

Chapter 2

Intersecting Attractors

Jose Francisco Morales

2.1 Introduction

The attractor mechanism [1–4], initially discovered in the context of $\mathcal{N} = 2$ black holes, has been recognized as a universal phenomenon governing any extremal flow in supergravity, i.e. a flow with an AdS horizon. It applies to both BPS and non-BPS black hole solutions in Einstein supergravities [5, 6], ungauged [7] and gauged [8] supergravities with higher derivative interactions and general intersections of brane solutions [9].

In these lectures we review a unifying framework [9, 10] for the attractor mechanism underlying any extremal flow in supergravity. More precisely, we consider general black p -brane solutions built out of intersections of branes (and or fluxes) with AdS_{p+2} near horizon geometry. In complete analogy with what happens in the case of extremal black holes $p = 0$, the solutions can be thought as scalar attractor flows from infinity to a horizon where active scalars becomes fixed to particular values determined entirely in terms of the black p -brane charges. Moreover the near horizon geometry encodes the thermodynamical content (entropy or central charges) of the boundary theory describing the microscopic degrees of freedom of the black p -brane. The scalar flow generalizes the more familiar black hole attractor mechanism to the case where a general set of branes charged under forms of various ranks intersect on an extended p -dimensional hyperplane.

We focus on static, asymptotically flat, spherically symmetric and extremal black p -brane solutions in supergravities at the two derivative level. The analysis combines standard attractor techniques based on the extremization of the black hole central charge [1–4] and the so-called “entropy function formalism” introduced in [7] (see [11, 12] for reviews and complete lists of references). Like for black

J.F. Morales (✉)
INFN, Section of Rome Tor Vergata, Rome, Italy
e-mail: morales@roma2.infn.it

holes carrying vector-like charges, we define the entropy function for black p -branes as the Legendre transform with respect to the brane charges of the supergravity action evaluated at the near-horizon geometry. The resulting entropy function can be written as a sum of a gravitational term and an effective potential V_{eff} given as a superposition of the kinetic energies of the forms under which the brane is charged. Extremization of this effective potential gives rise to the attractor equations which determine the values of the scalars at the horizon as functions of the brane charges. In particular, the entropy function itself can be expressed in terms of the U-duality invariants built from these charges and it is proportional to the central charge of the dual CFT $_{p+1}$ living on the AdS_{p+2} boundary. The attractor flow can then be thought of as a c -flow towards the minimum of the *supergravity c -function* [13, 14]. Interestingly, the central charges for extremal black p -branes satisfy an area law formula generalizing the famous Bekenstein-Hawking result for black holes.

We divide the review into two parts:

- In the first part, we illustrate the entropic attractor algorithm in the context of extremal black holes and black strings in $\mathcal{N} = (1, 1)$ supergravity in $D = 6$ dimensions (see [9] for similar results in $D = 7, 8$). We derive the entropy function F and the near-horizon geometry via extremization of F . At the extremum, the entropy function results into a U-duality invariant combination of the brane charges reproducing the black hole entropy and the black string central charge, respectively. Scalars fall into two classes: “fixed scalars” with strictly positive masses and “flat scalars” not fixed by the attractor equations. Flat scalars span the moduli space of the solution. The moduli spaces will be given by symmetric product spaces that can be interpreted as the intersection of the charge orbits of the various branes entering in the solution. In addition one finds extra “geometric moduli” (radii and Wilson lines) that are not fixed by the attractors. Entropy and central charges are entirely determined in terms of the black hole(string) charges and do not depend on the moduli of the solution.
- In the second part we apply the entropic formalism to the study of AdS_4 flux vacua in $\mathcal{N} = 2$ gauged supergravities with an arbitrary number of vector and hyper-multiplets [10]. We find generically a tower of AdS vacua with $\mathcal{N} = 1$ unbroken supersymmetry and two $\mathcal{N} = 0$ towers. We show that both supersymmetric and non-supersymmetric solutions solve a supersymmetry inspired system of linear differential equations. Finally we derive a U-duality invariants formula for the cosmological constant characterizing the vacuum solution.

2.2 Area Law for Central Charges

Before specifying to a particular supergravity theory, here we derive a universal Bekenstein-Hawking like formula underlying any gravity flow (supersymmetric or not) ending on an AdS point. Let $AdS_d \times \Sigma_m$, with Σ_m a product of Einstein spaces, be the near-horizon geometry of an extremal black $(d - 2)$ -brane solution in

$D = d + m$ dimensions. After reduction along Σ_m this solution can be thought as the vacuum of a gauged gravity theory in d dimensions. To keep the discussion, as general as possible, we analyze the solution from its d -dimensional perspective. The only fields that can be turned on consistently with the AdS_d symmetries are constant scalar fields. Therefore we can describe the near-horizon dynamics in terms of a gravity theory coupled to scalars φ^i with a potential V_d . The potential V_d depends on the details of the higher-dimensional theory. The “entropy function” is given by evaluating this action at the AdS_d near horizon geometry (with constant scalars $\varphi^i \approx u^i$)

$$F = -\frac{1}{16\pi G_d} \int d^d x \sqrt{-g} (R - V_d) = \frac{\Omega_{AdS_d} r_{AdS}^d}{16\pi G_d} \left\{ \frac{d(d-1)}{r_{AdS}^2} + V_d \right\}, \quad (2.1)$$

with r_{AdS} the AdS radius and Ω_{AdS_d} the regularized volume of an AdS slice of radius one. Following [15] we take for Ω_{AdS_d} the finite part of the AdS volume integral when the cut off is sent to infinity. More precisely we write the AdS metric

$$ds^2 = r_{AdS}^d (d\rho^2 - \sinh^2 \rho d\tau^2 + \cosh^2 \rho d\Omega_{d-2}^2), \quad (2.2)$$

with $\tau \in [0, 2\pi]$, $0 \leq \rho \leq \cosh^{-1} r_0$ and $d\Omega_{d-2}$ the volume form of a unitary (d-2)-dimensional sphere. The regularized volume Ω_{AdS_d} is then defined as the (absolute value of the) finite part of the volume integral $\int d^d x \sqrt{-g}$ in the limit $r_0 \rightarrow \infty$. This results into

$$\Omega_{AdS_d} = \frac{2\pi}{(d-1)} \Omega_{d-2}. \quad (2.3)$$

A different prescription for the volume regularization leads to a redefinition of the entropy function by a charge independent irrelevant constant. The “entropy” and near-horizon geometry follow from the extremization of the entropy function F with respect to the fixed scalars u^i and the radius r_{AdS}

$$\begin{aligned} \frac{\partial F}{\partial u^i} &\propto \frac{\partial V_d}{\partial u^i} \stackrel{!}{=} 0, \\ \frac{\partial F}{\partial r_{AdS}} &\propto r_{AdS}^2 V_d + (d-1)(d-2) \stackrel{!}{=} 0. \end{aligned} \quad (2.4)$$

The first equation determines the values of the scalars at the horizon. The second equation determines the radius of AdS in terms of the value of the potential at the minimum. Notice that solutions exist only if the potential V_d is negative. Indeed, as we will see in the next section, V_d is always composed from a part proportional to a positive definite effective potential V_{eff} generated by the higher dimensional brane charges and a negative contribution $-R_\Sigma$ related to the constant curvature of the internal space Σ (see Eq. 2.30 below). The “entropy” is given by evaluating F at the extremum and can be written in the suggestive form

$$F = \frac{\Omega_{d-2} r_{\text{AdS}}^{d-2}}{4 G_d} = \Omega_{d-2} r_{\text{AdS}}^{d-2} \frac{A}{4 G_D}, \quad (2.5)$$

where A denotes the area of Σ_m , Ω_{d-2} is the volume of the unit $(d-2)$ -sphere, and $G_D = A G_d$ the D -dimensional Newton constant. For black holes ($d = 2$), this formula is nothing than the well known Bekenstein-Hawking entropy formula $S = \frac{A}{4 G_D}$ and it shows that F can be identified with the black hole entropy. For black strings ($d = 3$), $\frac{3}{\pi} F = \frac{3 r_{\text{AdS}}}{2 G_3}$ reproduces the central charge c of the two-dimensional CFT living on the AdS_3 boundary [16]. In general, the scaling of (2.5) with the AdS radius matches that of the supergravity c -function introduced in [13] and it suggests that F can be interpreted as the critical value of the central charge c reached at the end of the attractor flow.

2.3 The Entropy Function

The bosonic action of supergravity in D -dimensions can be written as

$$S_{\text{SUGRA}} = \int (R * \mathbb{1} - \frac{1}{2} g_{ij}(\phi) d\phi^i \wedge * d\phi^j - \frac{1}{2} N_{\Lambda_n \Sigma_n}(\phi^i) F_n^{\Lambda_n} \wedge * F_n^{\Sigma_n} + \mathcal{L}_{\text{WZ}}), \quad (2.6)$$

with $F_n^{\Lambda_n}$, denoting a set of n -form field strengths, ϕ^i the scalar fields living on a manifold with metric $g_{ij}(\phi)$ and $\mathcal{L}_{\text{WZ}}$ some Wess-Zumino type couplings. The scalar-dependent positive definite matrix $N_{\Lambda_n \Sigma_n}(\phi^i)$ provides the metric for the kinetic term of the n -forms. The sum over n is understood. In the following we will omit the subscript n keeping in mind that both the rank of the forms and the range of the indices Λ depends on n . We will work in units where $16\pi G_D = 1$, and restore at the end the dependence on G_D . For simplicity we will restrict ourselves here to solutions with trivial Wess-Zumino contributions and this term will be discarded in the following.

We look for extremal black p -brane intersections with near-horizon geometry of topology $M_D = AdS_{p+2} \times S^m \times T^q$. Explicitly we look for solutions with near-horizon geometry

$$ds^2 = r_{\text{AdS}}^2 ds_{\text{AdS}_{p+2}}^2 + r_S^2 ds_{S^m}^2 + \sum_{k=1}^q r_k^2 d\theta_k^2, \\ F^\Lambda = p_a^\Lambda \alpha^a + e^{\Lambda r} \beta_r, \quad \phi^i = u^i, \quad (2.7)$$

with $\mathbf{r} = (r_{\text{AdS}}, r_S, r_k)$, describing the AdS and sphere radii, and u^i denoting the fixed values of the scalar fields at the horizon. α^a and β_r denote the volume forms of the compact $\{\Sigma^a\}$ and non-compact $\{\Sigma_r\}$ cycles, respectively, in M_D . The forms are normalized such as

$$\int_{\Sigma^a} \alpha^b = \delta_a^b, \quad \int_{\Sigma_r} \beta_s = \delta_s^r. \quad (2.8)$$

They define the volume dependent functions C^{ab}, C_{rs}

$$\int_{M_D} \alpha^a \wedge * \alpha^b = C^{ab}, \quad \int_{M_D} \beta_r \wedge * \beta_s = C_{rs}, \quad (2.9)$$

describing the cycle intersections. In particular, for the factorized products of AdS space and spheres we consider here, these functions are diagonal matrices with entries

$$C^{ab} = \delta^{ab} \frac{v_D}{\text{vol}(\Sigma^a)^2}, \quad C_{rs} = \delta_{rs} \frac{v_D}{\text{vol}(\Sigma^r)^2}, \quad (2.10)$$

with v_D the volume of M_D . Integrals over AdS spaces are cut off to a finite volume, according to the discussion around (2.3).

The solutions will be labeled by their electric q_{Ir} and magnetic charges p_a^I defined as

$$p_a^A = \int_{\Sigma^a} F^A, \\ q_{Ar} = \int_{*\Sigma^r} N_{A\Sigma} * F^\Sigma = C_{rs} N_{A\Sigma} e^{\Sigma s}, \quad (2.11)$$

where we denote by $*\Sigma^r$ the complementary cycle to Σ^r in M_D .

Let us now consider the “entropy function” associated to a black p -brane solution with near-horizon geometry (2.7). The entropy function F is defined as the Legendre transform in the electric charges q_{Ar} of S_{SUGRA} evaluated at the near-horizon geometry

$$F = e^{Ar} q_{Ar} - S_{\text{SUGRA}} \\ = e^{Ar} q_{Ar} - R v_D + \frac{1}{2} N_{A\Sigma} p_a^A p_b^\Sigma C^{ab} - \frac{1}{2} N_{A\Sigma} e^{Ar} e^{\Sigma s} C_{rs}, \quad (2.12)$$

The fixed values of $\mathbf{r}, u^i, e^{Ir}$ at the horizon can be found via extremization of F with respect to $\mathbf{r}, u^i$, and e^{Ir} :

$$\frac{\partial F}{\partial \mathbf{r}} = \frac{\partial F}{\partial u^i} = \frac{\partial F}{\partial e^{Ar}} = 0. \quad (2.13)$$

From the last equation one finds that

$$q_{Ar} = N_{A\Sigma} e^{\Sigma s} C_{rs}, \quad (2.14)$$

in agreement with the definition of electric charges (2.11). Solving this set of equations for $e^{\Lambda r}$ in favor of $q_{\Lambda r}$ one finds

$$F(Q, \mathbf{r}, u^i) = -R(\mathbf{r}) v_D(\mathbf{r}) + \frac{1}{2} Q^T \cdot M(\mathbf{r}, u^i) \cdot Q, \quad (2.15)$$

with

$$M(\mathbf{r}, u^i) = \begin{pmatrix} N_{\Lambda\Sigma}(u^i) C^{ab}(\mathbf{r}) & 0 \\ 0 & N^{\Lambda\Sigma}(u^i) C^{rs}(\mathbf{r}) \end{pmatrix}, \quad Q \begin{pmatrix} p_a^\Sigma \\ q_{\Sigma r} \end{pmatrix}, \quad (2.16)$$

and $N^{\Lambda\Sigma}$, C^{rs} denoting the inverse of $N_{\Lambda\Sigma}$ and C_{rs} respectively.

