

# Contents

|          |                                                                                                                                             |          |
|----------|---------------------------------------------------------------------------------------------------------------------------------------------|----------|
| <b>1</b> | <b>Introduction</b>                                                                                                                         | <b>1</b> |
| 1.1      | A General Background                                                                                                                        | 1        |
| 1.2      | Text Preview                                                                                                                                | 5        |
|          | References                                                                                                                                  | 7        |
| <b>2</b> | <b>Stability Loss Problems Related to Solid and Hollow Circular Cylinders</b>                                                               | <b>9</b> |
| 2.1      | Formulation of the Problem Related to the Global Stability Loss                                                                             | 9        |
| 2.2      | Method of Solution for the Global Stability Loss Problem                                                                                    | 13       |
| 2.3      | Approximate Equations for the Stability Loss of the Cylinder-Beam Obtained from Equations of the TDLTS by the Average-Integrating Procedure | 26       |
| 2.3.1    | Bernoulli Beam theory                                                                                                                       | 26       |
| 2.4      | The Third Order Refined Beam Theory                                                                                                         | 30       |
| 2.5      | Numerical Results and Discussions                                                                                                           | 32       |
| 2.5.1    | Solid Cylinder                                                                                                                              | 35       |
| 2.5.2    | Hollow Cylinder                                                                                                                             | 41       |
| 2.6      | Formulation of the Problem Related to the Rotationally Symmetric Local Stability Loss                                                       | 45       |
| 2.7      | Method of Solution for the Rotationally Symmetric Problem                                                                                   | 48       |
| 2.8      | Approximate Equations of the Stability Loss of the Cylinder-Shell Obtained from Equations of the TDLTS by the Average-Integrating Procedure | 55       |
| 2.8.1    | Kirchhoff-Love Shell Theory                                                                                                                 | 55       |
| 2.9      | The Third Order Refined Shell Theory                                                                                                        | 59       |
| 2.10     | Numerical Results Related to the Rotationally Symmetric Stability Loss Problem                                                              | 61       |
| 2.11     | Conclusions                                                                                                                                 | 68       |
|          | References                                                                                                                                  | 69       |

|          |                                                                                                                                                                                   |            |
|----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|
| <b>3</b> | <b>Stability Loss Problems for Viscoelastic Plates . . . . .</b>                                                                                                                  | <b>71</b>  |
| 3.1      | Formulation of the Problem and Basic Field Equations . . . . .                                                                                                                    | 71         |
| 3.2      | Approach for the Solution to the Stability Loss Problem<br>for Rectangular Plate . . . . .                                                                                        | 74         |
| 3.3      | Simply Supported Rectangular Plate. . . . .                                                                                                                                       | 80         |
| 3.3.1    | Deriving Approximate Equations of the Stability<br>Loss of the Simply Supported Rectangular<br>Plate from Equations of the TDLTS by the<br>Average-Integrating Procedure. . . . . | 80         |
| 3.3.2    | Solution for the Formulated Mathematical Problems<br>for the Simply Supported Plate . . . . .                                                                                     | 88         |
| 3.3.3    | Numerical Results and Discussions . . . . .                                                                                                                                       | 90         |
| 3.4      | Rectangular Plate Clamped at Two Opposite Ends<br>and Simply Supported at the Two Other Opposite Ends . . . . .                                                                   | 94         |
| 3.4.1    | Solution Procedure of the Equations of the TDLTS.<br>Semi-Analytical FEM Modeling . . . . .                                                                                       | 94         |
| 3.4.2    | Solution Procedure for the Approximate Stability<br>Loss Equations . . . . .                                                                                                      | 98         |
| 3.4.3    | Numerical Results and Discussions . . . . .                                                                                                                                       | 99         |
| 3.5      | Rectangular Plate Clamped at all Ends. . . . .                                                                                                                                    | 102        |
| 3.5.1    | Solution Procedure for the TDLTS Problem.<br>3D FEM Modeling. . . . .                                                                                                             | 102        |
| 3.5.2    | Solution Procedure for the Approximate<br>Stability Loss Problems . . . . .                                                                                                       | 104        |
| 3.5.3    | Numerical Results and Discussions . . . . .                                                                                                                                       | 105        |
| 3.6      | Symmetric Stability Loss of the Circular Plate . . . . .                                                                                                                          | 107        |
| 3.6.1    | Formulation of the Problem and Governing<br>Field Equations . . . . .                                                                                                             | 107        |
| 3.6.2    | Method of Solution . . . . .                                                                                                                                                      | 110        |
| 3.6.3    | Numerical Results and Discussions . . . . .                                                                                                                                       | 117        |
| 3.7      | Stability Loss of the Rotating Circular and Annular Discs . . . .                                                                                                                 | 125        |
| 3.7.1    | Formulation of the Problem. . . . .                                                                                                                                               | 125        |
| 3.7.2    | Method of Solution . . . . .                                                                                                                                                      | 127        |
| 3.7.3    | Numerical Results and Discussions . . . . .                                                                                                                                       | 129        |
|          | References . . . . .                                                                                                                                                              | 132        |
| <b>4</b> | <b>Buckling Delamination of Elastic and Viscoelastic<br/>Composite Plates with Cracks . . . . .</b>                                                                               | <b>135</b> |
| 4.1      | Background of Related Problems. . . . .                                                                                                                                           | 135        |
| 4.2      | Buckling Delamination Problems for Plate-Strips<br>with a Single Crack . . . . .                                                                                                  | 137        |
| 4.2.1    | Formulation of the Problems . . . . .                                                                                                                                             | 137        |
| 4.2.2    | Method of Solution . . . . .                                                                                                                                                      | 140        |
| 4.2.3    | Numerical Results and Discussions . . . . .                                                                                                                                       | 146        |

