying adaptation to a randomly evolving environment. Evolution **62**, 1965–1977 (2008)
96. Hemer, M., Church, J., Swail, V., Wang, X.: Coordinated global wave climate projections. In: Preprints of 11th International Workshop on Wave Hindcasting and Forecasting and Coastal Hazards, Symposium (2009)
97. Hemer, M.A., Fan, Y., Mori, N., Semedo, A., Wang, X.L.: Projected changes in wave climate from a multi-model ensemble. Nat. Clim. Change Adv. (online publication), 1–6 (2013)
98. Hemer, M.A., McInnes, K., Ranasinghe, R.: Future projections of the East Australian wave climate. In: Preprints of 11th International Workshop on Wave Hindcasting and Forecasting and Coastal Hazards, Symposium (2009)
99. Higdon, D.: A process-convolution approach to modelling temperatures in the North Atlantic Ocean. Environ. Ecol. Stat. **5**, 173–190 (1998)
100. Hoskins, J.R.M., Wallis, J.R.: Regional frequency analysis. Cambridge University Press, Cambridge (1997)
101. Huerta, G., Sansó, B.: Time-varying models for extreme values. Environ. Ecol. Stat. **14**, 285–299 (2007)
102. Hwang, P.A., Teague, W.J., Jacobs, G.A.: A statistical comparison of wind speed, wave height, and wave period derived from satellite altimeters and ocean buoys in the Gulf of Mexico region. J. Geophys. Res. **103**, 10, 451–10, 468 (1998)
103. IPCC: Climate change 2007: Synthesis report. Technical report. Intergovernmental Panel on Climate Change (2007)
104. IPCC: Managing the risks of extreme events and disasters to advance climate change adaptation. Cambridge University Press, Cambridge (2011)
105. Izadparast, A.H., Niedzwecki, J.M.: Estimating wave crest distributions using the method of L-moments. Appl. Ocean Res. **31**, 37–43 (2009)
106. Janssen, P.A.: Progress in ocean wave forecasting. J. Comput. Phys. **227**, 3572–3594 (2008)
107. Jenkins, A.D.: Wave duration/persistence statistics, recording interval and fractal dimension. Int. J. Offshore Polar Eng. **12**, 109–113 (2002)
108. Jönsson, A., Broman, B., Rahm, L.: Variations in the Baltic Sea wave fields. Ocean Eng. **30**, 107–126 (2002)
109. Jun, M., Stein, M.L.: An approach to producing space-time covariance functions on spheres. Technometrics **49**, 468–479 (2007)
110. Jun, M., Stein, M.L.: Nonstationary covariance models for global data. Ann. Appl. Stat. **2**, 1271–1289 (2008)
111. Kalnay, E., Kanamitsu, M., Kistler, R., Collins, W., Deaven, D., Gandin, L., Iredell, M., Saha, S., White, G., Woollen, J., Zhu, Y., Chelliah, M., Ebisuzaki, W., Higgins, W., Janowiak, J., Mo, K., Ropelewski, C., Wang, J., Leetmaa, A., Reynolds, R., Jenne, R., Joseph, D.: The NCEP/NCAR 40-year reanalysis project. Bull. Am. Meteorol. Soc. **77**, 437–471 (1996)

112. Kinsman, B.: Wind Waves: their generation and propagation on the ocean surface. Prentice-Hall, Englewood Cliffs (1965)
113. Kolovos, A., Christakos, G., Hristopulos, D., Serre, M.: Methods for generating non-separable spatiotemporal covariance models with potential environmental applications. Adv. Water Resour. **27**, 815–830 (2004)
114. Komar, P.D., Allan, J.C.: Increasing hurricane-generated wave heights along the U.S. east coast and their climate controls. J. Coastal Res. **24**, 479–488 (2008)
115. Komen, G., Cavaleri, L., Donelan, M., Hasselmann, K., Hasselmann, S., Janssen, P.: Dynamics and modelling of ocean waves. Cambridge University Press, Cambridge (1994)
116. Krogstad, H.E.: Height and period distributions of extreme waves. Appl. Ocean Res. **7**, 158–165 (1985)
117. Krogstad, H.E., Barstow, S.F.: Satellite wave measurements for coastal engineering applications. Coast. Eng. **37**, 283–307 (1999)
118. Krogstad, H.E., Barstow, S.F.: Analysis and applications of second-order models for maximum crest height. J. Offshore Mech. Arct. Eng. **126**, 66–71 (2004)
