Chapter 2
Does a Positive Perpetual Growth Rate Exist?

As discussed in Chap. 1, asset future risk and returns determine its current value. The so called risk is the uncertainty of the future returns. Thus the asset return forecast is very important for asset valuation and financial decisions. It is too difficult or inefficient to forecast asset returns from year to year. A more reasonable way is to forecast an average growth rate, so that all future (annual) returns can be derived based on a current (annual) return and the average (annual) growth rate. This is the convention in finance so far.

2.1 Average Annual Growth Rate Revisit

Forecast is a tough task. Asset returns (earnings or cash flows) forecast is not an exception. Although forecast is not the theme or core function of finance, finance is responsible for the forecast feasibility of the variables incorporated in its theories and models. This should be the minimum requirement or standard for financial theories and models, because finance as an independent science is decision or application oriented.

It is not feasible to forecast asset future returns year by year. A wiser choice is to forecast an average annual growth rate extended into the future. Combining this forecasted average annual growth rate and a (normalized) current return, one can derive all future returns of the asset. This has become a convention since the early days of finance as an independent science. The average annual growth rate is often referred to as a constant growth rate.

When we use an average annual growth rate in a financial model, an inevitable question is: how long is the time horizon considered for averaging out the growth rate. This is actually determined by the valuation method chosen. Within the context of DCF method, an asset value is the sum of the present values of its all future cash flows. As lifespan varies across assets, the infallible choice of time horizon is the infinite future. Hence the constant growth rate is the average annual growth

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rate over an infinite time horizon, which is usually called a perpetual growth rate in current financial community.

For example, Gordon (1962) derives a stock valuation model[1] with such a perpetual growth rate as an independent variable. Gordon's model takes the form:

$$\mathrm{p} = \frac{D_0\,(1+g)}{k-g} = \frac{D_1}{k-g} \tag{2.1}$$

Where D_0 is the dividend paid per share in current year, $D_1 = D_0\,(1+g)$ is the estimated dividend per share in next year, k is the market (investors) required rate of return on this stock, and g is the estimated constant perpetual growth rate of dividend.

Equation 2.1 is referred to as Gordon growth model or constant growth model in finance community. This model is widely used for stock valuation as well as for other asset valuation because of its multiple advantages, which include at least:

(1) Simplicity

Simplicity is necessary for a model to be feasible in application. As an application- oriented science, finance should care about the simplicities of its theory and model. The Gordon growth model (referred to as Gordon model hereafter) is an ideal model in such a sense. The (independent) variables taken into account are not much. The form of the model is very simple. In addition, its logic or derivation process is very easy to understand.

Viewing dividends as the cash flows of stock, the dividend in year 1, year 2, ..., year t, ... is $D_1 = D_0\,(1+g)^1$, $D_2 = D_0\,(1+g)^2, \ldots,$ $D_t = D_0\,(1+g)^t, \ldots$ respectively. Value a stock by add the present values of all its future dividends,

$$\mathrm{P} = \frac{D_0(1+g)^1}{(1+k)^1} + \frac{D_0(1+g)^2}{(1+k)^2} + \cdots \frac{D_0(1+g)^t}{(1+k)^t} \cdots \tag{2.2}$$

Thus,

$$\mathrm{p}\frac{(1+g)}{(1+k)} = \frac{D_0(1+g)^2}{(1+k)^2} + \frac{D_0(1+g)^3}{(1+k)^3} + \cdots \frac{D_0(1+g)^t}{(1+k)^t} \cdots \tag{2.3}$$

Equation 2.2 minus Eq. 2.3,

$$\mathrm{P}\left[1 - \frac{(1+g)}{(1+k)}\right] = \frac{D_0(1+g)^1}{(1+k)^1} \tag{2.4}$$

Then,

$$\mathrm{P} = \frac{D_0(1+g)}{k-g} = \frac{D_1}{k-g}$$

[1] We will discuss it further in Chap. 3 [1].

(2) Soundness

Finance as a science or financial theory should be correct exactly in concept. The Gordon model and the variables incorporated are all correct exactly in concept. For instance, g in the model is a perpetual growth rate rather than an average annual growth rate over any finite time horizon, which is in line exactly with the concept of DCF method. In addition, the asset value increases as the future return (determined by the D and g) goes up and decreases as the future risk (incorporated in the k) goes up, which is in line with the axiom of asset value determination. As discussed in Chap. 1, this axiom should be the basic standard to build and to judge a financial or valuation model.