|        |                                                                                                                                           |     |
|--------|-------------------------------------------------------------------------------------------------------------------------------------------|-----|
| 4.3    | Buckling Delamination of the Plate-Strip<br>with Two Parallel Cracks . . . . .                                                            | 153 |
| 4.3.1  | Mathematical Formulation of the Problem . . . . .                                                                                         | 153 |
| 4.3.2  | Method of Solution: FEM Modeling. . . . .                                                                                                 | 155 |
| 4.3.3  | Numerical Results and Discussions . . . . .                                                                                               | 156 |
| 4.4    | Buckling Delamination of the Plate-Strip<br>with Two Collinear Cracks . . . . .                                                           | 159 |
| 4.4.1  | Formulation of the Problem and Solution Method . . . . .                                                                                  | 159 |
| 4.4.2  | Numerical Results and Discussions . . . . .                                                                                               | 161 |
| 4.5    | Buckling Delamination of the Three-Layered (Sandwich)<br>Plate-Strip with Two Parallel Interface Cracks. . . . .                          | 164 |
| 4.5.1  | Formulation of the Problem. . . . .                                                                                                       | 164 |
| 4.5.2  | Method of Solution . . . . .                                                                                                              | 166 |
| 4.5.3  | Numerical Results and Discussions . . . . .                                                                                               | 172 |
| 4.6    | Buckling Delamination of the Three-Layered (Sandwich)<br>Plate-Strip with Two Collinear Interface Cracks . . . . .                        | 176 |
| 4.6.1  | Formulation of the Problem and Method<br>of Solution . . . . .                                                                            | 176 |
| 4.6.2  | Numerical Results and Discussions . . . . .                                                                                               | 179 |
| 4.7    | Buckling Delamination of the Elastic and Viscoelastic<br>Composite Circular Plate-Disc with a Penny-Shaped Crack. . . . .                 | 181 |
| 4.7.1  | Formulation of the Problem. . . . .                                                                                                       | 181 |
| 4.7.2  | Method of Solution . . . . .                                                                                                              | 183 |
| 4.7.3  | Numerical Results and Discussions . . . . .                                                                                               | 186 |
| 4.8    | Buckling Delamination of the Three-Layered (Sandwich)<br>Circular Plate-Disc with Two Parallel Interface<br>Penny-Shaped Cracks . . . . . | 190 |
| 4.8.1  | Formulation of the Problem. . . . .                                                                                                       | 190 |
| 4.8.2  | Method of Solution . . . . .                                                                                                              | 193 |
| 4.8.3  | Numerical Results and Discussions . . . . .                                                                                               | 200 |
| 4.9    | Remarks on the FEM Modeling of the Crack's Tips . . . . .                                                                                 | 202 |
| 4.10   | Buckling Delamination of a Rectangular Plate<br>Containing a Rectangular Crack. . . . .                                                   | 205 |
| 4.10.1 | Formulation of the Problems . . . . .                                                                                                     | 206 |
| 4.10.2 | Solution Method . . . . .                                                                                                                 | 209 |
| 4.10.3 | FEM Modeling . . . . .                                                                                                                    | 212 |
| 4.10.4 | Numerical Results and Discussions . . . . .                                                                                               | 213 |
| 4.11   | Buckling Delamination of a Sandwich Rectangular<br>Plate with Interface Rectangular Cracks . . . . .                                      | 231 |
| 4.11.1 | Formulation of the Problem. . . . .                                                                                                       | 231 |
| 4.11.2 | Solution Method . . . . .                                                                                                                 | 237 |
| 4.11.3 | FEM Modeling . . . . .                                                                                                                    | 241 |
| 4.11.4 | Numerical Results and Discussions . . . . .                                                                                               | 242 |
|        | References . . . . .                                                                                                                      | 265 |