119. Kundu, P.K., Bell, T.L.: Space-time scaling behaviour of rain statistics in a stochastic fractional diffusion model. J. Hydrol. **322**, 49–58 (2006)
120. Kushnir, Y., Cardone, V., Greenwood, J., Cane, M.: The recent increase in north atlantic wave heights. J. Clim. **10**, 2107–2113 (1997)
121. Leclerc, M., Ourada, T.B.: Non-stationary regional flood frequency analysis at ungauged sites. J. Hydrol. **343**, 254–265 (2007)
122. Lee, S.J., Balling, R., Gober, P.: Bayesian maximum entropy mapping and the soft data problem in urban climate research. Ann. Assoc. Am. Geogr. **98**, 309–322 (2008)
123. Lehner, S., Koenig, T., Schulz-Stellenfleth, J.: Global statistics of extreme windspeed and sea state from SAR. In: Proceedings of the Envisat Symposium 2007. European Space Agency (2007)
124. Lemos, R.T., Sansó, B.: A spatio-temporal model for mean, anomaly and trend fields of north atlantic sea surface temperature. J. Am. Stat. Assoc. **104**, 5–18 (2009)
125. Lindgren, G.: Slepian models for the stochastic shape of individual Lagrange sea waves. Adv. Appl. Probab. **38**, 430–450 (2006)
126. Lionello, P., Cogo, S., Galati, M., Sanna, A.: The Mediterranean surface wave climate inferred from future scenario simulations. Global Planet. Change **63**, 152–162 (2008)
127. Ma, C.: Recent developments on the construction of spatio-temporal covariance models. Stoch. Env. Res. Risk Assess. **22**, S39–S47 (2008)
128. MA, Q.S., Li, Y.B., Li, J.: Regional frequency analysis of significant wave heights based on L-moments. China Ocean Eng. **20**, 85–98 (2006)
129. Machado, U., Rychlik, I.: Wave statistics in non-linear random sea. Extremes **6**, 125–146 (2003)
130. Mahjoobi, J., Etemad-Shahidi, A.: An alternative approach for the prediction of significant wave heights based on classification and regression trees. Appl. Ocean Res. **30**, 172–177 (2008)
131. Mahjoobi, J., Mosabbeb, E.A.: Prediction of significant wave height using regressive support vector machines. Ocean Eng. **36**, 339–347 (2009)
132. Majumdar, A., Gelfand, A.E.: Multivariate spatial modeling for geostatistical data using convolved covariance functions. Math. Geol. **39**, 225–245 (2007)
133. Makarynskyy, O., Pires-Silva, A., Makarynska, D., Ventura-Soares, C.: Artificial neural networks in wave predictions at the west coast of Portugal. Comput. Geosci. **31**, 415–424 (2005)
134. Mandal, S., Prabaharan, N.: Ocean wave forecasting using recurrent neural networks. Ocean Eng. **33**, 1401–1410 (2006)
135. Martucci, G., Carniel, S., Chiggiato, J., Sclavo, M., Lionello, P., Galati, M.: Statistical trend analysis and extreme distribution of significant wave height from 1958 to 1999—an appliaction to the Italian seas. Ocean Sci. Discuss. **6**, 2005–2036 (2009)
136. Meath, S.E., Aye, L., Haritos, N.: Accuracy of satellite-measured wave heights in the Australian region for wave power applications. Bull. Sci. Technol. Soc. **28**, 244–255 (2008)

137. Méndez, F.J., Menéndez, M., Luceño, A., Losada, I.J.: Estimation of the long-term variability of extreme significant wave height using time-dependent Peak Over Threshold (POT) model. J. Geophys. Res. **111**, C07, 024 (2006)
138. Méndez, F.J., Menéndez, M., Luceño, A., Medina, R., Graham, N.E.: Seasonality and duration in extreme value distributions of significant wave height. Ocean Eng. **35**, 131–138 (2008)
139. Menéndez, M., Méndez, F.J., Izaguirre, C., Luceño, A., Losada, I.J.: The influence of seasonality on estimating return values of significant wave height. Coast. Eng. **56**, 211–219 (2009)
140. Middleton, J.F., Thompson, K.R.: Return periods of extreme sea levels from short records. J. Geophys. Res. **91**, 11, 707–11,716 (1986)