The perpetual growth rate of returns has been a necessary variable in financial analyses and asset valuations since the Gordon model which is widely used to value stock as well as other assets. Understandingly, the principle of value determination is the same for various assets, and the only different is the content of the cash flows or returns. For stock valuation, it is dividend or earnings; for firm valuation, it is operating cash flows or free cash flows.

2.2 Arithmetic or Geometric Average Growth Rate

Again, finance as a science or financial theory should be correct exactly in concept. There are two basic ways to obtain an average growth rate: arithmetic averaging and geometric averaging. We thus should make sure which is correct to choose.

Growth means the value change of a variable across times or periods. Let V_t and V_{t-1} to be the value of a variable in year t and year t-1 respectively, assuming they occur at the end of the year; define V_t/V_{t-1} as the growth factor of the variable in year t. Hence the growth rate in year t is "V_t/V_{t-1}-1". Use AAG and GAG to represent the arithmetic average growth rate and the geometric average growth rate respectively. Then,

$$\mathrm{AAG} = \frac{V_1/V_0 + V_2/V_1 + V_3/V_2 + \cdots + V_n/V_{n-1}}{n} - 1 \tag{2.5}$$

$$\mathrm{GAG} = \left(\frac{V_1}{V_0} \times \frac{V_2}{V_1} \times \frac{V_3}{V_2} \times \cdots \times \frac{V_n}{V_{n-1}}\right)^{\left(\frac{1}{n}\right)} - 1 = \left(\frac{V_n}{V_0}\right)^{\left(\frac{1}{n}\right)} - 1 \tag{2.6}$$

Consider an example. Investor W buys a stock today at a price of 100. The price goes up to 150 at the end of year 1 and goes down to 100 at the end of year 2. Then, what is the average annual growth rate of the stock price? Our intuition tells us that the average annual growth rate is 0 %. However, if we calculate it based on Eqs. 2.5 and 2.6, the average annual growth rate is 8.33 % and 0 % respectively.

This demonstrates that the GAG is correct, whereas the AAG is not. Please note that the annual growth rate under GAG is affected only by the beginning value and final value of the variable, and independent of all the intermediate ups and downs. Obviously, when the growth rate of every year is indeed constant, GAG is equal to AAG; otherwise, GAG is less than AAG. If the standard deviation of the yearly growth rates is SD, the relationship between AAG and GAG is:

$$(1 + \mathrm{AAG})^2 = (1 + \mathrm{GAG})^2 + \mathrm{SD}^2 \tag{2.7}$$

Actually, the arithmetic averaging is more suitable for measuring the absolute growth, i.e., the annual growth in value; whereas the geometric averaging is more suitable for measuring the relative growth, i.e., the annual growth rate. In other words, using AAG' to represent the average annual absolute growth, Eq. 2.5 should be rewritten as:

$$\mathrm{AAG'} = \frac{(V_1 - V_0) + (V_2 - V_1) + (V_3 - V_2) + \cdots + (V_n - V_{n-1})}{n} = \frac{V_n - V_0}{n} \tag{2.8}$$

Based on Eq. 2.8, the average annual absolute growth of the stock price is 0 dollars, which is in line with our intuition. Similarly, the annual absolute growth under Eq. 2.8 (AAG') is also affected only by the beginning value and final value of the variable, and independent of all the intermediate ups and downs. Therefore when we use average growth rate, keep it in mind that it should be a geometric average. This is also in line with the convention of compounding growth and compounding discounting in finance.

2.3 ZZ Growth Paradox

Mathematically, the Gordon model or Eq. 2.1 requires that k > g. This is often mentioned or stressed in finance textbooks. However, a neglected but very important question is: does a positive perpetual growth rate exist?

Surprisingly, the answer is "No". There is actually no positive perpetual growth rate for any firm. If we define it as a constant (average) growth rate extended into the infinite future, what we obtain can only be a negative growth rate.

The reasoning is very simple. No firm can live forever. Expected returns in any form (accounting earnings, operating or free cash flows, and dividends on stock, etc.) will be zero over a long enough time or infinite future, because a firm will surely go bankrupt or disappear given such a long enough time!

