

# Contents

<b>1</b>	<b>Introduction</b>	1
1.1	Background	1
1.1.1	Hilbert Space	1
1.1.2	Notations and Basic Facts	5
1.2	Metric Projection	16
1.2.1	Existence and Uniqueness of the Metric Projection	17
1.2.2	Characterization of the Metric Projection	19
1.2.3	First Applications of the Characterization Theorem	20
1.3	Convex Optimization Problems	23
1.3.1	Convex Minimization Problems	24
1.3.2	Variational Inequality	27
1.3.3	Common Fixed Point Problem	27
1.3.4	Convex Feasibility Problem	27
1.3.5	Linear Feasibility Problem	30
1.3.6	General Convex Feasibility Problem	33
1.3.7	Split Feasibility Problem	34
1.3.8	Linear Split Feasibility Problem	35
1.3.9	Multiple-Sets Split Feasibility Problem	35
1.4	Exercises	36
<b>2</b>	<b>Algorithmic Operators</b>	39
2.1	Basic Definitions and Properties	40
2.1.1	Nonexpansive Operators	41
2.1.2	Quasi-nonexpansive Operators	45
2.1.3	Cutters and Strongly Quasi-nonexpansive Operators	53
2.2	Firmly Nonexpansive Operators	65
2.2.1	Basic Properties of Firmly Nonexpansive Operators	66
2.2.2	Relationships Between Firmly Nonexpansive and Nonexpansive Operators	70
2.2.3	Further Properties of the Metric Projection	76

2.2.4	Metric Projection onto a Closed Subspace.....	80
2.2.5	Metric Projection onto a Closed Affine Subspace.....	82
2.2.6	Properties of Relaxed Firmly Nonexpansive Operators ....	84
2.2.7	Fixed Points of Firmly Nonexpansive Operators.....	90
2.3	Strongly Nonexpansive Operators.....	91
2.4	Generalized Relaxations of Algorithmic Operators.....	96
2.5	Exercises.....	102
<b>3</b>	<b>Convergence of Iterative Methods.....</b>	<b>105</b>
3.1	Iterative Methods.....	105
3.2	Properties of the Weak Convergence.....	106
3.3	Properties of Fejér Monotone Sequences.....	108
3.4	Asymptotically Regular Operators.....	111
3.5	Opial's Theorem and Its Consequences.....	114
3.6	Generalization of Opial's Theorem.....	116
3.7	Opial-Type Theorems for Cutters.....	118
3.8	Strong Convergence of Fejér Monotone Sequences.....	123
3.9	Relationships Among Algorithmic Operators.....	126
3.10	Exercises.....	127
<b>4</b>	<b>Algorithmic Projection Operators.....</b>	<b>129</b>
4.1	Examples of Metric Projections.....	129
4.1.1	Metric Projection onto a Hyperplane.....	129
4.1.2	Metric Projection onto a Finite Dimensional Affine Subspace.....	132
4.1.3	Metric Projection onto a Half-Space.....	133
4.1.4	Metric Projection onto a Band.....	133
4.1.5	Metric Projection onto the Orthant.....	134
4.1.6	Metric Projection onto Box Constraints.....	135
4.1.7	Metric Projection onto a Ball.....	137
4.1.8	Metric Projection onto an Ellipsoid.....	137
4.1.9	Metric Projection onto an Ice Cream Cone.....	140
4.2	Cutters.....	142
4.2.1	Characterization of Cutters.....	142
4.2.2	Cutters with Subsets of Fixed Points Being Affine Subspaces.....	143
4.2.3	Subgradient Projection.....	144
4.3	Alternating Projection.....	147
4.3.1	Basic Properties.....	147
4.3.2	Fixed Points of the Alternating Projection.....	148
4.3.3	Alternating Projection for a Closed Affine Subspace.....	151
4.3.4	Generalized Relaxation of the Alternating Projection.....	152
4.3.5	Averaged Alternating Reflection.....	160
4.4	Simultaneous Projection.....	162
4.4.1	Simultaneous Projection as an Alternating Projection in a Product Space.....	163

