

Preface

The section conjecture, as stated by Grothendieck [Gr83] in a letter to Faltings in 1983, speculates about a representation of rational points in the realm of anabelian geometry. Every k -rational point of a geometrically connected variety X/k gives rise to a conjugacy class of sections

$$s : \pi_1(\mathrm{Spec}(k)) \rightarrow \pi_1(X)$$

of the natural projection $\pi_1(X) \rightarrow \pi_1(\mathrm{Spec}(k))$ of étale fundamental groups. The section conjecture suggests that the converse also holds for smooth, projective curves of genus at least 2 over fields k that are finitely generated over \mathbb{Q} .

If the section conjecture turns out to be true, then it would shed new Galois theoretic light on the old Diophantine problem of describing rational points.

This volume of Lecture Notes consists of the author's Habilitationsschrift and aims to develop the foundations of the anabelian geometry of sections and to present our knowledge about the section conjecture with a natural bias towards the work of the author. In addition, we discuss the section conjecture over number fields from a local to a global point of view and provide detailed discussions of various analogues of the section conjecture, which might serve as supporting evidence in favour of the conjecture.

Acknowledgments

As this present book builds on several years of work, it is quite natural that over this period I have been influenced by many people and institutions and owe to them a certain debt of gratitude that I will now try to balance.

I have worked at several places: the Mathematisches Institut Bonn, the Institute for Advanced Study in Princeton for the academic year 2006/2007, the University of Pennsylvania in Philadelphia in 2007/2008, the Isaac Newton Institute in Cambridge during the summer of 2009, and most recently at the Mathematisches Institut Heidelberg and in particular the MAThematics Center Heidelberg (MATCH).

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There are certainly aspects of the section conjecture beyond what is covered in this book, and also some aspects that were originally intended to be included. But then, among other things, Antonia and Jaden would not sleep in their beloved bunk bed. So, by keeping me busy otherwise, my kids earned the honour of forcing me to focus on finding an end point for this piece of work. I am thankful to Antonia, Jaden, and Lucie for the joy that they are and give every day, and for bringing me coffee and breakfast while these final lines were being written in January 2011.

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