141. Minoura, M., Naito, S.: Stochastic sea climate simulation based on hindcast data. In: Proceedings of the 16th International Offshore and Polar Engineering conference (ISOPE 2006). The International Society of Offshore and Polar Engineering (ISOPE) (2006)
142. Monbet, V., Ailliot, P., Prevesto, M.: Survey of stochastic models for wind and sea state time series. Probab. Eng. Mech. **22**, 113–126 (2007)
143. Monbet, V., Marteau, P.F.: Continuous space discrete time markov models for multivariate sea state parameter processes. In: Proceedings of the 11th International Offshore and Polar Engineering conference (ISOPE 2001). The International Society of Offshore and Polar Engineering (ISOPE) (2001)
144. Monbet, V., Prevosto, M.: Bivariate simulation of non stationary and non Gaussian observed processes—application to sea state parameters. Appl. Ocean Res. **23**, 139–145 (2001)
145. Muraleedharan, G., Rao, A., Kurup, P., Unnikrishnan Nair, N., Sinha, M.: Modified Weibull distribution for maximum and significant wave height simulation and prediction. Coast. Eng. **54**, 630–638 (2007)
146. Nakićenović, N., Alcamo, J., Davis, G., de Vries, B., Fenhann, J., Gaffin, S., Gregory, K., Grügler, A., Jung, T.Y., Kram, T., La Rovere, E.L., Michaelis, L., Mori, S., Morita, T., Pepper, W., Pitcher, H., Price, L., Riahi, K., Roehrl, A., Rogner, H.H., Sankovski, A., Schlesinger, M., Shukla, P., Smith, S., Swart, R., van Rooijen, S., Victor, N., Dadi, Z.: Emissions scenarios. Cambridge University Press, Cambridge (2000)
147. Natvig, B., Tvete, I.F.: Bayesian hierarchical space-time modeling of earthquake data. Methodol. Comput. Appl. Probab. **9**, 89–114 (2007)
148. Nguyen, V., Nguyen, T., Ashkar, F.: Regional frequency analysis of extreme rainfalls. Water Sci. Technol. **45**, 75–81 (2002)
149. Ochi, M.K.: Ocean waves: the stochastic approach. Cambridge Ocean Technology series no. 6. Cambridge University Press, Cambridge (1998)
150. Olsen, A., Schrøter, C., Jensen, J.: Wave height distribution observed by ships in the North Atlantic. Ships Offshore Struct. **1**, 1–12 (2006)
151. Özger, M., Şen, Z.: Triple diagram method for the prediction of wave height and period. Ocean Eng. **34**, 1060–1068 (2007)
152. Panchang, V., Zhao, L., Demirbilek, Z.: Eestimation of extreme wave heights using GEOSAT measurements. Ocean Eng. **26**, 205–225 (1999)
153. Perrie, W., Jiang, J., Long, Z., Toulany, B., Zhang, W.: NW Atlantic wave estimates and climate change. In: Preprints of 8th International Workshop on Wave Hindcasting and Forecasting (2004)
154. Phillips, S.J., Anderson, R.P., Schapire, R.E.: Maximum entropy modeling of species geographic distributions. Ecol. Model. **190**, 231–259 (2006)
155. Prevesto, M., Krogstad, H.E., Robin, A.: Probability distributions for maximum wave and crest heights. Coast. Eng. **40**, 329–360 (2000)
156. Queffeulou, P.: Long-term validation of wave height measurements from altimeters. Mar. Geodesy **27**, 495–510 (2004)
157. Reitan, T., Schweder, T., Henderiks, J.: Phenotypic evolution studied by layered stochastic differential equations. Ann. Appl. Stat. **6**, 1531–1551 (2012)
158. Renard, B., Lang, M., Bois, P.: Statistical analysis of extreme events in a non-stationary context via a Bayesian framework: case study with peak-over-threshold data. Stoch. Env. Res. Risk Assess. **21**, 97–112 (2006)

159. Ribereau, P., Guillou, A., Naveau, P.: Estimating return levels from maxima of non-stationary random sequences using the Generalized PWM method. Nonlinear Process. Geophys. **15**, 1033–1039 (2008)