|          |                                                                                                                                                                                           |            |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|
| <b>5</b> | <b>Surface and Internal Stability Loss in the Structure of Elastic and Viscoelastic Layered Composites. . . . .</b>                                                                       | <b>269</b> |
| 5.1      | Background of Related Problems . . . . .                                                                                                                                                  | 269        |
| 5.2      | Stability Loss in the Structure of Elastic and Viscoelastic Layered Composites with Periodical Initial Imperfections. . . . .                                                             | 271        |
| 5.2.1    | Formulation of the Problem on the Determination of the Stress–Strain State in a Layered Composite with an Arbitrary Number of Layers with Initially Infinitesimal Imperfections . . . . . | 271        |
| 5.2.2    | Method of Solution . . . . .                                                                                                                                                              | 273        |
| 5.2.3    | Numerical Results and Discussions . . . . .                                                                                                                                               | 288        |
| 5.3      | Stability Loss in the Structure of the Elastic and Viscoelastic Layered Composites with Local Initial Imperfections. . . . .                                                              | 294        |
| 5.3.1    | Formulation of the Problem and Method of Solution . . . . .                                                                                                                               | 294        |
| 5.3.2    | Numerical Results and Discussions . . . . .                                                                                                                                               | 296        |
| 5.4      | The Influence of the Inclination of the Local Initial Imperfections of the Reinforcing Layers on the Values of the Critical Parameters . . . . .                                          | 302        |
| 5.4.1    | Formulation of the Problem and Solution Method . . . . .                                                                                                                                  | 302        |
| 5.4.2    | Results and Discussions . . . . .                                                                                                                                                         | 306        |
| 5.4.3    | Conclusions. . . . .                                                                                                                                                                      | 308        |
| 5.5      | Surface Undulation Instability of the Viscoelastic Half-Space Covered with a Stack of Layers in Bi-Axial Compression. . . . .                                                             | 309        |
| 5.5.1    | Formulation of the Problem. . . . .                                                                                                                                                       | 310        |
| 5.5.2    | Method of Solution . . . . .                                                                                                                                                              | 312        |
| 5.5.3    | Numerical Results and Discussions . . . . .                                                                                                                                               | 319        |
|          | References . . . . .                                                                                                                                                                      | 334        |
| <b>6</b> | <b>Stability Loss in the Structure of Unidirected Fibrous Elastic and Viscoelastic Composites . . . . .</b>                                                                               | <b>337</b> |
| 6.1      | Some General Remarks on the Field Equations, Problem Formulations and Method of Solution. . . . .                                                                                         | 337        |
| 6.1.1    | General Remarks on the Field Equations and Problem Formulations . . . . .                                                                                                                 | 337        |
| 6.1.2    | General Remarks on the Method of Solution. . . . .                                                                                                                                        | 341        |
| 6.2      | Micro Buckling of a Single Fiber in the Viscoelastic Matrix. . . . .                                                                                                                      | 348        |
| 6.2.1    | Formulation of the Problem and Method of Solution . . . . .                                                                                                                               | 348        |
| 6.2.2    | Numerical Results and Discussions . . . . .                                                                                                                                               | 350        |
| 6.3      | Internal Stability Loss of Two Neighboring Fibers in a Viscoelastic Matrix . . . . .                                                                                                      | 353        |
| 6.3.1    | Formulation of the Problem and Method of Solution . . . . .                                                                                                                               | 353        |
| 6.3.2    | Numerical Results and Discussion . . . . .                                                                                                                                                | 362        |
| 6.4      | Internal Stability Loss of a Row of Unidirected Periodically                                                                                                                              |            |

|                                                                                                                                                                             |            |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|
| Located Fibers in a Viscoelastic Matrix . . . . .                                                                                                                           | 368        |
| 6.4.1 Formulation of the Problem and Solution Method . . . .                                                                                                                | 368        |
| 6.4.2 Numerical Results and Discussions . . . . .                                                                                                                           | 373        |
| 6.5 Stability Loss of a Micro-Fiber in an Elastic and a Viscoelastic<br>Matrix Near the Free Convex Cylindrical Surface . . . . .                                           | 374        |
| 6.5.1 Formulation of the Problem. . . . .                                                                                                                                   | 374        |
| 6.5.2 Method of Solution . . . . .                                                                                                                                          | 377        |
| 6.5.3 Numerical Results and Discussions . . . . .                                                                                                                           | 391        |
| References . . . . .                                                                                                                                                        | 399        |
| <b>Supplement 1: Applications of the Approach Developed in Chap. 4<br/>on the Problems Related to the Stress Concentration<br/>in Initially Stressed Bodies . . . . .</b>   | <b>401</b> |
| <b>Supplement 2: Self-Balanced Stresses Caused by Periodical Curving<br/>of Two Neighbouring and Periodically Located Row<br/>of Fibers in an Infinite Matrix . . . . .</b> | <b>425</b> |
| <b>Index . . . . .</b>                                                                                                                                                      | <b>441</b> |

Stability Loss and Buckling Delamination  
Three-Dimensional Linearized Approach for Elastic and  
Viscoelastic Composites

Akbarov, S.D.

2013, XVIII, 450 p., Hardcover

ISBN: 978-3-642-30289-3