It is convenient to introduce the scalar and form intersection “vielbeine” $\mathcal{V}_\Lambda^M$, J^{ab} , J'^{rs} according to

$$N_{\Lambda\Sigma} = \mathcal{V}_\Lambda^M \mathcal{V}_\Sigma^N \delta_{MN}, \quad C^{ab} = J^{ac} J^{bc}, \quad C^{rs} = J'^{rt} J'^{st}. \quad (2.17)$$

From (2.10) one finds for the factorized products of AdS space and spheres

$$J^{ab} = \delta^{ab} \frac{v_D^{1/2}}{\text{vol}(\Sigma^a)}, \quad J'^{rs} = \delta^{rs} \frac{\text{vol}(\Sigma^r)}{v_D^{1/2}}. \quad (2.18)$$

The electric and magnetic central charges can be written in terms of these quantities as

$$Z_{\text{mag}}^{Ma} = \mathcal{V}_\Lambda^M J^{ba} p_b^\Lambda, \quad Z_{\text{el},M}^r = (\mathcal{V}^{-1})_M^\Lambda J'^{sr} q_{\Lambda s}. \quad (2.19)$$

Combining (2.17) and (2.19) one can rewrite the scalar dependent part of the entropy function as the effective potential

$$V_{\text{eff}} = \frac{1}{2} Q^T \cdot M(\mathbf{r}, u^i) \cdot Q = \frac{1}{2} Z_{\text{mag}}^{Ma} Z_{\text{mag}}^{Ma} + \frac{1}{2} Z_{\text{el},M}^r Z_{\text{el},M}^r. \quad (2.20)$$

For the $n = D/2$ -forms in even dimensions the argument is similar, except for the possibility of an additional topological term

$$S_{\text{SUGRA}} = \int \left(R * \mathbb{1} - \frac{1}{2} \mathcal{I}_{\Lambda\Sigma}(\phi^i) F_n^\Lambda \wedge * F_n^\Sigma - \frac{1}{2} \mathcal{R}_{\Lambda\Sigma}(\phi^i) F_n^\Lambda \wedge F_n^\Sigma \right), \quad (2.21)$$

(note that $\mathcal{R}_{\Lambda\Sigma} = \epsilon \mathcal{R}_{\Sigma\Lambda}$, with $\epsilon = (-1)^{[D/2]}$). Following the same steps as before one finds

$$V_{\text{eff}} \frac{1}{2} Q^T \cdot M(\mathbf{r}, u^i) \cdot Q, \quad (2.22)$$

with

$$M(\mathbf{r}, u^i) \equiv C^{ab} \begin{pmatrix} (\mathcal{I} + \epsilon \mathcal{R} \mathcal{I}^{-1} \mathcal{R})_{\Lambda\Sigma} & \epsilon (\mathcal{R} \mathcal{I}^{-1})_{\Lambda}^\Sigma \\ (\mathcal{I}^{-1} \mathcal{R})_{\Sigma}^\Lambda & (\mathcal{I}^{-1})^{\Lambda\Sigma} \end{pmatrix}, \quad Q \begin{pmatrix} p_a^\Lambda \\ q_{\Lambda a} \end{pmatrix}. \quad (2.23)$$

For $\mathcal{R} = 0$ we are back to the diagonal matrix (2.16). In general, thus we obtain for the $D/2$ -forms an effective potential

$$V_{\text{eff}} = \frac{1}{2} \mathcal{Q}^T \cdot M(\mathbf{r}, u^i) \cdot \mathcal{Q} = \frac{1}{2} Z^{Ma} Z^{Ma}, \quad (2.24)$$

with

$$Z^{Ma} = J^{ba} (\mathcal{V}_A^M p_b^A + \mathcal{V}^{\Lambda M} q_{\Lambda b}), \quad (2.25)$$

where $\mathcal{V}_I^M = (\mathcal{V}_A^M, \mathcal{V}^{\Lambda M})$ is the coset representative.

Summarizing, in the case of a general supergravity with bosonic action (2.6) the entropy function is given by

$$F(\mathcal{Q}, \mathbf{r}, u^i) = -R(\mathbf{r}) v_D(\mathbf{r}) + V_{\text{eff}}(u^i, \mathbf{r}), \quad (2.26)$$

with the *intersecting-branes effective potential*

$$\begin{aligned} V_{\text{eff}} &= \frac{1}{2} \sum_n \mathcal{Q}_n^T \cdot M_n(\mathbf{r}, u^i) \cdot \mathcal{Q}_n \\ &= \frac{1}{2} Z^{Ma} Z^{Ma} + \frac{1}{2} \sum_{n \neq D/2} \left(Z_{\text{mag}}^{M_n a_n} Z_{\text{mag}}^{M_n a_n} + Z_{\text{el}, M_n}^{r_n} Z_{\text{el}, M_n}^{r_n} \right), \end{aligned} \quad (2.27)$$

where the first contribution in the second line comes from the $n = D/2$ forms. Notice that there are two types of interference between the potentials coming from forms of different rank: First, they in general depend on a common set of scalar fields and second, they carry a non-trivial dependence on the AdS and the sphere radii. Besides this important difference the critical points of the effective potential can be studied with the standard attractor techniques for vector like charged black holes.

The near-horizon geometry follows from the extremization equations

$$\nabla V_{\text{eff}} \equiv \partial_{u^i} V_{\text{eff}} du^i \frac{1}{2} \sum_n \mathcal{Q}_n^T \cdot \nabla M_n(\mathbf{r}, u^i) \cdot \mathcal{Q}_n \stackrel{!}{=} 0, \quad (2.28)$$

$$\partial_r [-R(\mathbf{r}) v_D(\mathbf{r}) + V_{\text{eff}}(u^i, \mathbf{r})] \stackrel{!}{=} 0. \quad (2.29)$$

We conclude this section by noticing that after reduction to AdS_d , the D -dimensional effective potential V_{eff} combines with the contribution coming from the scalar curvature R_Σ of the internal manifold into the d -dimensional scalar potential

$$V_d = \frac{1}{v_D} V_{\text{eff}} - R_\Sigma \quad (2.30)$$

appearing in (2.1). Notice that the resulting potential is not positive defined and therefore an AdS vacuum is supported.

2.4 $\mathcal{N} = (1, 1)$ in $D = 6$

In this section we illustrate the entropy formalism in the context of $\mathcal{N} = (1, 1)$ in $D = 6$ dimensions. In $D = 6$ dimensions, supergravities contain scalars, vectors and two forms and one finds both black hole and black string extremal solutions.

2.4.1 $\mathcal{N} = (1, 1)$, $D = 6$ Supersymmetry Algebra

The half-maximal $(1, 1)$, $D = 6$ Poincaré supersymmetry algebra has Weyl pseudo-Majorana supercharges and $\mathcal{R}$ -symmetry $SO(4) \sim SU(2)_L \times SU(2)_R$. Its central extension reads as follows (see e.g. [17–19])

$$\{\mathcal{Q}_\gamma^A, \mathcal{Q}_\delta^B\} = \gamma_{\gamma\delta}^\mu Z_\mu^{[AB]} + \gamma_{\gamma\delta}^{\mu\nu\rho} Z_{\mu\nu\rho}^{(AB)}; \quad (2.31)$$

$$\{\mathcal{Q}_{\dot{\gamma}}^{\dot{A}}, \mathcal{Q}_{\dot{\delta}}^{\dot{B}}\} = \gamma_{\dot{\gamma}\dot{\delta}}^\mu Z_\mu^{[\dot{A}\dot{B}]} + \gamma_{\dot{\gamma}\dot{\delta}}^{\mu\nu\rho} Z_{\mu\nu\rho}^{(\dot{A}\dot{B})}; \quad (2.32)$$

$$\{\mathcal{Q}_\gamma^A, \mathcal{Q}_{\dot{\delta}}^{\dot{A}}\} = C_{\gamma\dot{\delta}} Z^{A\dot{A}} + \gamma_{\gamma\dot{\delta}}^{\mu\nu} Z_{\mu\nu}^{A\dot{A}}, \quad (2.33)$$

where $A, \dot{A} = 1, 2$, so that the (L,R) -chiral supercharges are $SU(2)_{(L,R)}$ -doublets.

Notice that $Z_{\mu\nu\rho}^{(AB)} = Z_{\mu\nu\rho}^{(\dot{A}\dot{B})} = 0$, because the presence of the term $Z_{\mu\nu\rho}^{(AB)}$ is inconsistent with the bound $p \leq D - 4$, due to the assumed asymptotical flatness of the (intersecting) black p -brane space-time background.

Strings can be dyonic, and are associated to the central charges $Z_\mu^{[AB]}, Z_\mu^{[\dot{A}\dot{B}]}$ in the $(\mathbf{1}, \mathbf{1})$ of the $\mathcal{R}$ -symmetry group. They are embedded in the $\mathbf{1}_\pm$ (here and below the subscripts denote the weight of $SO(1, 1)$) of the U -duality group $SO(1, 1) \times SO(4, n_V)$. On the other hand, black holes and their magnetic duals (black two-branes) are associated to $Z^{A\dot{A}}, Z_{\mu\nu}^{A\dot{A}}$ in the $(\mathbf{2}, \mathbf{2}')$ of $SO(4)$, and they are embedded in the $(\mathbf{n}_V + \mathbf{4})_{\pm \frac{1}{2}}$ of $SO(1, 1) \times SO(4, n_V)$.

In our analysis, the corresponding central charges are denoted respectively by Z_+ and Z_- for dyonic strings, and by $Z_{\text{el}, A\dot{A}}$ and $Z_{\text{mag}, A\dot{A}}$ for black holes and their magnetic duals.

2.4.2 $\mathcal{N} = (1, 1)$, $D = 6$ Supergravity

The bosonic field content of half-maximal $\mathcal{N} = (1, 1)$ supergravity in $D = 6$ dimensions coupled to n_V matter (*vector*) multiplets consists of a graviton, $(n_V + 4)$ vector fields with field strengths F_2^M , $M = 1, \dots, (n_V + 4)$, a three form field strength H_3 , and $4n_V + 1$ scalar fields parametrizing the scalar manifold

$$\mathcal{M} = SO(1, 1) \times \frac{SO(4, n_V)}{SO(4) \times SO(n_V)}, \quad \dim_{\mathbb{R}} \mathcal{M} = 4n_V + 1, \quad (2.34)$$

with the dilaton ϕ spanning $SO(1, 1)$, and the $4n_V$ real scalars z^i ($i = 1, \dots, 4n_V$) parametrisng the quaternionic manifold $\frac{SO(4, n_V)}{SO(4) \times SO(n_V)}$. The U-duality group is $SO(1, 1) \times SO(4, n_V)$ and the field strengths transform under this group in the representations

$$\begin{aligned} F_2^A &: (\mathbf{n}_V + \mathbf{4})_{+\frac{1}{2}}, \\ H_3 &: \mathbf{1}_{\pm 1}. \end{aligned} \quad (2.35)$$

The coset representative L_A^M , $A, M = 1, \dots, 4 + n_V$, of $\frac{SO(4, n_V)}{SO(4) \times SO(n_V)}$ sits in the $(\mathbf{4}, \mathbf{n}_V)$ representation of the stabilizer $H = SO(4) \times SO(n_V) \sim SU(2)_L \times SU(2)_R \times SO(n_V)$, and satisfies the defining relations

$$L_A^M \eta_{MN} L_{\Sigma}^N = \eta_{A\Sigma}, \quad L_A^M \eta^{A\Sigma} L_{\Sigma}^N = \eta^{MN}, \quad (2.36)$$

with the $SO(4, n_V)$ metric $\eta_{A\Sigma}$. It is related to the vielbein $\mathcal{V}_A^M$ from (2.17) by

$$\mathcal{V}_A^M = e^{-\phi/2} L_A^M, \quad (2.37)$$

and its inverse is defined by $L_M^A L_A^N = \delta_M^N$. The Maurer-Cartan equations take the form

$$P_{MN} = L_M^A d_{\tau} L_{AN} = L_M^A \partial_i L_{AN} dz^i, \quad (2.38)$$

where P_{MN} is a symmetric off-diagonal block matrix with non-vanishing entries only in the $(4 \times n_V)$ -blocks. Here and below we use δ_{MN} to raise and lower the indices M, N .

The solutions will be specified by the electric and magnetic three-form charges q , p , and the two-form charges p^A , q_{Σ} . The quadratic and cubic U-duality invariants that can be built from these charges are

$$\mathcal{I}_2 = pq, \quad \mathcal{I}_3 = \frac{1}{2} \eta_{A\Sigma} p^A p^{\Sigma} p, \quad \mathcal{I}'_3 = \frac{1}{2} \eta^{A\Sigma} q_A q_{\Sigma} q. \quad (2.39)$$

The central charges (2.19) and (2.25) are given by

$$\begin{aligned} Z_{\text{mag}, M} &= e^{-\phi/2} J_2 L_{AM} p^A, & Z_{\text{el}, M} &= e^{\phi/2} J'_2 L_M^A q_A, \\ Z_{\pm} &= \frac{1}{\sqrt{2}} J_3 (e^{\phi} p \pm e^{-\phi} q). \end{aligned} \quad (2.40)$$

Using (2.36), the U-duality invariants (2.39) can be rewritten in terms of the central charges as

$$\begin{aligned}
\frac{1}{2} (Z_+^2 - Z_-^2) &= J_3^2 \mathcal{I}_2, \\
\frac{1}{2\sqrt{2}} \eta^{MN} Z_{\text{mag},M} Z_{\text{mag},N} (Z_+ + Z_-) &= (J_3 J_2^2) \mathcal{I}_3, \\
\frac{1}{2\sqrt{2}} \eta^{MN} Z_{\text{el},M} Z_{\text{el},N} (Z_+ - Z_-) &= (J_3 J_2^2) \mathcal{I}_3'.
\end{aligned} \tag{2.41}$$

The effective potential V_{eff} (2.27) for this theory is given by

$$V_{\text{eff}} = \frac{1}{2} Z_+^2 + \frac{1}{2} Z_-^2 + \frac{1}{2} Z_{\text{el},M}^2 + \frac{1}{2} Z_{\text{mag},M}^2. \tag{2.42}$$

From the Maurer-Cartan equations (2.38) one derives

$$\begin{aligned}
\nabla Z_{\text{mag},M} &= -P_{MN} Z_{\text{mag},N} - \frac{1}{2} P_\phi Z_{\text{mag},M}, \\
\nabla Z_{\text{el},M} &= P_{MN} Z_{\text{el},N} + \frac{1}{2} P_\phi Z_{\text{el},M}, \\
\nabla Z_\pm &= P_\phi Z_\mp.
\end{aligned} \tag{2.43}$$

with $P_\phi = d\phi$. The attractor equations (2.28) thus translate into

$$P_{MN} (Z_{\text{el},M} Z_{\text{el},N} - Z_{\text{mag},M} Z_{\text{mag},N}) + P_\phi (2Z_+ Z_- - \frac{1}{2} Z_{\text{mag},M}^2 + \frac{1}{2} Z_{\text{el},M}^2) \stackrel{!}{=} 0. \tag{2.44}$$

Splitting the index M into $(A\dot{A}) = 1, \dots, 4$, $(A, \dot{A} = 1, 2)$ (*central charges sector*) and $I = 5, \dots, (n_V + 4)$ (*matter charges sector*), and using the fact that only the components $P_{I,A\dot{A}} = P_{A\dot{A},I}$ are non-vanishing, the attractor equations can be written as

$$\begin{aligned}
Z_{\text{el},A\dot{A}} Z_{\text{el},I} - Z_{\text{mag},A\dot{A}} Z_{\text{mag},I} &= 0, \\
4Z_+ Z_- - Z_{\text{mag},A\dot{A}} Z_{\text{mag}}^{A\dot{A}} + Z_{\text{el},A\dot{A}} Z_{\text{el}}^{A\dot{A}} - Z_{\text{mag},I}^2 + Z_{\text{el},I}^2 &= 0.
\end{aligned} \tag{2.45}$$

Indices $A, \dot{A}$ are raised and lowered by $\epsilon_{AB}, \epsilon_{\dot{A}\dot{B}}$. We will study the solutions of these equations, their supersymmetry-preserving features, and the corresponding moduli spaces. BPS solutions correspond to the solutions of (2.45) satisfying

$$Z_{\text{mag},I} = Z_{\text{el},I} = 0, \tag{2.46}$$

as follows from the Killing spinor equation $\delta\lambda_A^I \sim T_{\mu\nu}^I \gamma^{\mu\nu} \epsilon_A = 0$ with $T_{\mu\nu}^I$ the matter central charge densities.