Bankruptcy or disappearance is the inevitable destination of every firm in reality. Even being absorbed into another firm via purchase and acquisition, a firm finally cannot escape from disappearance together with the buyers. How strong and brilliant are the corporate behemoths like Barings Bank (1762–1995), Lemon Brothers (1950–2008), Eastman Kodak (1880–2012), etc. used to be? But where are they today?

Most of firms in the world so far are less than 1000 years old. Unfortunately, the average lifespan of leading US companies listed in the S&P 500 index has decreased by more than 50 years in the last century, from 67 years in the 1920 s to just 15 years today, according to Professor Richard Foster from Yale University [2].

Obviously, it needs not an infinite time for a firm to devalue to zero. Based on Eq. 2.6, in a long enough but finite time horizon, i.e., before $n \to \infty$, the value of returns will goes from a positive number (V_0) to a number close to zero ($V_n \to 0$). This implies that $(V_n/V_0) < 1$, hence $(V_n/V_0)^{(1/n)} < 1$. GAG or geometric average growth rate thus can only be negative.

The prevailing convention in finance (practical research and theoretical research), however, is to assume (or forecast or estimate) a positive perpetual growth rate when valuing an asset with the Gordon model. This implies probably that most (if not all) of the applications of Gordon model are just for show. Obviously, when one of the variables input of the model are wrong about positive or negative, it is enough to result to an intolerant mistake. We thus cannot expect a correct result based on such inputs.

An explanation for such a convention is that it is helpful for simplifying the calculation. The returns of a firm are likely to increase for a short or long time and decline thereafter. Within the context of DCF valuation, cash flows in far away are less important because of the discounting. Thus, making a mistake over the declining period (treating a negative growth as a positive growth) will not affect the valuation result too much. This explanation is obviously not convincing. Simplification is not a good excuse for changing the sign of growth rate from negative to positive. Put it another way, we can simplify the calculation while keeping the growth rate negative rather than changing it to a wrongly positive.

There is another convention in applying Gordon model, which is discounting the returns of an asset over years in near future and applying Gordon model for the returns thereafter with a constant positive growth rate. Although most financial researchers are used to such a way, it is real wrong to assume a constant positive growth rate in such circumstances, because the far future (after the near future years) is more likely to grow negatively.

Anyway, it is rather difficult for current finance community to accept the negative long term or perpetual growth rate. In addition, the valuation result with a negative growth rate will be unacceptable lower than that with a positive growth rate.

Consider an acceptable stock with a perpetual growth rate of dividend of 7 % and a discount rate of 10 %. If we suppose the current dividend (year 0) of this stock is 1 dollar, based on the Gordon model, value of a share will be:

$$\mathrm{p} = \frac{D_0(1+g)}{k-g} = \frac{1 \times (1+7\,\%)}{10\,\% - 7\,\%} = 35$$

This is the prevailing valuation.

Now let us take the lifespan of the firm into account. Assume this is a typical firm listed in the S&P 500 index, and its life expectancy is 41 years ($= 67 \times 50\,\% + 15 \times 50\,\%$). Assuming the dividend at the end of year 41 is zero will result in a -100 % annual growth rate of dividend. To make our calculation more meaningful, let us assume a dividend at the end of year 41 is something close to zero, say, 1/1000000 dollars. Thus, the average annual growth rate is:

$$\begin{aligned} \text{GAG} &= \left(\frac{1/1000000}{1}\right)^{\left(\frac{1}{41}\right)} - 1 \\ &= 0.7139 - 1 = -28.6\,\% \end{aligned}$$

Take the −28.6 % (an average annual growth rate over 41 years) approximately as the perpetual growth rate, based on the Gordon model, the share value,

$$\text{p} = \frac{D_0(1+g)}{k-g} = \frac{1 \times (1 - 28.6\,\%)}{10\,\% + 28.6\,\%} = 1.85$$

Obviously, the valuation difference between 35 and 1.85 is too large to be reconciled by prevailing financial wisdom. The most important and most urgent problem, however, is not to explain the difference, but to judge which one is correct, or which one is more correct.

Unfortunately, it is rather difficult to answer such a “simple” question. On one hand, the valuation result of “1.85”, which is 95 % lower than the “normal valuation” result of 35, is too low for most people to accept. On the other hand, the valuation based on a positive perpetual growth rate seems specious, because no firms will grow positively forever; the average growth rate, based on correct concept and logic, can only be negative.