160. Richardson, K., Steffen, W., Schellnhuber, H.J., Alcamo, J., Barker, T., Kammen, D.M., Leemans, R., Liverman, D., Munasinghe, M., Osman-Elasha, B., Stern, N., Wæver, O.: International scientific congress climate change: global risks, challenges & decisions—synthesis report. Technical report, International Alliance of Research Universities (2009)
161. Robert, C.P., Casella, G.: Monte Carlo statistical methods, 2nd edn. Springer, Heidelberg (2004)
162. Różyński, G.: Long-term evolution of Baltic Sea wave climate near a coastal segment in Poland; its drivers and impacts. Ocean Eng. **37**, 186–199 (2010)
163. Ruggiero, P., Komar, P.D., Allan, J.C.: Increasing wave heights and extreme value projections: the wave climate of the U.S. Pacific Northwest. Coast. Eng. **57**, 539–552 (2010)
164. Rychlik, I.: On some reliability applications of Rice's formula for the intensity of level crossings. Extremes **3**, 331–348 (2000)
165. Rydén, J.: A note on asymptotic approximations of distributions for maxima of wave crests. Stoch. Env. Res. Risk Assess. **20**, 238–242 (2006)
166. Saf, B.: Assessment of the effects of discordant sites on regional flood frequency analysis. J. Hydrol. **380**, 362–375 (2010)
167. Sahu, S.K., Gelfand, A.E., Holland, D.M.: High resolution space-time ozone modeling for assessing trends. J. Am. Stat. Assoc. **102**, 1212–1220 (2007)
168. Sang, H., Gelfand, A.E.: Hierarchical modeling for extreme values observed over space and time. Environ. Ecol. Stat. **16**, 407–426 (2009)
169. Sansó, B., Schmidt, A.M., Nobre, A.A.: Bayesian spatio-temporal models based on discrete convolutions. Can. J. Stat. **36**, 239–258 (2008)
170. Sasaki, W., Hibiya, T., Kayahara, T.: Interannual variability and future projections of summertime ocean wave heights in the western North Pacific. Ocean Sci. Discuss. **3**, 1637–1651 (2006)
171. Sasaki, W., Iwasaki, S., Matsuura, T., Iizuka, S.: Recent increase in summertime extreme wave heights in the western North Pacific. Geophys. Res. Lett. **32**, L15, 607 (2005)
172. Savelieva, E., Demyanov, V., Kanevski, M., Serre, M., Christakos, G.: BME-based uncertainty assessment of the Chernobyl fallout. Geoderma **128**, 312–324 (2005)
173. Schmidt, A.M., Gelfand, A.E.: A Bayesian coregionalization approach for multivariate pollutant data. J. Geophys. Res. **108**, STS 10 (2003)
174. Scotto, M., Guedes Soares, C.: Modelling the long-term time series of significant wave height with non-linear threshold models. Coast. Eng. **40**, 313–327 (2000)
175. Scotto, M., Guedes Soares, C.: Bayesian inference for long-term prediction of significant wave height. Coast. Eng. **54**, 393–400 (2007)
176. Smith, E.L., Stephenson, A.G.: An extended gaussian max-stable process model for spatial extremes. J. Stat. Plan. Inference **139**, 1266–1275 (2009)
177. Soukissian, T.H., Kalantzi, G., Karagali, I.: De-clustering of Hs-time series for applying the peaks-over-threshold method. In: Proceedings of the 16th International Offshore and Polar Engineering conference (ISOPE 2006). The International Society of Offshore and Polar Engineering (ISOPE) (2006)
178. Soukissian, T.H., Kalantzi, G.D.: Extreme value analysis methods used for extreme wave prediction. In: Proceedings of the 16th International Offshore and Polar Engineering conference (ISOPE 2006). The International Society of Offshore and Polar Engineering (ISOPE) (2006)
179. Soukissian, T.H., Kalantzi, G.D.: A new method for applying the r-largest maxima model for design sea-state prediction. In: Proceedings of the 17th International Offshore and Polar Engineering Conference (ISOPE 2007). The International Society of Offshore and Polar Engineering (ISOPE) (2007)
180. Soukissian, T.H., Photiadou, C.S.: A sensitivity analysis of the bottom-up algorithm for the segmentation of H_s-time series. In: Proceedings of the 16th International Offshore and Polar Engineering conference (ISOPE 2006). The International Society of Offshore and Polar Engineering (ISOPE) (2006)