Let us finally consider the *moduli space* of the attractor solutions, i.e. the scalar degrees of freedom which are *not* stabilized by the attractor mechanism at the classical level. For homogeneous scalar manifolds this space is spanned by the vanishing eigenvalues of the Hessian matrix $\nabla\nabla V_{\text{eff}}$ at the critical point. Using the Maurer-Cartan equations (2.43) one can write $\nabla\nabla V_{\text{eff}}$ at the critical point as

$$\begin{aligned}
\nabla\nabla V_{\text{eff}} &= P_{I,AA} P_J^{AA} (2 Z_{\text{el},I} Z_{\text{el},J} + 2 Z_{\text{mag},I} Z_{\text{mag},J}) \\
&\quad + P^{I,AA} P^{J,B\dot{B}} (2 Z_{\text{el},A\dot{A}} Z_{\text{el},B\dot{B}} + 2 Z_{\text{mag},A\dot{A}} Z_{\text{mag},B\dot{B}}) \\
&\quad + P_\phi P_\phi (2 Z_+^2 + 2 Z_-^2 + \frac{1}{2} Z_{\text{mag},M}^2 + \frac{1}{2} Z_{\text{el},M}^2) \\
&\quad + 2 P_\phi P^{I,AA} (Z_{\text{el},I} Z_{\text{el},A\dot{A}} + Z_{\text{mag},I} Z_{\text{mag},A\dot{A}}) \\
&= H_{IA\dot{A},JB\dot{B}} P^{I,AA} P^{J,B\dot{B}} + 2 H_{IA\dot{A},\phi} P^{I,AA} P_\phi + H_{\phi,\phi} P_\phi P_\phi, \quad (2.47)
\end{aligned}$$

which defines the Hessian symmetric matrix $\mathbf{H}$ with components $H_{IA\dot{A},JB\dot{B}}, H_{IA\dot{A},\phi}, H_{\phi,\phi}$. By explicit evaluation of the Hessian matrix for both BPS and non-BPS solutions we will show that eigenvalues are always zero or positive implying the stability (at the classical level) of the solutions under consideration here. We will now specify to the different near-horizon geometries and study the BPS and non-BPS solutions of the attractor equations.

2.4.3 $AdS_3 \times S^3$

Let us start with an $AdS_3 \times S^3$ near-horizon geometry, in which only the three-form charges (magnetic p and electric q) are switched on (*dyonic black string*). There are no closed two-forms supported by this geometry and therefore two-form charges are not allowed. The near-horizon geometry ansatz can then be written as

$$ds^2 = r_{\text{AdS}}^2 ds_{\text{AdS}_3}^2 + r_S^2 ds_{S^3}^2, \quad H_3 = p \alpha_{S^3} + e \beta_{\text{AdS}_3}. \quad (2.48)$$

The attractor equations (2.45) are solved by

$$Z_{\text{mag},M} = Z_{\text{el},M} = Z_- = 0, \quad \text{or equivalently,} \quad (2.49)$$

$$Z_{\text{mag},M} = Z_{\text{el},M} = Z_+ = 0. \quad (2.50)$$

Solution (2.49) has $\mathcal{I}_2 > 0$, whereas solution (2.50) has $\mathcal{I}_2 < 0$; they are both $\frac{1}{4}$ -BPS, and they are equivalent, because the considered theory is non-chiral.

Plugging the solutions (2.49) or (2.50) into (2.42) one can write the effective potential at the horizon in the scalar independent form

$$V_{\text{eff}} = \frac{1}{2} Z_+^2 + \frac{1}{2} Z_-^2 = \frac{1}{2} |Z_+^2 - Z_-^2| J_3^2 |\mathcal{I}_2| = \left(\frac{v_{AdS_3}}{v_{S^3}} \right) |\mathcal{I}_2|, \quad (2.51)$$

Extremizing F in $\mathbf{r}$, one finds the entropy function and near-horizon AdS and sphere radii

$$V_{\text{eff}} = \frac{v_{\text{AdS}_3}}{v_{S^3}} |\mathcal{I}_2|, \quad r_{\text{AdS}} = r_S = \frac{|\mathcal{I}_2|^{1/4}}{2\pi},$$

$$F = v_{\text{AdS}_3 \times S^3} \left(\frac{6}{r_{\text{AdS}}^2} - \frac{6}{r_S^2} \right) + V_{\text{eff}} = |\mathcal{I}_2|. \quad (2.52)$$

Now let us consider the moduli space of the solutions. Plugging (2.49) and (2.50) into (2.47) one finds that the only non-trivial component of the Hessian matrix is

$$H_{\phi\phi} = 2Z_+^2 + 2Z_-^2 = 4V_{\text{eff}} > 0. \quad (2.53)$$

Therefore, the Hessian matrix $\mathbf{H}$ for the $AdS_3 \times S^3$ solution has $4n_V$ vanishing eigenvalues and one strictly positive eigenvalue, corresponding to the dilaton direction. Consequently, the moduli space of non-degenerate attractors with near-horizon geometry $AdS_3 \times S^3$ is the quaternionic symmetric manifold

$$\mathcal{M}_{\text{BPS}} = \frac{SO(4, n_V)}{SO(4) \times SO(n_V)}. \quad (2.54)$$

This result is also evident from the explicit form of the attractor solution $Z_- = 0$: only the dilaton is stabilized, while all other scalars are not fixed since the remaining equations $Z_{\text{el},M} = Z_{\text{mag},M} = 0$ are automatically satisfied for $p^\Lambda = q_\Lambda = 0$.

2.4.4 $AdS_3 \times S^2 \times S^1$

For solutions with near-horizon geometry $AdS_3 \times S^2 \times S^1$, there is no support for electric two-form charges and therefore $e^\Lambda = 0$. We set also the electric three-form charge e to zero otherwise no solutions are found. The near-horizon ansatz becomes

$$ds^2 = r_{\text{AdS}}^2 ds_{\text{AdS}_3}^2 + r_S^2 ds_{S^2}^2 + r_1^2 d\theta^2,$$

$$F_2^\Lambda = p^\Lambda \alpha_{S^2}, \quad H_3 = p \alpha_{S^2 \times S^1}. \quad (2.55)$$

The attractor equations (2.45) admit two types of solutions with non trivial central charges

$$\text{BPS} : Z_+ = Z_-, \quad Z_{\text{mag},A\dot{A}} Z_{\text{mag}}^{A\dot{A}} = 4Z_+^2; \quad (2.56)$$

$$\text{non-BPS} : Z_+ = Z_- \quad Z_{\text{mag},I}^2 = 4Z_+^2. \quad (2.57)$$

Plugging the solution into (2.41) one finds the relation

$$|J_2^2 J_3 \mathcal{I}_3| = 2\sqrt{2} Z_+^3. \quad (2.58)$$

that allows us to write the effective potential (2.42) at the horizon in the scalar independent form

$$V_{\text{eff}} = 3Z_+^2 = \frac{3}{2} |J_2^2 J_3 \mathcal{I}_3|^{\frac{2}{3}}, \quad (2.59)$$

with

$$(J_2^2 J_3)^{\frac{2}{3}} = \frac{v_{\text{AdS}_3} v_{S^1}^{\frac{1}{3}}}{v_{S^2}}, \quad (2.60)$$

The black string central charge and the near-horizon radii follow from r -extremization of the entropy function F and are given by

$$V_{\text{eff}} = \frac{3}{2} \frac{v_{\text{AdS}_3}}{v_{S^2}} v_{S^1}^{1/3} |\mathcal{I}_3|^{\frac{2}{3}}, \quad r_{\text{AdS}} = 2r_S = \frac{|\mathcal{I}_3|^{1/3}}{2\pi v_{S^1}^{1/3}},$$

$$F = v_{\text{AdS}_3 \times S^2 \times S^1} \left(\frac{6}{r_{\text{AdS}_3}^2} - \frac{2}{r_{S^2}^2} \right) + V_{\text{eff}} = |\mathcal{I}_3|. \quad (2.61)$$

Note that the radius of the extra S^1 is not fixed by the extremization equations. Besides this geometric modulus the solutions can be also deformed by turning on Wilson lines for the vector field potentials $A_5^A = c^A$. This is in contrast with the more familiar case of black holes in $D = 4, 5$ where the near-horizon geometry is completely fixed at the end of the attractor flow. As we shall see in the following, this will be always the case for extremal black p -branes. “Geometric moduli” describing the shapes and volumes of the extra circles and constant values of field potentials along these circles remain unfixed at the horizon. It is important to stress, that the extreme value of the entropy function does not depend on the moduli of the solution and is given entirely by a U-duality invariant combination of the black brane charges.

Now, let us consider the moduli spaces of the two solutions. The BPS solution (2.56) has remaining symmetry $SO(3) \times SO(n_V)$, because by using an $SO(4)$ transformation this solution can be recast in the form

$$Z_{\text{mag}, A\dot{A}} = 2z \delta_{A1} \delta_{\dot{A}1}, \quad Z_+ = Z_- = z, \quad Z_{\text{el}, M} = 0. \quad (2.62)$$

Notice that both choices of sign satisfy the Killing spinor relations (2.46) and therefore correspond to supersymmetric solutions. Plugging (2.62) into the Hessian matrix (2.47) one finds

$$\mathbf{H} = z^2 \begin{pmatrix} 8\delta_{IJ}\delta_{A1}\delta_{B1}\delta_{\dot{A}1}\delta_{\dot{B}1} & 0_{4n_V \times 1} \\ 0_{1 \times 4n_V} & 6 \end{pmatrix}. \quad (2.63)$$

This matrix has $3n_V$ vanishing eigenvalues and $n_V + 1$ strictly positive eigenvalues, corresponding to the dilaton direction plus the n_V directions $P_{I,11}$. Consequently, the moduli space of the BPS attractor solution (2.56) with near-horizon geometry $AdS_3 \times S^2 \times S^1$ is the symmetric manifold

$$\mathcal{M}_{\text{BPS}} = \frac{SO(3, n_V)}{SO(3) \times SO(n_V)} . \quad (2.64)$$

More precisely, the scalars along $P_{I,A\dot{A}}$ in the $(\mathbf{4}, \mathbf{n}_V)$ of the group H decompose with respect to the symmetry group $SO(3) \times SO(n_V)$ as:

$$(\mathbf{4}, \mathbf{n}_V) \rightarrow (\mathbf{3}, \mathbf{n}_V) \oplus (\mathbf{1}, \mathbf{n}_V) , \quad (2.65)$$

and only the $(\mathbf{1}, \mathbf{n}_V)$ representation is massive, together with the dilaton. The $(\mathbf{3}, \mathbf{n}_V)$ representation remains massless and contains all the massless Hessian modes of the attractor solutions.

The analysis of the moduli space for the non-BPS solution follows closely that for the BPS one. Now the symmetry is $SO(4) \times SO(n_V - 1)$ and using an $SO(n_V)$ transformation such a solution can be recast as follows:

$$Z_{\text{mag}, I} = 2z\delta_{I1}, \quad Z_+ = Z_- = z, \quad Z_{\text{el}, M} = Z_{\text{mag}, A\dot{A}} = Z_{\text{el}, A\dot{A}} = 0. \quad (2.66)$$

Plugging (2.66) into the Hessian matrix (2.47), now one finds

$$\mathbf{H} = z^2 \begin{pmatrix} 8\delta_{A\dot{A}}\delta_{B\dot{B}}\delta_{J1}\delta_{I1} & 0_{4n_V \times 1} \\ 0_{1 \times 4n_V} & 6 \end{pmatrix}. \quad (2.67)$$

This Hessian matrix has $4(n_V - 1)$ vanishing eigenvalues and $4 + 1$ strictly positive eigenvalues, corresponding to the dilaton direction plus the 4 $P_{1,A\dot{A}}$ directions. Consequently, the *moduli space* of the non-BPS attractor solution with near-horizon geometry $AdS_3 \times S^2 \times S^1$ is the symmetric manifold

$$\mathcal{M}_{\text{nonBPS}} = \frac{SO(4, n_V - 1)}{SO(4) \times SO(n_V - 1)} . \quad (2.68)$$

More precisely, the scalars along $P_{I,A\dot{A}}$ in the $(\mathbf{4}, \mathbf{n}_V)$ of the group H decompose with respect to the symmetry group $SO(4) \times SO(n_V - 1)$ as:

$$(\mathbf{4}, \mathbf{n}_V) \rightarrow (\mathbf{4}, \mathbf{n}_V - 1) \oplus (\mathbf{4}, \mathbf{1}) , \quad (2.69)$$

and only the $(\mathbf{4}, \mathbf{1})$ representation is massive, together with the dilaton. The $(\mathbf{4}, \mathbf{n}_V - 1)$ representation remains massless, and it contains all the massless Hessian modes of the attractor solution.