The plausible negative growth rate thus brings us a dilemma: we can hardly accept a negative perpetual growth rate; but the perpetual growth rate can only be negative in logic. As such a dilemma is hard to be explained or solved; we refer to it as the “ZZ growth paradox”. It seems urgent, because the long-term or perpetual growth rate is an inevitable variable in many financial and valuation models. Challenging by the ZZ growth paradox, many prevailing financial models and analyses seem on the verge of collapse.

The challenge from the ZZ growth paradox, however, is not as terrible as it seems to be at the first sight. We will further our discussion to reveal the implications of the ZZ growth paradox to valuation and finance, rather than to persuade readers to accept the negative growth rate. The numerical examples in the following discussions should not be regarded as practical guidance, but just for stimulating fundamental rethinking, and hopefully such rethinking can lead to more insights beyond the existing financial theory.

2.4 Bankruptcy Probability and Firm Life

The ZZ growth paradox reminds us that when valuing an asset or just estimating the growth of the asset returns, asset life should be considered as an important factors. Assets or stocks have finite lifespan, because firms will go bankrupt for sure someday in the future. Let us explore the bankruptcy probability and the firm life determination in this section based on Moody’s data of actual corporate accumulated default rates.

Moody's often publishes actual accumulated default rates over one year to ten year's periods for various rated firms, as shown in Table 2.1.

Please note that Moody's "default" by definition is different from "bankruptcy". In academic concepts and common understanding, bankruptcy means the end of the firm life hence vanishing of the stock value. Moody's default includes the bankruptcy as well as some situations of financial distress. This implies that the actual cumulative bankruptcy probability is lower than the corresponding percentage in Table 2.1.

Let b represent the bankruptcy probability over one year, hence (1-b) is the survival probability over one year. Thus the probability that a firm will survive over

Table 2.1 Moody's historical average cumulative default rates: 1970-2006

years	1	2	3	4	5	6	7	8	9	10
Aaa (%)	0.000	0.000	0.000	0.026	0.099	0.172	0.250	0.334	0.424	0.520
Aa (%)	0.008	0.019	0.042	0.106	0.177	0.260	0.343	0.415	0.463	0.522
A (%)	0.021	0.095	0.220	0.344	0.472	0.614	0.759	0.925	1.105	1.286
Baa (%)	0.181	0.506	0.929	1.433	1.937	2.449	2.956	3.448	4.013	4.633
Ba (%)	1.203	3.222	5.568	7.953	10.207	12.226	13.992	15.690	17.371	19.095
B (%)	5.235	11.298	17.044	22.054	26.791	30.976	34.762	37.972	40.908	43.322
Caa-C (%)	19.466	30.509	39.731	46.935	52.659	56.841	59.965	63.289	66.359	69.251

Source Richard Cantor, David T. Hamilton, Jennifer Tennant [3]

Table 2.2 Expected firm life and cumulative bankruptcy probability

	Firm ratings						
	Aaa	Aa	A	Baa	Ba	B	Caa-C
5-year cumulative default rates (Moody's) (%)	0.099	0.177	0.472	1.937	10.207	26.791	52.659
5-year cumulative bankruptcy probability (%)	0.050	0.089	0.236	0.969	5.104	13.396	26.330
One-year bankruptcy probability (%)	0.010	0.018	0.047	0.194	1.042	2.835	5.928
Constant one-year bankruptcy probability (%)	0.510	0.515	0.534	0.630	1.181	2.347	4.357
Expected firm life[a] (years)	196.00	194.07	187.10	158.69	84.66	42.61	22.95
10-year CBP[b] (%)	4.986	5.035	5.218	6.126	11.203	21.138	35.949
20-year CBP (%)	9.724	9.816	10.164	11.876	21.152	37.808	58.975
50-year CBP (%)	22.567	22.764	23.506	27.100	44.795	69.498	89.220
100-year CBP (%)	40.041	40.346	41.486	46.855	69.524	90.696	98.838
200-year CBP (%)	64.049	64.414	65.761	71.756	90.712	99.134	99.986
500-year CBP (%)	92.250	92.445	93.140	95.761	99.737	99.999	100.00
1000-year CBP (%)	99.399	99.429	99.529	99.820	99.999	100.00	100.00