181. Soukissian, T.H., Samalekos, P.E.: Analysis of the duration and intensity os sea states using segmentation of significant wave height time series. In: Proceedings of the 16th International Offshore and Polar Engineering conference (ISOPE 2006). The International Society of Offshore and Polar Engineering (ISOPE) (2006)
182. Stefanakos, C.N., Athanassoulis, G.A.: Extreme value predictions based on nonstationary time series of wave data. Environmetrics **17**, 25–46 (2006)
183. Stefanakos, C.N., Monbet, V.: Estimation of wave height return periods using a nonstationary time series modelling. In: Proceedings of the 25th International Conference on Offshore Mechanics and Arctic Engineering (OMAE 2006). American Society of Mechanical Engineers (ASME) (2006)
184. Stein, E.M., Stein, J.C.: Stock price distributions with stochastic volatility: an analytic approach. Rev. Financial Stud. **4**, 727–752 (1991)
185. Sterl, A., Caires, S.: Climatology, variability and extrema of ocean waves: the web-based KNMI/ERA-40 wave atlas. Int. J. Climatol. **25**, 963–977 (2005)
186. Sterl, A., Komen, G., Cotton, P.: Fifteen years of global wave hindcasts using winds from the European Centre for Medium-Range Weather Forecast reanalysis: validating the reanalyzed winds and assessing the wave climate. J. Geophys. Res. **103**, 5477–5492 (1998)
187. Swail, V.R., Cox, A.T.: On the use of NCEP-NCAR reanalysis surface marine wind fields for a long-term North Atlantic wave hindcast. J. Atmospheric Ocean. Technol. **17**, 532–545 (2000)
188. Todorov, V.: Estimation of continuous-time stochastic volatility models with jumps using high-frequency data. J. Econom. **148**, 131–148 (2009)
189. Torsethaugen, K.: A two-peak wave spectral model. In: Proceedings of the 12th International Conference on Offshore Mechanics and Arctic Engineering (OMAE 1993). American Society of Mechanical Engineers (ASME) (1993)
190. Torsethaugen, K., Haver, S.: Simplified double peak spectral model for ocean waves. In: Proceedings of the 14th International Offshore and Polar Engineering conference (ISOPE 2004). The International Society of Offshore and Polar Engineering (ISOPE) (2004)
191. Trulsen, K.: Weakly nonlinear and stochastic properties of ocean wave fields. application to an extreme wave event. In: J. Grue, K. Trulsen (eds.) Waves in geophysical fluids—Tsunamis, rogue waves, internal waves and internal tides, CISM Courses and lectures No. 489, pp. 49–106. Springer, Heidelberg (2006)
192. Unami, K., Abagale, F.K., Yangyuoru, M., Alam, A.H.M.B., Kranjac-Berisavljevic, G.: A stochastic differential equation model for assessing drought and flood risks. Stoch. Env. Res. Risk Assess. **24**, 725–733 (2010)
193. Uppala, S.M., Kållberg, P.W., Simmons, A.J., Andrae, U., Da, Costa Bechtold, V., Fiorino, M., Gibson, J.K., Haseler, J., Hernandez, A., Kelly, G.A., Li, X., Onogi, K., Saarinen, S., Sokka, N., Allan, R.P., Andersson, E., Arpe, K., Balmaseda, M.A., Beljaars, A.C.M., Van de Berg, L., Bidlot, J., Bormann, N., Caires, S., Chevallier, F., Dethof, A., Dragosavac, M., Fisher, M., Fuentes, M., Hagemann, S., Hólm E. Hoskins, B.J., Isaksen, L., Janssen, P.A.E.M., Jenne, R., McNally, A.P., Mahfouf, J.F., Morcrette, J.J., Rayner, N.A., Saunders, R.W., Simon, P., Sterl, A., Trenberth, K.E., Untch, A., Vasiljevic, D., Vitebro, P., Woolen, J.: The ERA-40 re-analysis. Q. J. R. Meteorol. Soc. **131**, 2961–3012 (2005)
194. Vaccaro, S.: Position-dependent stochastic diffusion model of ion channel gating. Phys. Rev. E **78**, 061, 915 (2008)