2.4.5 $AdS_2 \times S^3 \times S^1$

For solutions with $AdS_2 \times S^3 \times S^1$ near-horizon geometry, there is no support for magnetic two-form charges and therefore $Z_{\text{mag},M} = 0$. The near-horizon ansatz becomes

$$\begin{aligned} ds^2 &= r_{\text{AdS}}^2 ds_{\text{AdS}_2}^2 + r_S^2 ds_{S^3}^2 + r_1^2 d\theta^2, \\ F_2^A &= e^A \beta_{\text{AdS}_2}, \quad H_3 = e \beta_{\text{AdS}_2 \times S^1}. \end{aligned} \quad (2.70)$$

The fixed-scalar equations (2.44) admit two type of solutions

$$\begin{aligned} \text{BPS : } Z_{\text{mag},M} &= Z_{\text{el},I} = 0, \quad Z_+ = -Z_-, \quad Z_{\text{el},A\dot{A}} Z_{\text{el}}^{A\dot{A}} = 4Z_+^2, \\ \text{non-BPS : } Z_{\text{mag},M} &= Z_{\text{el},A\dot{A}} = 0, \quad Z_+ = -Z_-, \quad Z_{\text{el},I}^2 = 4Z_+^2. \end{aligned} \quad (2.71)$$

Now one finds

$$J_2'^2 J_3' \mathcal{I}_3' = 2\sqrt{2} Z_+^3, \quad (2.72)$$

and the effective potential (2.42) at the horizon can be written in the scalar independent form

$$V_{\text{eff}} = 3Z_+^2 = \frac{3}{2} |J_2'^2 J_3' \mathcal{I}_3'|^{\frac{2}{3}}, \quad (2.73)$$

with

$$(J_2'^2 J_3')^{\frac{2}{3}} = \frac{v_{\text{AdS}_2}}{v_{S^1}^{1/3} v_{S^3}}, \quad (2.74)$$

Extremizing F in the radii r one finds

$$\begin{aligned} V_{\text{eff}} &= \frac{3}{2} \frac{v_{\text{AdS}_2}}{v_{S^3} v_{S^1}^{1/3}} |\mathcal{I}_3'|^{\frac{2}{3}}, \quad r_{\text{AdS}} = \frac{1}{2} r_S = \frac{|\mathcal{I}_3'|^{1/6}}{2\pi v_{S^1}^{1/3}}, \\ F &= v_{\text{AdS}_2 \times S^3 \times S^1} \left(\frac{2}{r_{\text{AdS}_2}^2} - \frac{6}{r_{S^3}^2} \right) + V_{\text{eff}} = |\mathcal{I}_3'|^{1/2}. \end{aligned} \quad (2.75)$$

for the black hole entropy and AdS and sphere radii. Again, the radius of the extra S^1 is not fixed by the extremization equations. The analysis of the moduli spaces follows *mutatis mutandis* that of the $AdS_3 \times S^2$ attractors (replacing magnetic by electric charges) and the results are again given by the symmetric manifolds (2.64) and (2.68).

Table 2.1 Supersymmetric M-intersections

	0	1	2	3	4	5	6	7	8	9	10	Near-horizon
M2	—	—	•	•	•	•	•	•	•	•	—	$AdS_3 \times S^3 \times T^5$
M5	—	—	•	•	•	•	—	—	—	—	•	
M2	—	•	•	•	•	•	•	•	•	—	—	$AdS_2 \times S^3 \times T^6$
M2	—	•	•	•	•	•	•	—	—	•	•	
M2	—	•	•	•	•	—	—	•	•	•	•	
M5	—	—	•	•	•	•	•	—	—	—	—	$AdS_3 \times S^2 \times T^6$
M5	—	—	•	•	•	—	—	•	•	—	—	
M5	—	—	•	•	•	—	—	—	—	•	•	

2.5 The Lift to Eleven Dimensions

The attractor solutions we have discussed have a simple lift to higher dimensions all the way up to 11-dimensional supergravity. The black string solution with $AdS_3 \times S^3$ near-horizon geometry follow from dimensional reduction of M2M5 branes intersecting on a string. The solution with $AdS_3 \times S^2 \times S^1$ follow from the reduction of a triple M2-intersection on a string. Finally the $AdS_2 \times S^3 \times S^1$ near-horizon geometry corresponds to a triple M5 intersection on a time-like line. The orientations of the M2,M5 branes in the three cases are summarized in Table 2.1.

After dimensional reductions down to $D = 6, 7, 8$ -dimensions the solutions expose a variety of charges with respect to forms of various rank. Indeed, a single brane intersection in $D = 11$ leads to different solutions after reduction to D -dimensions depending on the orientation of the M-branes along the internal space. Different solutions carry charges with respect to a different set of forms in the D -dimensional supergravity. They can be fully characterized by U-duality invariants built out of the brane charges. The list of U-duality invariants leading to extremal black p -brane solutions in $D = 6, 7, 8$ dimensions is displayed in Table 2.2. As expected, there is a one-to-one correspondence between the entries in this table and the solutions that follow from extremization of the entropy function in the corresponding D dimensional supergravity. We list also the U-duality groups, the representation content and the corresponding moduli spaces.

2.6 AdS_4 Flux Vacua

AdS_4 vacua arise in string theory compactifications with fluxes and can be thought as the near horizon geometries of extremal black brane solutions. In fact, the vacuum conditions of flux compactifications display many analogies with the attractor equations defining the near-horizon limit of extremal black holes. This was first shown in [5] in the context of $\mathcal{N} = 1$ orientifolds of type IIB on Calabi-Yau three-folds. In that paper, first it was observed that the $\mathcal{N} = 1$ scalar potential has a

Table 2.2 Electric and magnetic charges for M-brane intersections. $p = 0, 1$ corresponds to intersections on a black hole and a black string respectively. $q_n(p_n)$ denotes the electric(magnetic) charge of the brane solution and n specifies the rank of the form

d	p	U-duality	Charges	Reps	Moduli space
6	1	$SO(5, 5)$	$p_3 q_3$	$\mathbf{10} \times \mathbf{10}$	$\frac{SO(5,4)}{SO(5) \times SO(4)}$
	1		$p_2 p_2 p_3$	$\mathbf{16}_s \times \mathbf{16}_s \times \mathbf{10}$	$\frac{SO(4,3)}{SO(4) \times SO(3)}$
	0		$q_2 q_2 q_3$	$\mathbf{16}_c \times \mathbf{16}_c \times \mathbf{10}$	$\frac{SO(4,3)}{SO(4) \times SO(3)}$
7	1	$SL(5)$	$p_3 q_3$	$\mathbf{5} \times \mathbf{5}'$	$\frac{SL(4)}{SO(4)}$
	1		$p_2 p_2 q_3$	$\mathbf{10}' \times \mathbf{10}' \times \mathbf{5}$	$\frac{SO(3,2)}{SO(3) \times SO(2)}$
	1		$p_3 p_3 p_2$	$\mathbf{5}' \times \mathbf{5}' \times \mathbf{10}'$	$\frac{SL(3)}{SO(3)}$
	0		$q_2 q_2 p_3$	$\mathbf{10} \times \mathbf{10} \times \mathbf{5}'$	$\frac{SO(3,2)}{SO(3) \times SO(2)}$
	0		$q_3 q_3 q_2$	$\mathbf{5} \times \mathbf{5} \times \mathbf{10}$	$\frac{SL(3)}{SO(3)}$
8	1	$SL(3) \times SL(2)$	$p_3 q_3$	$(\mathbf{3}', \mathbf{1}) \times (\mathbf{3}, \mathbf{1})$	$\left(\frac{SL(2)}{SO(2)} \right)^2$
	1		$p_4 q_4$	$(\mathbf{1}, \mathbf{2}) \times (\mathbf{1}, \mathbf{2})$	$\frac{SL(3)}{SO(3)}$
	1		$p_2 p_3 q_4$	$(\mathbf{3}, \mathbf{2}) \times (\mathbf{3}', \mathbf{1}) \times (\mathbf{1}, \mathbf{2})$	$\frac{SL(2)}{SO(2)}$
	1		$p_3 p_3 p_3$	$(\mathbf{3}', \mathbf{1}) \times (\mathbf{3}', \mathbf{1}) \times (\mathbf{3}', \mathbf{1})$	$\frac{SL(2)}{SO(2)}$
	0		$q_2 q_3 p_4$	$(\mathbf{3}', \mathbf{2}) \times (\mathbf{3}, \mathbf{1}) \times (\mathbf{1}, \mathbf{2})$	$\frac{SL(2)}{SO(2)}$
	0		$q_3 q_3 q_3$	$(\mathbf{3}, \mathbf{1}) \times (\mathbf{3}, \mathbf{1}) \times (\mathbf{3}, \mathbf{1})$	$\frac{SL(2)}{SO(2)}$

form close to the $\mathcal{N} = 2$ black hole potential, with the black hole central charge Z replaced by the combination $e^{\frac{K}{2}} W$ involving the $\mathcal{N} = 1$ Kähler potential K and superpotential W , and with the NSNS and RR fluxes playing the role of the black hole electric charges. Then, exploiting the underlying special Kähler geometry of the Calabi-Yau complex structure moduli space, the differential equations both for black hole attractors and for flux vacua were recast in a set of algebraic equations. Explicit attracting Calabi-Yau solutions were derived. A recent review on the subject, including a complete list of references, can be found in [11].

Here we develop the above ideas focussing on a four-dimensional $\mathcal{N} = 2$ setup. We consider general $\mathcal{N} = 2$ gauged supergravities related to type II theories via flux compactification on non-Calabi-Yau manifolds. At the four dimensional level, the family of supergravities we consider can be obtained by deforming the Calabi-Yau effective action. The latter is characterized by the choice of two special Kähler manifolds $\mathcal{M}_1$ and $\mathcal{M}_2$: while $\mathcal{M}_2$ defines the vector multiplet scalar manifold, $\mathcal{M}_1$ determines via c-map the quaternionic manifold $\mathcal{M}_Q$ parameterized by the scalars in the hypermultiplets [20, 21]. The deformation we study is the most general abelian gauging of the Heisenberg algebra of axionic isometries which is always admitted by $\mathcal{M}_Q$ [22–24]. From the point of view of compactifications, this deformation is expected to correspond to a dimensional reduction of type IIA/IIB on $SU(3)$ and $SU(3) \times SU(3)$ structure manifolds (and possibly non-geometric backgrounds) in

the presence of general NSNS and RR fluxes [25–30]. In this perspective, our setup provides a unifying framework for the study of $\mathcal{N} = 2$ flux compactifications of type II theories on generalized geometries.

We start our analysis in Sect. 2.6.1 by reconsidering the expression derived in [24] for the $\mathcal{N} = 2$ scalar potential associated with the gauging described above. This expression is manifestly invariant under the symplectic transformations rotating the flux charges together with the symplectic sections of both the special Kähler geometries on $\mathcal{M}_1$ and $\mathcal{M}_2$. We derive a convenient reformulation of the scalar potential in terms of a triplet of Killing prepotentials $\mathcal{P}_x$, $x = 1, 2, 3$, and their covariant derivatives. The $\mathcal{P}_x$ encode the informations about the gauging, and play a role analogous to the central charge Z of $\mathcal{N} = 2$ black holes, or to the covariantly holomorphic superpotential $e^{\frac{K}{2}}W$ of $\mathcal{N} = 1$ compactifications. Interestingly, in our reformulation the only derivatives appearing are the special Kähler covariant derivatives with respect to the coordinates on $\mathcal{M}_1$ and $\mathcal{M}_2$, with no explicit contributions from the $\mathcal{M}_Q$ coordinates orthogonal to $\mathcal{M}_1$. This rewriting is advantageous since it allows to study the hypersector with the powerful techniques of special Kähler geometry.

In Sect. 2.6.2 we move to the study of the extremization equations for the potential. These are spelled out in Sect. 2.6.2, and resemble the attractor equations of $\mathcal{N} = 2$ black holes, though they are more complicated to solve. In Sect. 2.6.3 we present a supersymmetry inspired set of linear differential equations that imply the vacuum equations and encode all (supersymmetric or not) extremal solutions we found. In Sect. 2.6.4 we find explicit AdS solutions for the wide class of supergravities whose scalar manifolds $\mathcal{M}_1$ and $\mathcal{M}_2$ are symmetric with cubic prepotentials. These solutions generalize those derived in [31] in the context of flux compactifications of massive type IIA on coset spaces with $SU(3)$ structure, allowing for an arbitrary number of vector multiplets (Sect. 2.6.4) and hypermultiplets (Sect. 2.6.4), as well as for a large set of fluxes. In Sect. 2.6.5 we derive a manifestly U-invariant expression for the AdS cosmological constant. Appendix A.1 illustrates how the gaugings we consider can arise from dimensional reduction of type II theories on manifolds with $SU(3)$ structure.

2.6.1 Revisiting the $\mathcal{N} = 2$ Flux Potential

In this section we reconsider the scalar potential V derived in [24] by gauging the $\mathcal{N} = 2$ effective action for type II theories on Calabi-Yau three-folds, and we reformulate it in terms of Killing prepotentials $\mathcal{P}_x$ and their special Kähler covariant derivatives. From a dimensional reduction perspective, V arises in general type II flux compactifications on 6d manifolds with $SU(3)$ and $SU(3) \times SU(3)$ structure (and non-geometric backgrounds) preserving eight supercharges [25–30]. An explicit dictionary between the gauging and the fluxes is presented in Appendix A.1 for type IIA on $SU(3)$ structures, together with a further list of references.

Scalars in $\mathcal{N} = 2$ supergravity organize in vector multiplets and hypermultiplets, respectively parameterizing a special Kähler manifold $\mathcal{M}_2$ and a quaternionic manifold $\mathcal{M}_Q$. In the cases of our interest, $\mathcal{M}_Q$ is derived via c-map from a special Kähler submanifold $\mathcal{M}_1$ [20, 21]. For both type IIA/IIB compactifications, we denote by $h_1 + 1$ the number of hypermultiplets (where the 1 is associated with the universal hypermultiplet), and by $h_2 + 1$ the number of vector fields (including the graviphoton in the gravitational multiplet), so that $h_1 = \dim_{\mathbb{C}} \mathcal{M}_1$ and $h_2 = \dim_{\mathbb{C}} \mathcal{M}_2$. Furthermore, we introduce complex coordinates z^i , $i = 1, \dots, h_1$ on $\mathcal{M}_1$, and x^a , $a = 1, \dots, h_2$ on $\mathcal{M}_2$, and we denote by

$$\begin{aligned} \Pi_1^{\mathbb{I}} &= e^{\frac{\kappa_1}{2}} \begin{pmatrix} Z^I \\ \mathcal{G}_I \end{pmatrix}, & I = (0, i) = 0, 1, \dots, h_1, \\ \Pi_2^{\mathbb{A}} &= e^{\frac{\kappa_2}{2}} \begin{pmatrix} X^A \\ \mathcal{F}_A \end{pmatrix}, & A = (0, a) = 0, 1, \dots, h_2 \end{aligned} \quad (2.76)$$

the covariantly holomorphic symplectic sections of the special Kähler geometry on $\mathcal{M}_1$ and $\mathcal{M}_2$ respectively. All along the paper, indices in a double font like $\mathbb{I}$ and $\mathbb{A}$ correspond to symplectic indices. The respective ranges are $\mathbb{I} = 1, \dots, 2(h_1 + 1)$ and $\mathbb{A} = 1, \dots, 2(h_2 + 1)$.