[a] According to Queuing Theory, The expected firm life is simply the reciprocal of the corresponding constant one-year bankruptcy probability
[b] CBP = cumulative bankruptcy probability

n consecutive years is $(1\text{-}b)^n$. So the cumulative bankruptcy probability B over n consecutive years is:

$$B = 1 - (1 - b)^n \tag{2.9}$$

Considering the difference between the bankruptcy and the default, take half of the default rates in Table 2.1 as the cumulative bankruptcy probabilities. To reduce the uncertainty of the actual bankruptcy probabilities in any one year, we can work out the average one-year bankruptcy probability based on the five-year cumulative bankruptcy probability (column 6 of Table 2.1)[2] by using Eq. 2.9 inversely, as shown in row 1–3 of Table 2.2.

To calculate the cumulative bankruptcy probabilities over various time horizons in the future, we need a constant one-year bankruptcy probability. The average one-year bankruptcy probability in row 3 of Table 2.2 cannot be used for such a purpose, because it is under the assumption that the firm keeps its original rating unchanged in the future. Therefore, this average one-year bankruptcy probability should be adjusted further according to the likelihood of rating shift[3] of the firm in the future, because the firm's rating shift will affect the one-year bankruptcy probability significantly.

To make things simple, assume any year in the future, a firm is 70 % likely to keep its rating in the previous year and 5 % likely to shift to every other rating.[4] Under such an assumption, the constant one-year bankruptcy probabilities of various rated firms can be worked out as shown in row 4 of Table 2.2. Based on such constant one-year bankruptcy probabilities, we further derive the life expectancies of various rated firms and cumulative bankruptcy probabilities over various time horizons, shown in row 5–12 of Table 2.2.

To be simple, refer to firms rated from Aaa to A as A firms, firms rated from Baa to B as B firms and firms rated from Caa to C as C firms. In Table 2.2, expected life of A firms is around 190 years; expected life of B firms is 50–150 years; expected life of C firms is under 30 years. Over 50 % of B and C firms cannot avoid bankruptcy in 100 years. Most (92–100 %) firms cannot avoid bankruptcy in 500 years. In a time horizon of 1000 years, the cumulative bankruptcy probability is over 99 % even for the Aaa firms. This reflects the reality to a large extent, since we rarely see a firm living over 1000 years.[5]

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2 Choice of other column will not change the results too much. Note that most of the standpoints here related to the negative growth rate are logically sound. The calculations are mainly for illustrative purposes, and some impreciseness in assumptions or data will not hurt the conclusions. Therefore, we prefer the simpler assumption or data processing for simplifying the illustrating as long as it is not far from reality.

3 Although we need not to stress on the preciseness, we do consider the significant differences or changes in various concepts and calculations.

4 Note that there are 7 ratings listed here, and $70\ \% \times 1 + 5\% \times 6 = 100\ \%$, and adjusting downward of the "70 %" and consequently adjusting upward of the "5 %" will increase the constant one year bankruptcy probabilities of A firms and decrease the constant one year bankruptcy probabilities of C firms.

5 According to the survey of O'Hara (2002), only 3 companies have been living over 1000 years so far.

2.5 Is Bankruptcy Loss Incorporated into Discount Rate?

Since the negative growth rate is hard to be accepted, we try to seek some reasons for the positive growth rate in this section. First, is it possible that the bankruptcy risk is already incorporated into the discount rate k, so we do not need to take bankruptcy into account when we estimate the growth rate? If it is true, the positive perpetual growth rate is possible.

In finance, risk is uncertainty of future returns. The bankruptcy expectation actually has double effects. One is reducing the returns and the other is increasing the total risk. The former comprises expected losses resulting from suspension of operations due to bankruptcy, as represented by a certain negative growth rate. The later is the uncertainty of bankruptcy and can be represented by the increase of the discount rate k.

As discussed above, firms cannot escape from bankruptcy. When the bankruptcy occurs, a firm normally left nothing to its stock value. Therefore, viewing from the shareholders point at any time before the actual bankruptcy, the expected bankruptcy loss is simply the total value of the equity. Viewing from the whole firm point, the expected bankruptcy loss is even larger as it may also include the loss of the debt-holders. For the purpose of stock valuation, it is correct to define the (expected) bankruptcy loss as the current value of the stock or equity.