195. Van Gelder, P., De Ronde, J., Neykov, N.M., Neytchev, P.: Regional frequency analysis of extreme wave heights: trading space for time. In: B.L. Edge (ed.) Coastal engineering (2000), Proceedings of the 27th International Conference on Coastal Engineering Held in Sydney, Australia, July 16–21, 2000, pp. 1099–1112. American Society of Civil Engineers (2001)
196. Vanem, E.: Long-term time-dependent stochastic modelling of extreme waves. Stoch. Env. Res. Risk Assess. **25**, 185–209 (2011)
197. de Waal, D., van Gelder, P.: Modelling of extreme wave heights and periods through copulas. Extremes **8**, 345–356 (2005)

198. Walton, T.L., Borgman, L.E.: Simulation of nonstationary, non-Gaussian water levels on Great Lakes. J. Waterw. Port Coast. Ocean Eng. **116**, 664–685 (1990)
199. Wang, X.J., Zwiers, F.W., Swail, V.R.: North Atlantic ocean wave climate change scenarios for the twenty-first century. J. Clim. **17**, 2368–2383 (2004)
200. Wang, X.L., Swail, V.R.: Changes of extreme wave heights in northern hemisphere oceans and related atmospheric circulation regimes. J. Clim. **14**, 2204–2221 (2001)
201. Wang, X.L., Swail, V.R.: Trends of Atlantic wave extremes as simulated in a 40-yr wave hindcast using kinematically reanalyzed wind fields. J. Clim. **15**, 1020–1035 (2002)
202. Wang, X.L., Swail, V.R.: Climate change signal and uncertainty in projections of ocean wave heights. Clim. Dyn. **26**, 109–126 (2006)
203. Wang, X.L., Swail, V.R.: Historical and possible future changes of wave heights in northern hemisphere oceanc. In: W. Perrie (ed.) Atmosphere-ocean interactions, Advances in fluid mechanics, vols. 2, 39, pp. 185–218. WIT Press, Southampton (2006)
204. Wang, X.L., Swail, V.R., Cox, A.: Dynamical versus statistical downscaling methods for ocean wave heights. Int. J. Climatol. **30**, 317–332 (2010)
205. Wang, X.L., Swail, V.R., Zwiers, F.W., Zhang, X., Feng, Y.: Detection of external influences on trends of atmospheric storminess and northern oceans wave heights. Clim. Dyn. **32**, 189–203 (2009)
206. Weisse, R., Stawarz, M.: Long-term changes and potential for future developments of the North Sea wave climate. In: Preprints of 8th International Workshop on Wave Hindcasting and Forecasting (2004)
207. West, M., Harrison, J.: Bayesian forecasting and dynamic models, 2nd edn. Springer, Heidelberg (1997)
208. Wikle, C.K.: Hierarchical models in environmental science. Int. Stat. Rev. **71**, 181–199 (2003)
209. Wikle, C.K., Berliner, L.M., Cressie, N.: Hierarchical Bayesian space-time models. Environ. Ecol. Stat. **5**, 117–154 (1998)
210. Wikle, C.K., Milliff, R.F., Nychka, D., Berliner, L.M.: Spatiotemporal hierarchical Bayesian modeling: tropical ocean surface winds. J. Am. Stat. Assoc. **96**, 382–397 (2001)
211. Wolf, J., Woolf, D.K.: Waves and climate change in the north-east Atlantic. Geophys. Res. Lett. **33**, L06, 604 (2006)
212. Woolf, D., Challenor, P., Cotton, P.: Variability and predictability of the North Atlantic wave climate. J. Geophys. Res. **107**, 9(1–14) (2002)
213. Yang, T., Shao, Q., Hao, Z.C., Chen, X., Zhang, Z., Xu, C.Y., Sun, L.: Regional frequency analysis and spatio-temporal pattern characterization of rainfall extremes in the Pearl River Basin, China. J. Hydrol. **380**, 386–405 (2010)
214. Yoon, S., Cho, W., Heo, J.H.: A full Bayesian approach to generalized maximum likelihood estimation of generalized extreme value distribution. Stoch. Env. Res. Risk Assess. **24**, 761–770 (2010)
215. Zhang, H.: Maximum-likelihood estimation for multivariate spatial linear coregionalization models. Environmetrics **18**, 125–139 (2007)

http://www.springer.com/978-3-642-30252-7

Bayesian Hierarchical Space-Time Models with
Application to Significant Wave Height
Vanem, E.
2013, XX, 262 p. 85 illus., 14 illus. in color., Hardcover
ISBN: 978-3-642-30252-7