To complete the geometric data on $\mathcal{M}_1$ and $\mathcal{M}_2$, we introduce the respective Kähler potentials K_1 , K_2 , Kähler metrics $g_{i\bar{j}}$, $g_{a\bar{b}}$, and symplectic invariant metrics $\mathbb{C}_1$, $\mathbb{C}_2$:

$$\begin{aligned} K_1 &= -\log i(\bar{Z}^I \mathcal{G}_I - Z^I \bar{\mathcal{G}}_I), & g_{i\bar{j}} &= \partial_i \partial_{\bar{j}} K_1 \\ K_2 &= -\log i(\bar{X}^A \mathcal{F}_A - X^A \bar{\mathcal{F}}_A), & g_{a\bar{b}} &= \partial_a \partial_{\bar{b}} K_2 \\ \mathbb{C}_{1\mathbb{I}\mathbb{J}} &= \mathbb{C}_1^{\mathbb{I}\mathbb{J}} = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix} = \mathbb{C}_{2\mathbb{A}\mathbb{B}} = \mathbb{C}_2^{\mathbb{A}\mathbb{B}} \Rightarrow \mathbb{C}_{\mathbb{A}\mathbb{B}} \mathbb{C}^{\mathbb{B}\mathbb{C}} = -\delta_{\mathbb{A}}^{\mathbb{C}}, \\ & & \mathbb{C}_{\mathbb{I}\mathbb{J}} \mathbb{C}^{\mathbb{J}\mathbb{K}} &= -\delta_{\mathbb{I}}^{\mathbb{K}}. \end{aligned}$$

Finally, the coordinates $z^i \in \mathcal{M}_2$ are completed to coordinates on $\mathcal{M}_Q$ by the real axions $\xi^{\mathbb{I}} = (\xi^I, \tilde{\xi}_I)^T$ arising from the expansion of the higher dimensional RR potentials, together with the 4d dilaton φ and the axion a dual to the NSNS 2-form along the 4d spacetime.

The quaternionic manifold $\mathcal{M}_Q$ always admits a Heisenberg algebra of axionic isometries [22], which are gauged once fluxes are turned on in the higher-dimensional background [23, 24]. The NSNS fluxes, including geometric (and possibly non-geometric) fluxes, are encoded in a real ‘bisymplectic’ matrix $Q_{\mathbb{A}}^{\mathbb{I}}$, while the RR fluxes are encoded in a real symplectic vector $c^{\mathbb{A}}$. Explicitly,

$$Q_{\mathbb{A}}^{\mathbb{I}} = \begin{pmatrix} e_A^{\mathbb{I}} & e_{AI} \\ m^{AI} & m^A_I \end{pmatrix}, \quad c^{\mathbb{A}} = \begin{pmatrix} p^A \\ q_A \end{pmatrix}. \quad (2.77)$$

Abelianity and consistency of the gauging impose the following constraints on the charges [23, 24]

$$Q^T \mathbb{C}_2 Q = Q \mathbb{C}_1 Q^T = c^T Q = 0. \quad (2.78)$$

In the context of dimensional reductions, these can be traced back to the Bianchi identities for the higher-dimensional field strengths, together with the nilpotency of the exterior derivative on the compact manifold [26, 28] (cf. Appendix A.1).

The scalar potential generated by the gauging can be written as the sum of two contributions: $V = V_{\text{NS}} + V_{\text{R}}$, where V_{NS} can be seen to come from the reduction of the NSNS sector of type II theories, while V_{R} derives from the RR sector. Both these contributions take a symplectically invariant form, and read [24]

$$\begin{aligned} V_{\text{NS}} = & -2e^{2\varphi} \left[\bar{\Pi}_1^T \tilde{Q}^T \mathbb{M}_2 \tilde{Q} \Pi_1 + \bar{\Pi}_2^T Q \mathbb{M}_1 Q^T \Pi_2 \right. \\ & \left. + 4\bar{\Pi}_1^T \mathbb{C}_1^T Q^T (\Pi_2 \bar{\Pi}_2^T + \bar{\Pi}_2 \Pi_2^T) Q \mathbb{C}_1 \Pi_1 \right] \\ V_{\text{R}} = & -\frac{1}{2} e^{4\varphi} (c + \tilde{Q} \xi)^T \mathbb{M}_2 (c + \tilde{Q} \xi), \end{aligned} \quad (2.79)$$

with

$$\tilde{Q}_{\mathbb{I}}^{\mathbb{A}} = (\mathbb{C}_2^T Q \mathbb{C}_1)_{\mathbb{I}}^{\mathbb{A}}, \quad (2.80)$$

while $(\mathbb{M}_1)_{\mathbb{I}\mathbb{J}}$ and $(\mathbb{M}_2)_{\mathbb{A}\mathbb{B}}$ are symmetric, negative-definite matrices built respectively from the period matrices $(\mathcal{N}_1)_{IJ}$ and $(\mathcal{N}_2)_{AB}$ of the special Kähler geometries on $\mathcal{M}_1$ and $\mathcal{M}_2$ via the relation

$$\mathbb{M} = \begin{pmatrix} 1 & -\text{Re}\mathcal{N} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \text{Im}\mathcal{N} & 0 \\ 0 & (\text{Im}\mathcal{N})^{-1} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\text{Re}\mathcal{N} & 1 \end{pmatrix}. \quad (2.81)$$

Nicely, expressions (2.79) for V_{NS} and V_{R} can be recast in a form that reminds that of the $\mathcal{N} = 2$ black hole potential, with the NSNS and RR fluxes $Q_{\mathbb{A}}^{\mathbb{I}}$ and $c^{\mathbb{A}}$ playing a role analogous to the black hole charges. The black hole central charge will here be replaced by the triplet of $\mathcal{N} = 2$ Killing prepotentials $\mathcal{P}_x$, $x = 1, 2, 3$ which describe the gauging under study. These read

$$\begin{aligned} \mathcal{P}_+ & \equiv \mathcal{P}_1 + i\mathcal{P}_2 = 2e^\varphi \Pi_2^T Q \mathbb{C}_1 \Pi_1 \\ \mathcal{P}_- & \equiv \mathcal{P}_1 - i\mathcal{P}_2 = 2e^\varphi \Pi_2^T Q \mathbb{C}_1 \bar{\Pi}_1 \\ \mathcal{P}_3 & = e^{2\varphi} \Pi_2^T \mathbb{C}_2 (c + \tilde{Q} \xi). \end{aligned} \quad (2.82)$$

Here, $\mathcal{P}_\pm$ encode the contribution of the NSNS sector, while $\mathcal{P}_3$ describes the contribution of the RR sector [26, 28].

We find that the NSNS and RR potentials (2.79) can be recast in the suggestive form

$$\begin{aligned} V_{\text{NS}} &= g^{a\bar{b}} D_a \mathcal{P}_+ \bar{D}_{\bar{b}} \bar{\mathcal{P}}_+ + g^{i\bar{j}} D_i \mathcal{P}_+ D_{\bar{j}} \bar{\mathcal{P}}_+ - 2|\mathcal{P}_+|^2 \\ V_{\text{R}} &= g^{a\bar{b}} D_a \mathcal{P}_3 \bar{D}_{\bar{b}} \bar{\mathcal{P}}_3 + |\mathcal{P}_3|^2, \end{aligned} \quad (2.83)$$

whose main benefit is to involve only special Kähler covariant derivatives of the $\mathcal{P}_x$, which are defined as

$$D_i \mathcal{P}_x = (\partial_i + \frac{1}{2} \partial_i K_1) \mathcal{P}_x, \quad D_a \mathcal{P}_x = (\partial_a + \frac{1}{2} \partial_a K_2) \mathcal{P}_x. \quad (2.84)$$

Equation (2.83) are the expressions for V_{NS} and V_{R} we are going to employ in the next sections. An expression closely related to the above rewriting of V_{NS} in terms of $\mathcal{P}_+$ appeared in [24], and our derivation of V_{R} in terms of $\mathcal{P}_3$ follows the same logic. In order to prove the equivalence between (2.79) and (2.83) we employ the following useful identities of special Kähler geometry [32]:

$$g^{i\bar{j}} D_i \Pi_1 D_{\bar{j}} \bar{\Pi}_1^T = -\frac{1}{2} (\mathbb{C}_1^T \mathbb{M}_1 \mathbb{C}_1 + i \mathbb{C}_1) - \bar{\Pi}_1 \Pi_1^T \quad (2.85)$$

$$g^{a\bar{b}} D_a \Pi_2 D_{\bar{b}} \bar{\Pi}_2^T = -\frac{1}{2} (\mathbb{C}_2^T \mathbb{M}_2 \mathbb{C}_2 + i \mathbb{C}_2) - \bar{\Pi}_2 \Pi_2^T. \quad (2.86)$$

These yield

$$\begin{aligned} g^{a\bar{b}} D_a \mathcal{P}_+ \bar{D}_{\bar{b}} \bar{\mathcal{P}}_+ &= -2 e^{2\varphi} (\bar{\Pi}_1^T \tilde{Q}^T \mathbb{M}_2 \tilde{Q} \Pi_1 + 2 \bar{\Pi}_1^T \mathbb{C}_1^T Q^T \Pi_2 \bar{\Pi}_2^T Q \mathbb{C}_1 \Pi_1) \\ g^{i\bar{j}} D_i \mathcal{P}_+ D_{\bar{j}} \bar{\mathcal{P}}_+ &= -2 e^{2\varphi} (\bar{\Pi}_2^T Q \mathbb{M}_1 Q^T \Pi_2 + 2 \bar{\Pi}_1^T \mathbb{C}_1^T Q^T \Pi_2 \bar{\Pi}_2^T Q \mathbb{C}_1 \Pi_1) \\ -2|\mathcal{P}_+|^2 &= -8 e^{2\varphi} \bar{\Pi}_1^T \mathbb{C}_1^T Q^T \bar{\Pi}_2 \Pi_2^T Q \mathbb{C}_1 \Pi_1 \\ g^{a\bar{b}} D_a \mathcal{P}_3 \bar{D}_{\bar{b}} \bar{\mathcal{P}}_3 + |\mathcal{P}_3|^2 &= -\frac{1}{2} e^{4\varphi} (c + \tilde{Q} \xi)^T \mathbb{M}_2 (c + \tilde{Q} \xi), \end{aligned} \quad (2.87)$$

and the equivalence between (2.79) and (2.83) is seen by addition of these four lines.

It is instructive to compare the quantities in (2.82) with the black hole central charge, which reads $Z_{\text{BH}} = \Pi_2^T \mathbb{C}_2 c$, where here $c = (p^A, q_A)$ is to be interpreted as the symplectic vector of electric and magnetic charges of the black hole. In addition to the fact that we are dealing with two quantities ($\mathcal{P}_+$ and $\mathcal{P}_3$) instead of a single one (Z_{BH}), we face here the further complication that these do not depend just on the covariantly holomorphic symplectic section Π_2 of the vector multiplet special Kähler manifold $\mathcal{M}_2$, but also on the scalars in the hypermultiplets. However, the constrained structure of the quaternionic manifold, determined via

c-map from the special Kähler submanifold $\mathcal{M}_1$, comes to the rescue, yielding a relatively simple dependence of $\mathcal{P}_+$ and $\mathcal{P}_3$ on the 4d dilaton e^φ (appearing as a multiplicative factor) and on the axionic variables $\xi^\mathbb{I}$ (appearing in $\mathcal{P}_3$ as a scalar-dependent shift of the charge vector c). Finally, the $\mathcal{M}_1$ coordinates enter in $\mathcal{P}_+$ via the covariantly holomorphic symplectic section Π_1 , making $\mathcal{P}_+$ a covariantly ‘biholomorphic’ object.

2.6.2 The Vacuum Equations

The extremization of the scalar potential (2.83) corresponds to the equations

$$\partial_\varphi V = 0 \Leftrightarrow V_{\text{NS}} + 2V_{\text{R}} = 0 \quad (2.88)$$

$$\partial_{\xi^\mathbb{I}} V = 0 \Leftrightarrow \widetilde{Q}^T \mathbb{M}_2 (c + \widetilde{Q} \xi) = 0 \quad (2.89)$$

$$\partial_i V = 0 \Leftrightarrow i C_{ijk} g^{j\bar{j}} g^{k\bar{k}} \bar{D}_{\bar{j}} \mathcal{P}_- \bar{D}_{\bar{k}} \bar{\mathcal{P}}_+ - D_i \mathcal{P}_+ \bar{\mathcal{P}}_+ + g^{a\bar{b}} D_a D_i \mathcal{P}_+ \bar{D}_{\bar{b}} \bar{\mathcal{P}}_+ = 0 \quad (2.90)$$

$$\begin{aligned} \partial_a V = 0 \Leftrightarrow i C_{abc} g^{b\bar{b}} g^{c\bar{c}} (\bar{D}_{\bar{b}} \bar{\mathcal{P}}_- \bar{D}_{\bar{c}} \bar{\mathcal{P}}_+ + \bar{D}_{\bar{b}} \bar{\mathcal{P}}_3 \bar{D}_{\bar{c}} \bar{\mathcal{P}}_3) - D_a \mathcal{P}_+ \bar{\mathcal{P}}_+ + 2 D_a \mathcal{P}_3 \bar{\mathcal{P}}_3 \\ + g^{i\bar{j}} D_i D_a \mathcal{P}_+ D_{\bar{j}} \bar{\mathcal{P}}_+ = 0. \end{aligned} \quad (2.91)$$

To write (2.90), we used the following characterizing relations of special Kähler geometry [33]

$$D_i D_j \Pi_1 = i C_{ijk} g^{k\bar{k}} \bar{D}_{\bar{k}} \bar{\Pi}_1, \quad D_i \bar{D}_{\bar{j}} \bar{\Pi}_1 = g_{i\bar{j}} \bar{\Pi}_1, \quad (2.92)$$

where C_{ijk} is the completely symmetric, covariantly holomorphic 3-tensor of the special Kähler geometry on $\mathcal{M}_1$. The analogous identities obtained by sending $1 \rightarrow 2$ and $i, j, k \rightarrow a, b, c$ have been used to derive Eq. (2.91). In particular, these relations imply

$$D_i D_j \mathcal{P}_+ = i C_{ijk} g^{k\bar{k}} \bar{D}_{\bar{k}} \bar{\mathcal{P}}_-, \quad (2.93)$$

as well as

$$D_a D_b \mathcal{P}_+ = i C_{abc} g^{c\bar{c}} \bar{D}_{\bar{c}} \bar{\mathcal{P}}_- \quad \text{and} \quad D_a D_b \mathcal{P}_3 = i C_{abc} g^{c\bar{c}} \bar{D}_{\bar{c}} \bar{\mathcal{P}}_3. \quad (2.94)$$

The system of equations above, in particular Eqs. (2.90) and (2.91), take a form which reminds the attractor equations for black holes in $\mathcal{N} = 2$ supergravity. In the remaining of this section we will show how this set of equations is solved by a supersymmetry inspired set of first order conditions accounting for both supersymmetric and non-supersymmetric solutions.