Theoretically, the uncertainty of bankruptcy should definitely be factored into the required or expected rate of return, k. In practice, however, the convention is to estimate k using the capital assets pricing model (CAPM) or some of its modifications, such as the Fama/French Three Factor Model[6] In determining the risk premium of a stock, CAPM and its modifications only account for systematic risk, as represented by the betas in the models, and assume non-systematic risks of all the individual stocks will offset each other in a rationally or perfectly diversified portfolio investment.

On the other hand, results of various researches[7] demonstrate that bankruptcy risk is not rewarded by higher returns and there are ongoing debates upon whether or not bankruptcy risk belongs to systematic risk. Therefore, the discount rate k derived from CAPM accounts at most for bankruptcy risk, but definitely does not account for expected bankruptcy loss. Risk is uncertainty. As bankruptcy will occur for sure over a long enough time horizon, the 100 % possible bankruptcy loss is virtually a negative return rather than a risk. Even if the bankruptcy risk is incorporated into the discount rate, the final expected loss still has to be incorporated into the growth rate. Hence a negative growth rate is unavoidable.

Nevertheless, it is interesting to calculate how much the conventional discount rate should be adjusted to account for all risks as well as the expected bankruptcy loss and thereby to justify a conventional positive perpetual growth rate. Let's just do that based on the example assumed in Sect. 2.3, in which the discount rate is 10 %, and current dividend is 1 dollar, with a positive perpetual growth rate of 7 %.

[6] See Fama Eugene F. and French Kenneth R. [4, 5].

[7] See among others, Dichev, Ilia D [6].

Firstly, we assume the bankruptcy loss is incorporated into the negative growth rate and all the risks, including bankruptcy risk, are included in the discount rate k. As shown in the Sect. 2.3, the valuation result is 1.85.

The Gordon model implies,

$$k = [D_0(1 + g)]/p + g \tag{2.10}$$

Now, based on the share value of 1.85 dollars and using the "conventional perpetual growth rate" of 7 % instead of the negative growth rate of −28.6 %, we can find an implied k which accounts for not only the relevant risks but also the bankruptcy loss:

$$k = [D_0(1 + g)]/p + g = 1(1 + 7\,\%)/1.85 + 7\,\% = 64.84\,\%$$

Thus, for a typical firm, if we incorporate the expected bankruptcy loss into the discount rate k, it should be as high as 64.84 %. This "surprising high discount rate" further confirms that conventional discount rates of around 10 % really have not accounted for bankruptcy loss. Hence the positive perpetual growth rate has no chance to be justified.

Viewing from a stand point, a firm has two kinds of risk: one is operating risk, shown as the fluctuations of earnings and value of the firm along with the change of the conditions within and outside the firm; another is bankruptcy risk, shown as the suspension of operation once the firm's value at the debt maturity falls below its debt book value. If a firm has no debt in its capital mix, it has only operating risk and has no bankruptcy risk.

Therefore, it is important to distinguish the bankruptcy cost and bankruptcy loss. The bankruptcy cost is corresponding to debt financing and bankruptcy risk. We will discuss it further in the fifth chapter for solving the problem of optimal capital structure. The bankruptcy loss is corresponding to the firm's total risk, including operating risk and bankruptcy risk. Obviously, for stock valuation, we should consider all risks rather than only bankruptcy risk, i.e. we should consider the bankruptcy loss rather than bankruptcy cost. If we forget the expected bankruptcy loss in valuing a stock, whether bankruptcy risk (bankruptcy cost) is incorporated into the discount rate is actually not so important, because the bankruptcy loss is much bigger in size than the bankruptcy cost.

2.6 The Implied Perpetual Growth Rate

A natural way to estimate a perpetual growth rate is using Gordon growth model inversely based on the price data of the relevant asset or its comparables.

$$g = \frac{Pk - D_0}{P + D_0} \tag{2.11}$$

The growth rate derived based on Eq. 2.11 is referred to as implied perpetual growth rate. For example, analysts often derive the perpetual growth rate of a stock based on the implied growth rates of comparable stocks. This is virtually wrong because, it actually has a strict premise that the prices of the comparable stocks used as the estimation base must be correct, i.e. the comparable stocks must be fair-priced in the market.