4.4.2	Properties of the Simultaneous Projection .....	165
4.4.3	Simultaneous Projection for a System of Linear Equations .....	168
4.4.4	Simultaneous Projection for the Linear Feasibility Problem .....	169
4.5	Cyclic Projection .....	171
4.5.1	Cyclic Relaxed Projection .....	172
4.5.2	Cyclic-Simultaneous Projection .....	173
4.5.3	Projections with Reflection onto an Obtuse Cone .....	174
4.5.4	Cyclic Cutter .....	176
4.6	Landweber Operator .....	176
4.6.1	Main Properties .....	177
4.6.2	Landweber Operator for Linear Systems .....	178
4.6.3	Extrapolated Landweber Operator for a System of Linear Equations .....	181
4.7	Projected Landweber Operator .....	184
4.8	Simultaneous Cutter .....	185
4.9	Extrapolated Simultaneous Cutter .....	187
4.9.1	Properties of the Extrapolated Simultaneous Cutter .....	187
4.9.2	Extrapolated Simultaneous Projection .....	189
4.9.3	Extrapolated Simultaneous Projection for LFP .....	190
4.9.4	Surrogate Projection .....	191
4.9.5	Surrogate Projection with Residual Selection .....	196
4.9.6	Extrapolated Simultaneous Subgradient Projection .....	198
4.10	Extrapolated Cyclic Cutter .....	199
4.10.1	Useful Inequalities .....	200
4.10.2	Properties of the Extrapolated Cyclic Cutter .....	201
4.11	Exercises .....	202
<b>5</b>	<b>Projection Methods .....</b>	<b>203</b>
5.1	Alternating Projection Methods .....	204
5.1.1	General Case .....	204
5.1.2	Alternating Projection Method for Closed Linear Subspaces .....	206
5.2	Extrapolated Alternating Projection Methods .....	208
5.2.1	Acceleration Techniques for Consistent Problems .....	209
5.2.2	Acceleration Techniques for Inconsistent Problems .....	210
5.2.3	Douglas–Rachford Algorithm .....	212
5.3	Projected Gradient Method .....	213
5.4	Simultaneous Projection Method .....	215
5.4.1	Convergence of the SPM .....	215
5.4.2	Projected Simultaneous Projection Methods .....	217

5.5	Cyclic Projection Methods .....	218
5.5.1	Convergence .....	219
5.5.2	Projection-Reflection Method .....	220
5.6	Successive Projection Methods .....	222
5.6.1	Convergence .....	222
5.6.2	Control Sequences .....	223
5.6.3	Examples .....	227
5.7	Landweber Method and Projected Landweber Method .....	228
5.8	Simultaneous Cutter Methods .....	230
5.8.1	Assumptions on Weight Functions .....	231
5.8.2	Convergence Theorem .....	242
5.8.3	Examples .....	245
5.8.4	Block Iterative Projection Methods .....	249
5.9	Sequential Cutter Methods .....	250
5.9.1	Convergence Theorem .....	251
5.9.2	Control Sequences for Sequential Cutter Methods .....	251
5.9.3	Examples .....	252
5.10	Extrapolated Simultaneous Cutter Methods .....	253
5.10.1	Assumptions on Step Sizes .....	254
5.10.2	Convergence Theorem .....	255
5.10.3	Extrapolated Simultaneous Subgradient Projection Method .....	258
5.11	Extrapolated Cyclic Cutter Method .....	259
5.11.1	Convergence .....	260
5.11.2	Accelerated Kaczmarz Method for a System of Linear Equations .....	262
5.12	Surrogate Constraints Methods .....	263
5.12.1	Proper Control .....	264
5.12.2	Convergence Theorem .....	265
5.12.3	Examples of Proper Control .....	266
5.13	SCM with Residual Selection .....	268
5.13.1	General Properties .....	268
5.13.2	Description of the Method .....	271
5.13.3	Obtuse Cone Selection .....	272
5.13.4	Regular Obtuse Cone Selection .....	273
5.14	Exercises .....	274
<b>References</b> .....		275
<b>Index</b> .....		295

Iterative Methods for Fixed Point Problems in Hilbert  
Spaces

Cegielski, A.

2013, XVI, 298 p. 61 illus., 3 illus. in color., Softcover

ISBN: 978-3-642-30900-7