2.6.3 First Order Conditions

In this subsection we propose a first order ansatz which generalizes the supersymmetry conditions and implies the vacuum equations (2.88)–(2.91). We refer the reader to [10] for the proof that vacuum equations are solved by this linear ansatz. We will later display some explicit examples of supersymmetric and non-supersymmetric vacua satisfying this first order ansatz.

We consider the following set of equations, linear in the $\mathcal{P}_x$ (and their derivatives), and hence in the flux charges:

$$\begin{aligned} \pm e^{\pm i\gamma - i\theta} \mathcal{P}_{\pm} &= u \mathcal{P}_3 \\ \pm e^{\pm i\gamma + i\theta} D_a \mathcal{P}_{\pm} &= v D_a \mathcal{P}_3 \\ D_i \mathcal{P}_+ &= 0 = \bar{D}_{\bar{i}} \mathcal{P}_-, \end{aligned} \quad (2.95)$$

where γ, u, v, θ are real positive parameters. While γ will be just a free phase, the other three parameters will need to satisfy certain constraints given below. The ansatz (2.95) generalizes the AdS $\mathcal{N} = 1$ supersymmetry condition, which corresponds to the particular choice.¹

$$\text{susy} \quad \Leftrightarrow \quad u = 2, \quad v = 1, \quad \theta = 0. \quad (2.96)$$

Our aim is to implement the first order ansatz (2.95) to derive non-supersymmetric solutions of the vacuum equations. For simplicity, we will restrict our analysis for the case in which $\mathcal{M}_2$ is a special Kähler manifold with a cubic prepotential. The ansatz (2.95) extremizes the scalar potential if the parameters u, v satisfy

$$\frac{1}{2}uv^2 - u^2v + u + v = 0, \quad (2.97)$$

and—under the assumption that $\mathcal{M}_2$ is cubic—if we further require that

$$D_a \mathcal{P}_3 = \alpha_3 \partial_a K_2 \mathcal{P}_3, \quad (2.98)$$

with α_3 given by

$$e^{3i \text{Arg}(\alpha_3) + 2i \text{Arg}(\mathcal{P}_3)} = -\sqrt{\frac{4u}{3v}} \frac{1 - v^2 e^{2i\theta}}{2 - uv e^{-2i\theta}}, \quad |\alpha_3|^2 = \frac{u}{3v}. \quad (2.99)$$

¹The $\mathcal{N} = 1$ conditions are completed by $\mathcal{P}_3 = -i\hat{\mu}$, where $\hat{\mu} \neq 0$ is the parameter appearing in the Killing spinor equation on AdS, related to the AdS cosmological constant Λ via $\Lambda = -3|\hat{\mu}|^2$.

Notice that by evaluating the modulus square of both its sides, the first of (2.99) yields a constraint involving u, v, θ only:

$$4uv^2 \cos(2\theta) = 3u^2v^3 - 4uv^4 - 4u + 12v. \quad (2.100)$$

We remark that Eq. (2.95) can be rephrased in the symplectically covariant algebraic form

$$0 = Q^T \Pi_2 - iu\mathcal{P}_3 e^{i\theta-\varphi} \text{Re}(e^{i\gamma} \Pi_1), \quad (2.101)$$

$$0 = Q\mathbb{C}_1 \text{Re}(e^{i\gamma} \Pi_1), \quad (2.102)$$

$$\begin{aligned} 0 = & 2Q\mathbb{C}_1 \text{Im}(e^{i\gamma} \Pi_1) + 2e^{-\varphi}(u+v) \text{Re}[e^{-i\theta} \bar{\mathcal{P}}_3 \mathbb{C}_2 \Pi_2] \\ & - e^\varphi (\mathbb{M}_2 v \cos \theta - \mathbb{C}_2 v \sin \theta)(c + \tilde{Q}\xi). \end{aligned} \quad (2.103)$$

Notice that, precisely as in the supersymmetric case, the second equation actually follows from the first one.

2.6.3.1 Explicit Solutions

In the next section we will present explicit vacuum solutions **A**, **B**, **C** (cf. (2.125)–(2.127)) satisfying the linear ansatz, with the parameters u, v, θ given by

$$\mathbf{A} \ (\mathcal{N} = 0) : \quad u = 3v = 2\sqrt{\frac{6}{5}}, \quad e^{i\theta} = \sqrt{\frac{1}{6}}(\sqrt{5} - i)$$

$$\mathbf{B} \ (\mathcal{N} = 0) : \quad u = v = 2, \quad e^{i\theta} = \frac{1}{2}(1 - i\sqrt{3})$$

$$\mathbf{C} \ (\mathcal{N} = 1) : \quad u = 2v = 2, \quad e^{i\theta} = 1.$$

2.6.4 Vacuum Solutions

In the following we present explicit $\mathcal{N} = 0$ and $\mathcal{N} = 1$ AdS₄ vacuum solutions of the $\mathcal{N} = 2$ gauged supergravities under study.

For simplicity we focus on the case where the special Kähler scalar manifolds $\mathcal{M}_1$ and $\mathcal{M}_2$ are symmetric manifolds G/H with cubic prepotentials. The complete list of symmetric special Kähler manifolds with cubic prepotentials is given by the cosets G/H displayed in the first row of the following table (see e.g. [22])

$\frac{G}{H}$	$\frac{SU(1,1)}{U(1)}$	$\frac{SU(1,1)}{U(1)} \times \frac{SO(2,n)}{SO(n) \times U(1)}$	$\frac{Sp(6, \mathbb{R})}{SU(3) \times U(1)}$	$\frac{SU(3,3)}{SU(3) \times SU(3) \times U(1)}$	$\frac{SO^*(12)}{SU(6) \times U(1)}$	$\frac{E_{7(-25)}}{E_6 \times SO(2)}$
$\mathcal{M}_Q$	$\frac{G_{2(2)}}{SO(4)}$	$\frac{SO(4, n+2)}{SO(n+2) \times SO(4)}$	$\frac{F_{4(4)}}{USp(6) \times SU(2)}$	$\frac{E_{6(2)}}{SU(6) \times SU(2)}$	$\frac{E_{7(-5)}}{SO(12) \times SU(2)}$	$\frac{E_{8(-24)}}{E_7 \times SU(2)}$
$\mathbf{R}_G$	2	(2, n + 2)	14	20	32	56
$\mathbf{R}_H$	0	n + 1	6	(3, 3)	15	27

The second row shows the quaternionic manifolds $\mathcal{M}_Q$ related to the special Kähler manifolds in the first row via the c-map [20]. The third row displays the symplectic G -representation under which the sections $\Pi_1^{\mathbb{I}}$ or $\Pi_2^{\mathbb{A}}$ transform. Finally, the last row shows the H -representation under which the scalar coordinates (z^i or x^a) on G/H transform.

We denote the cubic prepotentials on $\mathcal{M}_1$ and $\mathcal{M}_2$ respectively by

$$\mathcal{F} = \frac{1}{6} d_{abc} \frac{X^a X^b X^c}{X^0}, \quad \mathcal{G} = \frac{1}{6} d_{ijk} \frac{Z^i Z^j Z^k}{Z^0}, \quad (2.104)$$

where d_{abc} and d_{ijk} are scalar-independent, totally symmetric real tensors.

Choosing special coordinates, for $\mathcal{M}_2$ we have

$$\begin{aligned} X^A &= (1, x^a), & \mathcal{F}_A &= (-f, f_a), & f &= \frac{1}{6} d_{abc} x^a x^b x^c, \\ & & & & f_a &= \frac{1}{2} d_{abc} x^b x^c \\ \mathcal{V} &= \frac{1}{6} d_{abc} x_2^a x_2^b x_2^c, & \mathcal{V}_a &= \frac{1}{2} d_{abc} x_2^b x_2^c, & C_{abc} &= e^{K_2} d_{abc} \\ K_2 &= -\log(-8\mathcal{V}), & \partial_a K_2 &= \frac{i\mathcal{V}_a}{2\mathcal{V}}, \end{aligned} \quad (2.105)$$

where the complex coordinates x^a are split into real and imaginary parts as $x^a = x_1^a + i x_2^a$, with $x_2^a < 0$. Analogously, for $\mathcal{M}_1$ we have

$$\begin{aligned} Z^I &= (1, z^i), & \mathcal{G}_I &= (-g, g_i), & g &= \frac{1}{6} d_{ijk} z^i z^j z^k, \\ & & & & g_i &= \frac{1}{2} d_{ijk} z^j z^k \\ \tilde{\mathcal{V}} &= \frac{1}{6} d_{ijk} z_2^i z_2^j z_2^k, & \tilde{\mathcal{V}}_i &= \frac{1}{2} d_{ijk} z_2^j z_2^k, & C_{ijk} &= e^{K_1} d_{ijk} \\ K_1 &= -\log(-8\tilde{\mathcal{V}}), & \partial_i K_1 &= \frac{i\tilde{\mathcal{V}}_i}{2\tilde{\mathcal{V}}}. \end{aligned} \quad (2.106)$$

with $z_2^i < 0$. For the case of supergravity with no hypermultiplets other than the universal one, expressions (2.106) are replaced simply by $Z^0 = 1$, $\mathcal{G}_0 = -i$ and $e^{K_1} = \frac{1}{2}$.

Moreover, the following relations involving the metric $g_{a\bar{b}} = \partial_a \partial_{\bar{b}} K_2$ on $\mathcal{M}_2$ are valid:

$$\begin{aligned} g_{a\bar{b}} &= \frac{1}{4\mathcal{V}^2} (\mathcal{V}_a \mathcal{V}_{\bar{b}} - \mathcal{V} \mathcal{V}_{ab}), & g^{a\bar{b}} &= 2(x_2^a x_2^{\bar{b}} - 2\mathcal{V} \mathcal{V}^{ab}) \\ g^{ab} \mathcal{V}_b &= 4\mathcal{V} x_2^a, & g^{ab} \mathcal{V}_a \mathcal{V}_b &= 12\mathcal{V}^2, \end{aligned} \quad (2.107)$$

where $\mathcal{V}_{ab} = d_{abc} x_2^c$, and $\mathcal{V}^{ab}$ is its inverse. Similar relations hold for the corresponding quantities on $\mathcal{M}_1$ with $a, b \rightarrow i, j$ and $\mathcal{V} \rightarrow \tilde{\mathcal{V}}$.

To perform the forthcoming computations, it is convenient to introduce the following holomorphic prepotentials $W_{\pm}$, W_3 :

$$\mathcal{P}_{\pm} = e^{\frac{K_1 + K_2}{2} + \varphi} W_{\pm}, \quad \mathcal{P}_3 = e^{\frac{K_2}{2} + 2\varphi} W_3, \quad (2.108)$$

whose Kähler covariant derivatives are defined via

$$\begin{aligned} D_a W_x &= (\partial_a + \partial_a K_2) W_x, \\ D_i W_+ &= (\partial_i + \partial_i K_1) W_+, & D_i W_- &\equiv \partial_i W_- = 0, \\ D_{\bar{i}} W_- &= (\partial_{\bar{i}} + \partial_{\bar{i}} K_1) W_-, & D_{\bar{i}} W_+ &\equiv \partial_{\bar{i}} W_+ = 0, \end{aligned} \quad (2.109)$$

with $\partial_a K_2$ and $\partial_i K_1$ given in (2.105) and (2.106) respectively. Explicitly, recalling (2.108) and (2.82), and taking $m^{AI} = m^A{}_I = 0$ for simplicity, one finds

$$\begin{aligned} W_+ &= 2X^A (e_A{}^I \mathcal{G}_I - e_{AI} Z^I), & W_- &= 2X^A (e_A{}^I \tilde{\mathcal{G}}_I - e_{AI} \bar{Z}^I), \\ W_3 &= X^A (q_A + e_A{}^I \tilde{\xi}_I - e_{AI} \xi^I) - \mathcal{F}_A p^A, \end{aligned} \quad (2.110)$$

Recalling (2.105) and defining $f_{ab} = d_{abc} x^c$, we preliminarily compute:

$$\begin{aligned} \partial_a W_+ &= 2(e_a{}^I \mathcal{G}_I - e_{aI} Z^I), & \partial_a W_- &= 2(e_a{}^I \tilde{\mathcal{G}}_I - e_{aI} \bar{Z}^I), \\ \partial_a W_3 &= q_a + e_a{}^I \tilde{\xi}_I - e_{aI} \xi^I + f_a p^0 - f_{ab} p^b. \end{aligned} \quad (2.111)$$

To solve the vacuum equations (2.88)–(2.91) in full generality is a challenging problem that goes beyond the scope of this paper. In the following we present some simple solutions as prototypes of the general case.

2.6.4.1 Only Universal Hypermultiplet

We start by considering the case of a gauged supergravity with a single hypermultiplet, which we identify with the universal hypermultiplet of string

compactifications. Concerning the vector multiplets, we allow for an arbitrary number of them, and we just require that the associated special Kähler scalar manifold is symmetric and has a cubic prepotential,² specified by the 3-tensor d_{abc} . As it will be clear in the following, the latter assumption will allow us to perform computations in a more explicit fashion. In addition we assume that the only non-vanishing entries of the charge matrix Q be $e_{a0} \equiv e_a$. We remark that this choice might be generalized by using U-duality rotations. The constraints (2.78) require that $e_a p^a = 0$. For this choice of charges the prepotentials become

$$\begin{aligned} W_+ &= W_- = -2e_a x^a, \\ W_3 &= q_0 + (q_a - e_a \xi) x^a + p^0 f - p^a f_a, \end{aligned} \quad (2.112)$$

where all along this subsection we denote $\xi \equiv \xi^0$.