Theoretically, we cannot judge a stock or an asset is fairly priced or not before we value it based on convincing method and model as well as reliable inputs. It is easy to work out an implied growth rate based on Eq. 2.11, but it is not easy to determine whether it is right, because it is not easy to make sure whether the comparable assets are fair-valued in the market. Since stocks are mispriced in the market more often than not, it is rather difficult to find a correct or reliable implied growth rate.

There is another common way to estimate the perpetual growth rate, which is based on a stock index. Based on Standard & Poors 500 over 1960–2007, for instance,[8] we may work out an average rate of return (viewed as discount rate) of 10 % or so and an average growth rate (viewed as perpetual growth rate) of 7 % or so. The discount rate of 10 % and growth rate of 7 % are so "typical" that they are often used in financial classes and textbooks to exemplify the DCF valuations and calculations.

This is virtually wrong at least in two aspects.

On one hand, every specific stock relies on the life of the relevant firm. The life of any firm is much different from that of any stock index. A stock index survives when failed firms replaced by new firms, whereas the stock of the failed firm and its related cash flows are worthless. The growth rate derived this way is just a growth rate of a stock index over a finite time horizon. The problem is: the growth rate of a stock index cannot represent that of a stock, because the life of a stock index is much different from that of a stock. Even the life of typical or average stock is much shorter than that of a stock index.

On the other hand, forgetting the difference between the life expectancy of a stock index and a specific stock, i.e., assuming both a stock and the index have infinite life and that the stock is a typical stock in the index, this is still not correct in most circumstances as a way to obtain a growth rate for a stock, because it requires the index data must be correct, which implied that all stocks in the index were always fairly priced in the market, or the stocks being over priced is always cancel out exactly by the stocks being under priced. This is not possible. Thus, the growth rate derived this way is hardly to be right.

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[8] Based on the data of Aswath Damodaran (2008), yearly data of the S&P 500 from 1960 to 2007 (http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/spearn.htm), the author find that the compound annual growth rate (capital gain) of the S&P 500 was 7.12 %, the average annual dividend yield was 3.26 % and the average annual total return was 10.38 %.

2.7 The Stage of Finance as a Science

More and more financial papers and books argue that our financial theory is so advanced that no new discoveries can be found without interdisciplinary studies. An interesting question is: which stage is our financial theory in over its life cycle?

If a subject is characterized as most fundamental problems remain unsolved, it must be in its initial stage or start phase. Obviously, the most urgent task for a subject in initial stage is to solve the fundamental problems within the field rather than to search for outside cooperation or interdisciplinary studies.

We find in last chapter that most of the fundamental problems in finance remain unsolved. Now, we further find that we even cannot make sure whether the perpetual growth rate, a very basic input in finance, is positive or negative!

We may say that Astronomy as a science now is in its advanced stage; but we have to say that Astronomy was in its initial stage 500 years ago, because most fundamental problems remained unsolved at the time. For instance, mankind could not make sure whether earth goes around sun or sun goes around earth. Now, in finance, the perpetual growth rate is not sure to be positive or negative!

Therefore, finance as a science (not as a practice) now is similar to Astronomy 500 years ago. This is not blamable. Just think that 500 years ago, in the time of Nicolaus Copernicus (1473−1543), how many years had been spent on the research of Astronomy. Comparably, how many years have been spent on the research of finance until today!

Thus, finance as a science is just in its initial stage or start phase. Nevertheless, finance is a science and we should try our best to solve the problems (especially the fundamental problems) one by one. The most important thing is that we should put our efforts on the right direction and go along the right way, so that we can really find effective and efficient solutions with correct and simple methods. Whether our efforts are on the right direction and right way is determined by the essential features of finance as a science, i.e. an application—oriented decision science based on valuation, as revealed in last chapter.

After all, we fail to justify a positive perpetual or long term growth rate. Combining with the conclusion of last chapter, we further find that finance as a science is just in its initial stage. Again, we do not necessarily agree with the negative growth rate, so we prefer to call the logic puzzle as “ZZ growth paradox”. This paradox, unfortunately, is unavoidable for thinking and solving the unsolved fundamental financial problems, because the growth rate actually determines the future returns of an asset, which further determine the value of the asset. So, it is time for us to rethink fundamentally about finance.

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