It is convenient to introduce the following shifted variables (assuming $p^0 \neq 0$):

$$\mathbf{x}^a = x^a - \frac{p^a}{p^0}, \quad \mathbf{q}_0 = q_0 + \frac{q_a p^a}{p^0} - \frac{2P}{(p^0)^2}, \quad \mathbf{q}_a = q_a - \frac{P_a}{p^0}, \quad (2.113)$$

with

$$P = \frac{1}{6} d_{abc} p^a p^b p^c, \quad P_a = \frac{1}{2} d_{abc} p^b p^c. \quad (2.114)$$

In terms of these variables one finds

$$\begin{aligned} W_+ &= W_- = -2e_a \mathbf{x}^a, & \partial_a W_+ &= \partial_a W_- = -2e_a, \\ W_3 &= \mathbf{q}_0 + (\mathbf{q}_a - \xi e_a) \mathbf{x}^a + p^0 \mathbf{f}, & \partial_a W_3 &= \mathbf{q}_a - \xi e_a + p^0 \mathbf{f}_a, \end{aligned} \quad (2.115)$$

with

$$\mathbf{f} = \frac{1}{6} d_{abc} \mathbf{x}^a \mathbf{x}^b \mathbf{x}^c, \quad \mathbf{f}_a = \frac{1}{2} d_{abc} \mathbf{x}^b \mathbf{x}^c. \quad (2.116)$$

In writing (2.115) we have used that $e_a p^a = 0$. Notice that since $\mathbf{x}_2^a \equiv x_2^a$, the expressions in (2.107) can be equivalently written with the bold variable. The advantage of using the bold variables introduced above is that the explicit dependence on p^a is entirely removed. Finally we introduce the following quantities built from the geometric fluxes e_a :

$$R = \frac{1}{6} d^{abc} e_a e_b e_c, \quad R^a = \frac{1}{2} d^{abc} e_b e_c, \quad (2.117)$$

²In particular, the second property is relevant for type IIA compactifications on 6d manifolds M_6 with $SU(3)$ structure, where the Kähler potential K_2 is expected to take the cubic form $e^{-K_2} = \frac{4}{3} \int_{M_6} J \wedge J \wedge J$, where J is the almost symplectic 2-form on M_6 . See Appendix A.1 for more details.

where d^{abc} is the contravariant tensor of the symmetric special Kähler geometry satisfying

$$d_{abc} d^{b(d_1 d_2} d^{d_3 d_4)c} = \frac{4}{3} \delta_a^{(d_1} d^{d_2 d_3 d_4)}. \quad (2.118)$$

Solutions of the vacuum equations can be found starting from the simple ansatz

$$\mathbf{x}^a = x R^a, \quad \mathbf{q}_a = 0, \quad (2.119)$$

with $x = x_1 + i x_2$ a complex function of the charges to be determined. This ansatz can be motivated by noticing that once $\mathbf{q}_a$ is taken to zero, the only contravariant vector one can build with e_a 's variables is R^a . Using (2.118) one finds the following relations:

$$\begin{aligned} d_{abc} R^b R^c &= 2 R e_a, & R^a e_a &= 3 R, \\ \mathcal{V} &= (x_2)^3 R^2, & \mathcal{V}_a &= (x_2)^2 R e_a, & \mathbf{f} &= x^3 R^2, & \mathbf{f}_a &= x^2 R e_a, \\ W_{\pm} &= -6 R x, & W_3 &= (\mathbf{q}_0 - 3 R \xi x + p^0 R^2 x^3), \\ \partial_a W_+ &= \partial_a W_- = -2 e_a, & \partial_a W_3 &= (p^0 R x^2 - \xi) e_a, \\ e^{-K_1} &= 2, & e^{-K_2} &= -8 \mathcal{V}. \end{aligned} \quad (2.120)$$

Moreover, the covariant derivatives of the prepotentials take the form

$$D_a W_x = \frac{i \mathcal{V}_a}{2 \mathcal{V}} \alpha_x W_x, \quad (2.121)$$

with

$$\alpha_{\pm} = 1 - \frac{2 i x_2}{3 x}, \quad \alpha_3 = 1 - \frac{2 i x_2 R (p^0 R x^2 - \xi)}{\mathbf{q}_0 - 3 R \xi x + p^0 R^2 x^3}. \quad (2.122)$$

Using the relations (here there is no sum over x):

$$\begin{aligned} g^{a\bar{b}} D_a \mathcal{P}_x \bar{D}_{\bar{b}} \overline{\mathcal{P}}_x &= 3 |\alpha_x \mathcal{P}_x|^2, \\ i C_{abc} g^{b\bar{b}} g^{c\bar{c}} \bar{D}_{\bar{b}} \overline{\mathcal{P}}_x \bar{D}_{\bar{c}} \overline{\mathcal{P}}_x &= 2 \bar{\alpha}_x^2 \overline{\mathcal{P}}_x^2 \frac{i \mathcal{V}_a}{2 \mathcal{V}}, \end{aligned} \quad (2.123)$$

the scalar potential reads

$$V = e^{K_1 + K_2 + 2\varphi} (3 |\alpha_+|^2 - 2) |W_+|^2 + e^{K_2 + 4\varphi} (3 |\alpha_3|^2 + 1) |W_3|^2. \quad (2.124)$$

Extremizing with respect to $x_{1,2}, \xi, \varphi$ one finds the solutions

- **Solution A** ($\mathcal{N} = 0$) :

$$x_1 = \xi = 0, \quad x_2 = -5^{-1/6} \left(\frac{\mathbf{q}_0}{p^0 R^2} \right)^{1/3}, \quad e^\varphi = \frac{1}{2\sqrt{2}} 5^{\frac{5}{6}} \left(\frac{R}{p^0 \mathbf{q}_0^2} \right)^{1/3},$$

$$V = -\frac{75}{64} 5^{\frac{5}{6}} \left(\frac{R^4}{p^0 \mathbf{q}_0^5} \right)^{1/3}. \quad (2.125)$$

- **Solution B** ($\mathcal{N} = 0$) :

$$x_1 = -\frac{1}{\sqrt{3}} x_2 = \left(\frac{\mathbf{q}_0}{20 p^0 R^2} \right)^{1/3}, \quad \xi = \left(\frac{4 p^0 \mathbf{q}_0^2}{25 R} \right)^{1/3}, \quad e^\varphi = \frac{\sqrt{2}}{\sqrt{3}} \left(\frac{25 R}{4 p^0 \mathbf{q}_0^2} \right)^{1/3},$$

$$V = -\frac{5}{\sqrt{3}} \left(\frac{25 R^4}{4 p^0 \mathbf{q}_0^5} \right)^{1/3}. \quad (2.126)$$

- **Solution C** ($\mathcal{N} = 1$) :

$$x_1 = \frac{1}{\sqrt{15}} x_2 = -\frac{1}{2} \left(\frac{\mathbf{q}_0}{20 p^0 R^2} \right)^{1/3}, \quad \xi = -\left(\frac{p^0 \mathbf{q}_0^2}{50 R} \right)^{1/3}, \quad e^\varphi = \frac{\sqrt{2}}{\sqrt{3}} 5^{\frac{1}{6}} \left(\frac{2 R}{p^0 \mathbf{q}_0^2} \right)^{1/3},$$

$$V = -8\sqrt{3} 5^{-\frac{5}{6}} \left(\frac{2 R^4}{p^0 \mathbf{q}_0^5} \right)^{1/3}. \quad (2.127)$$

One also has to ensure $e^\varphi > 0$ and the positivity of the metric on the compact space M_6 . The latter condition in this case amounts to $x_2^a < 0$. The above solutions generalize to an arbitrary number of vector multiplets the ones derived in [31] in the context of flux compactifications of massive type IIA on coset manifolds. Furthermore, they allow for non-vanishing charges p^a, q_a (satisfying $p^0 q_a = P_a$).

One can easily check that the solutions we found fall into the linear ansatz described in the last section with u, v, θ given by

$$\mathbf{A} \ (\mathcal{N} = 0) : \quad u = 3v = 2\sqrt{\frac{6}{5}}, \quad e^{i\theta} = \sqrt{\frac{1}{6}}(\sqrt{5} - i)$$

$$\mathbf{B} \ (\mathcal{N} = 0) : \quad u = v = 2, \quad e^{i\theta} = \frac{1}{2}(1 - i\sqrt{3})$$

$$\mathbf{C} \ (\mathcal{N} = 1) : \quad u = 2v = 2, \quad e^{i\theta} = 1.$$

2.6.4.2 Adding Hypermultiplets

The three solutions above can be generalized to the case of a cubic supergravity with arbitrary number of vector multiplets and hypermultiplets. For simplicity we focus again to the case where the vector multiplet scalar manifold is symmetric. Here we consider a vacuum configuration with non-trivial charges: e_a^i , $e_a \equiv e_{a0}$, p^0 , $\mathbf{q}_0$, while $\mathbf{q}_a$ and all remaining charges in $Q_{\mathbb{A}}^{\mathbb{I}}$ are set to zero. Recalling (2.113), the prepotentials and their derivatives are now given by

$$\begin{aligned} W_+ &= 2\mathbf{x}^a (e_a^i g_i - e_a), & \partial_a W_+ &= 2(e_a^i g_i - e_a), & \partial_i W_+ &= 2\mathbf{x}^a e_a^j g_{ij}, \\ W_- &= 2\mathbf{x}^a (e_a^i \bar{g}_i - e_a), & \partial_a W_- &= 2(e_a^i \bar{g}_i - e_a), & \partial_{\bar{i}} W_- &= 2\mathbf{x}^a e_a^j \bar{g}_{ij}, \\ W_3 &= \mathbf{q}_0 + \mathbf{x}^a (e_a^i \tilde{\xi}_i - e_a \xi^0) + p^0 \mathbf{f}, & \partial_a W_3 &= e_a^i \tilde{\xi}_i - e_a \xi^0 + p^0 \mathbf{f}_a, \end{aligned} \quad (2.128)$$

where $g_{ij} = d_{ijk} z^k$. Again we follow an educated ansatz

$$\mathbf{x}^a = x R^a, \quad z^i = z S^i, \quad \tilde{\xi}_i = \zeta T_i, \quad (2.129)$$

where we defined

$$\begin{aligned} R^a &= \frac{1}{2} d^{abc} e_b e_c, & R &= \frac{1}{6} d^{abc} e_a e_b e_c \\ S^i &= e_a^i R^a, & T &= \frac{1}{6} d_{ijk} S^i S^j S^k, & T_i &= \frac{1}{2} d_{ijk} S^j S^k. \end{aligned} \quad (2.130)$$

In addition we impose the following relation between the NSNS charges

$$d_{ijk} e_a^i e_b^j e_c^k = \beta d_{abc} \quad \Rightarrow \quad T_i e_a^i = \beta R e_a, \quad T = \beta R^2, \quad (2.131)$$

for a positive number β . With these assumptions, one has the following simplifications

$$\begin{aligned} \mathcal{V} &= (x_2)^3 R^2, & \mathcal{V}_a &= (x_2)^2 R e_a, & \mathbf{f} &= x^3 R^2, & \mathbf{f}_a &= x^2 R e_a, \\ \tilde{\mathcal{V}} &= (z_2)^3 T, & \tilde{\mathcal{V}}_i &= (z_2)^2 T_i, & g &= z^3 T, & g_i &= z^2 T_i \\ W_+ &= 6 R x (z^2 \beta R - 1), & \partial_a W_+ &= 2(z^2 \beta R - 1) e_a, & \partial_i W_+ &= 4 x z T_i \\ W_3 &= \mathbf{q}_0 - 3 R x \hat{\xi} + p^0 R^2 x^3, & \partial_a W_3 &= (p^0 R x^2 - \hat{\xi}) e_a, \end{aligned} \quad (2.132)$$

where we introduced

$$\hat{\xi} = \xi^0 - \beta R \zeta. \quad (2.133)$$

The new equations in the presence of additional hypermultiplets can be solved by taking

$$z = -i \sqrt{\frac{3}{\beta R}}. \quad (2.134)$$

Plugging this into $W_{\pm}$ and $D_a W_{\pm}$ one gets $W_{\pm} = -24xR$ and $D_a W_{\pm} = -8e_a$. Comparing with (2.120), one finds that the Killing prepotentials $\mathcal{P}_x$ are related to those found in the last section after the replacements

$$\varphi \rightarrow \hat{\varphi} \quad \xi \rightarrow \hat{\xi} \quad V \rightarrow e^{2c} V \quad (2.135)$$

with

$$e^{2\varphi} = e^{2\hat{\varphi}+c} \quad \hat{\xi} = \xi^0 - \beta R \zeta \quad e^c = 64 e^{K_1} = \left(\frac{64 \beta}{3^3 R} \right)^{\frac{1}{2}} \quad (2.136)$$

More precisely the Killing potentials in the cases of $h_1 > 0$ and $h_1 = 0$ are related via $\mathcal{P}_x^{h_1>0}(\varphi, \xi) = e^c \mathcal{P}_x^{h_1=0}(\hat{\varphi}, \hat{\xi})$. This implies that the three solutions (2.125)–(2.127) generalize to the case of arbitrary number of vector and hypermultiplets after the replacements (2.135) leading to

$$\begin{aligned} x &\sim \left(\frac{\mathbf{q}_0}{p^0 R^2} \right)^{1/3}, \quad \hat{\xi} \sim \left(\frac{p^0 \mathbf{q}_0^2}{R} \right)^{1/3} \quad e^{\varphi} \sim \left(\frac{R^{\frac{1}{4}} \beta^{\frac{3}{4}}}{p^0 \mathbf{q}_0^2} \right)^{\frac{1}{3}}, \\ V &\sim \left(\frac{R \beta^3}{p^0 \mathbf{q}_0^5} \right)^{\frac{1}{3}}. \end{aligned} \quad (2.137)$$

where we omit numerical coefficients depending on the particular solution **A**, **B**, **C**. Notice that the combination $\xi^0 + \beta R \zeta$ is a flat direction of the potential.

2.6.5 *U-Invariant Cosmological Constant*

In this section, we propose a U-duality invariant formula for the dependence on NSNS and RR fluxes of the scalar potential V at its critical points. This defines the cosmological constant $\Lambda = V|_{\partial V=0}$.

As considered in the treatment above, the setup is the following. The vectors' and hypers' scalar manifolds are given by $\mathcal{M}_2 = G/H$ and $\mathcal{M}_Q$. Here, G/H is a *symmetric* special Kähler manifold with cubic prepotential (d -special Kähler space, see e.g. [22]), with complex dimension h_2 , coinciding with the number of (abelian) vector multiplets. On the other hand, $\mathcal{M}_Q$ is a *symmetric* quaternionic manifold, with quaternionic dimension $h_1 + 1$, corresponding to the number of hypermultiplets.

The manifold $\mathcal{M}_Q$ is the c-map [20] of the symmetric d -special Kähler space $\mathcal{M}_1 = \mathcal{G}/\mathcal{H} \subsetneq \mathcal{M}_Q$, with complex dimension h_1 . Thus, the overall U-duality group is given by

$$U \equiv G \times \mathcal{G} \subsetneq Sp(2h_2 + 2, \mathbb{R}) \times Sp(2h_1 + 2, \mathbb{R}). \quad (2.138)$$

The Gaillard-Zumino [34] embedding of G and $\mathcal{G}$ is provided by the symplectic representations $\mathbf{R}_G$ and $\mathbf{R}_{\mathcal{G}}$, respectively spanned by the symplectic indices $\mathbb{A}$ and $\mathbb{I}$. The RR fluxes sit in the $(2h_2 + 2)$ vector representation $\mathbf{R}_G$ (as above, $a = 1, \dots, h_2$ and $i = 1, \dots, h_1$ throughout)

$$c^{\mathbb{A}} \equiv \begin{pmatrix} p^0 \\ p^a \\ q_0 \\ q_a \end{pmatrix}, \quad (2.139)$$

whereas the NSNS fluxes fit into the $(2h_2 + 2) \times (2h_1 + 2)$ bi-vector representation $\mathbf{R}_G \times \mathbf{R}_{\mathcal{G}}$

$$Q^{\mathbb{A}\mathbb{I}} = \mathbb{C}_2^{\mathbb{A}\mathbb{B}} Q_{\mathbb{B}}^{\mathbb{I}} = \begin{pmatrix} m^{AI} & m_I^A \\ -e_A^I & -e_{AI} \end{pmatrix}. \quad (2.140)$$

A priori, in presence of $c^{\mathbb{A}}$ and $Q^{\mathbb{A}\mathbb{I}}$, various $(G \times \mathcal{G})$ -invariants, of different orders in RR and NSNS fluxes, can be constructed. Below we focus our analysis on invariants of total order 4 and 16 in fluxes, which respectively turn out to be relevant for the U-invariant characterization of Λ for the solutions **A**, **B**, **C** of Sects. 2.6.4.1 and 2.6.4.2.

2.6.5.1 Only Universal Hypermultiplet

Special Kähler symmetric spaces are characterized by a constant completely symmetric symplectic tensor $d_{\mathbb{A}_1\mathbb{A}_2\mathbb{A}_3\mathbb{A}_4}$. This tensor defines a quartic G -invariant $\mathcal{I}_4(c^4)$ given by

$$\begin{aligned} \mathcal{I}_4(c^4) &= d_{\mathbb{A}_1\mathbb{A}_2\mathbb{A}_3\mathbb{A}_4} c^{\mathbb{A}_1} \dots c^{\mathbb{A}_4} \\ &= -(p^0 q_0 + p^a q_a)^2 + \frac{2}{3} d_{abc} q_0 p^a p^b p^c - \frac{2}{3} d^{abc} p^0 q_a q_b q_c \\ &\quad + d_{abc} d^{aef} p^b p^c q_e q_f. \end{aligned} \quad (2.141)$$

A similar definition holds for the symplectic tensor $d_{\mathbb{I}_1\mathbb{I}_2\mathbb{I}_3\mathbb{I}_4}$ of the symmetric coset $\mathcal{G}/\mathcal{H}$.

For the explicit solutions we have found, we take RR fluxes $c^{\mathbb{A}}$ with $p^0 \neq 0$ and

$$q_a \equiv \frac{1}{2} \frac{d_{abc} p^b p^c}{p^0}. \quad (2.142)$$

Plugging this into (2.141) and using the relation (2.118) (holding in *homogeneous symmetric d-special Kähler geometries*) one finds

$$\mathcal{I}_4(c^4) = -(p^0)^2 \mathbf{q}_0^2 \quad \text{with} \quad \mathbf{q}_0 \equiv q_0 + \frac{1}{6} \frac{d_{abc} p^a p^b p^c}{(p^0)^2}. \quad (2.143)$$

Notice that the full dependence on p^a is encoded in the shift $q_0 \rightarrow \mathbf{q}_0$ and therefore we can, without loosing in generality, restrict ourselves to the simple choice $c^{\mathbb{A}} = (p^0, 0, \mathbf{q}_0, 0)$. The NSNS and RR fluxes are then given by

$$c^{\mathbb{A}} \equiv \begin{pmatrix} p^0 \\ 0 \\ \mathbf{q}_0 \\ 0 \end{pmatrix}, \quad Q_0^{\mathbb{A}} \equiv \begin{pmatrix} 0 \\ 0 \\ 0 \\ -e_a \end{pmatrix}, \quad Q^{\mathbb{A}0} = 0. \quad (2.144)$$

Besides $\mathcal{I}_4(c^4)$ one can build the following non-trivial quartic invariant

$$\mathcal{I}_4(c Q^3) = d_{\mathbb{A}_1 \mathbb{A}_2 \mathbb{A}_3 \mathbb{A}_4} c^{\mathbb{A}_1} Q_0^{\mathbb{A}_2} Q_0^{\mathbb{A}_3} Q_0^{\mathbb{A}_4} = -\frac{1}{6} p^0 d^{abc} e_a e_b e_c = -p^0 R. \quad (2.145)$$

Let us remark that $\mathcal{I}_4(c Q^3)$ is also invariant under the group $SO(2) = U(1)$ which, due to the absence of special Kähler scalars z^i in the hypersector, gets promoted to global symmetry. $\mathcal{G} = SO(2)$ is thus embedded into the symplectic group $Sp(2, \mathbb{R})$ via its irrep. **2**, through which it acts on symplectic sections $(Z^0, \mathcal{G}_0)$. The $SO(2)$ -invariance of $\mathcal{I}_4(c Q^3)$ is manifest, because the latter depends on the $SO(2)$ -invariant $\mathcal{I}_2((Q_{\mathbb{A}})^2) = (Q_{\mathbb{A}0})^2 + (Q_{\mathbb{A}}^0)^2 = (Q_{\mathbb{A}0})^2$ (no sum over $\mathbb{A}$ is understood).

It is easy to see that solutions **A**, **B**, **C** given in Eqs. (2.125)–(2.127) depend only on the two combinations $\mathcal{I}_4(c^4)$ and $\mathcal{I}_4(c Q^3)$ given in (2.141) and (2.145) respectively. Upgrading these combinations to their U-duality invariant forms we can then write a manifestly $(G \times U(1))$ -invariant formula for the AdS cosmological constant at the critical points

$$\Lambda = V \sim -\frac{\mathcal{I}_4^{4/3}(c Q^3)}{|\mathcal{I}_4(c^4)|^{5/6}} \sim \frac{Q^4}{c^2}. \quad (2.146)$$

Notice that RR and NSNS fluxes play very different roles in their contribution to Λ . Indeed, the cosmological constant grows quartically on NSNS fluxes and fall off quadratically on RR charges. It would be nice to understand whether this is a general scaling feature of the gauged supergravities under study.

2.6.5.2 Many Hypermultiplets

Next let us consider the case with arbitrary number of hypermultiplets. From (2.137), it follows that the solutions in this case depend only on the combinations $\mathcal{I}_4 = (p^0 \mathbf{q}_0)^2$ and $\mathcal{I}_{16} \sim (p^0)^4 R \beta^3 \sim c^4 Q^{12}$. In order to write the solutions in a U-duality invariant form we should then find an invariant $\mathcal{I}_{16}$ built out of 12 Q 's and 4 c 's that reduce to $(p^0)^4 R \beta^3$ on our choice of RR and NSNS fluxes. The following $(G \times \mathcal{G})$ -invariant quantity does the job

$$\begin{aligned} \mathcal{I}_{16} (c^4 Q^{12}) \equiv & d_{\mathbb{I}_1 \mathbb{I}_2 \mathbb{I}_3 \mathbb{I}_4} d_{\mathbb{I}_5 \mathbb{I}_6 \mathbb{I}_7 \mathbb{I}_8} d_{\mathbb{I}_9 \mathbb{I}_{10} \mathbb{I}_{11} \mathbb{I}_{12}} d_{\mathbb{A}_1 \mathbb{B}_1 \mathbb{B}_2 \mathbb{B}_3} d_{\mathbb{A}_2 \mathbb{B}_5 \mathbb{B}_6 \mathbb{B}_7} d_{\mathbb{A}_3 \mathbb{B}_9 \mathbb{B}_{10} \mathbb{B}_{11}} \times \\ & \times d_{\mathbb{A}_4 \mathbb{B}_4 \mathbb{B}_8 \mathbb{B}_{12}} c^{\mathbb{A}_1} c^{\mathbb{A}_2} c^{\mathbb{A}_3} c^{\mathbb{A}_4} Q^{\mathbb{B}_1 \mathbb{I}_1} \dots Q^{\mathbb{B}_{12} \mathbb{I}_{12}}. \end{aligned} \quad (2.147)$$

The explicit expression of $\mathcal{I}_{16} (c^4 Q^{12})$ is rather intricate. Nevertheless, this formula undergoes a dramatic simplification when considering the configuration of NSNS and RR fluxes, supporting the solutions found in Sect. 2.6.4. As before we encode the full dependence on p^a in the shift $q_0 \rightarrow \mathbf{q}_0$ and therefore we restrict ourselves to the charge vector choice $c^{\mathbb{A}} = (p^0, 0, \mathbf{q}_0, 0)$. More precisely, we take NSNS and RR fluxes with all components of $Q^{\mathbb{A}\mathbb{I}}$, $c^{\mathbb{A}}$ zero except for

$$Q_a^i = -e_a^i, \quad Q_{a0} = -e_a, \quad c^0 = p^0, \quad c_0 = \mathbf{q}_0, \quad (2.148)$$

where e_a^i satisfy the constraint (2.131) for some $\beta \in \mathbb{R}_+$. A simple inspection to (2.147), shows that contributions to $\mathcal{I}_{16}$ come only from the components $d_{ijk}^0 = -\frac{1}{6} d_{ijk}$ of $d_{\mathbb{I}_1 \dots \mathbb{I}_4}$ and $d_0^{b_1 b_2 b_3} = -\frac{1}{6} d^{abc}$ of $d_{\mathbb{A}\mathbb{B}_1 \mathbb{B}_2 \mathbb{B}_3}$. Indeed using (2.118) one finds

$$\mathcal{I}_{16} (c^4 Q^{12}) = \gamma (p^0)^4 R \beta^3, \quad (2.149)$$

with³

$$\gamma \equiv \frac{1}{6} (d^{abc} d_{abc} + h_2 + 3)^3. \quad (2.150)$$

We conclude that the explicit vacuum solutions **A**, **B**, **C** obtained in Sect. 2.6.4 can be written in a manifestly U-duality invariant form in terms of the U-duality invariants $\mathcal{I}_4(c^4)$ and $\mathcal{I}_{16}(c^4 Q^{12})$. In particular the value of the cosmological constant Λ is given by the $(G \times \mathcal{G})$ -invariant formula

$$\Lambda = V \sim \frac{\mathcal{I}_{16}^{1/3} (c^4 Q^{12})}{|\mathcal{I}_4 (c^4)|^{5/6}} \sim \frac{Q^4}{c^2}. \quad (2.151)$$

³Interestingly, the quantity $d^{abc} d_{abc}$, appearing in (2.150) is related to the Ricci scalar curvature $\mathcal{R}$ of the vector multiplets' scalar manifold G/H , whose general expression for a d -special Kähler space reads $\mathcal{R} = -(h_2 + 1) h_2 + d^{abc} d_{abc}$, see [35–37].

Appendix

A.1 Flux/Gauging Dictionary for IIA on $SU(3)$ Structure

Gauged $\mathcal{N} = 2$ supergravities with a scalar potential of the form studied in this paper can be derived by flux compactifications of type II theories on $SU(3)$ and $SU(3) \times SU(3)$ structure manifolds. While we refer to the literature (see e.g. [25, 27, 29–31, 38–40]) for a detailed study of such general $\mathcal{N} = 2$ dimensional reductions and the related issues,¹ in this appendix we provide a practical dictionary between the 10d and the 4d quantities, with a focus on the scalar potential derived from $SU(3)$ structure compactifications of type IIA. In particular, we illustrate how the expressions one derives for V_{NS} and V_{R} are consistent with the scalar potential (2.79) studied in the main text.

A.1.1 $SU(3)$ Structures and Their Curvature

An $SU(3)$ structure on a 6d manifold M_6 is defined by a real 2-form J and a complex, decomposable² 3-form Ω , satisfying the compatibility relation $J \wedge \Omega = 0$ as well as the non-degeneracy (and normalization) condition

$$\frac{i}{8} \Omega \wedge \bar{\Omega} = \frac{1}{6} J \wedge J \wedge J = \text{vol}_6 \neq 0 \quad \text{everywhere.} \quad (\text{A.1})$$

Ω defines an almost complex structure I , with respect to which is of type $(3, 0)$. In turn, I and J define a metric on M_6 via $g = JI$. The latter is required to be positive-definite, and vol_6 above denotes the associated volume form.

$SU(3)$ structures are classified by their torsion classes W_i , $i = 1, \dots, 5$, defined via [46]:

$$\begin{aligned} dJ &= \frac{3}{2} \text{Im}(\bar{W}_1 \Omega) + W_4 \wedge J + W_3 \\ d\Omega &= W_1 \wedge J \wedge J + W_2 \wedge J + \bar{W}_5 \wedge \Omega, \end{aligned} \quad (\text{A.2})$$

where W_1 is a complex scalar, W_2 is a complex primitive $(1,1)$ -form (primitive means $W_2 \wedge J \wedge J = 0$), W_3 is a real primitive $(1,2) + (2,1)$ -form (primitive $\Leftrightarrow W_3 \wedge J = 0$), W_4 is a real 1-form, and W_5 is a complex $(1,0)$ -form.

Reference [47] provides a formula for the Ricci scalar R_6 in terms of the torsion classes. We will restrict to $W_4 = W_5 = 0$, in which case the formula is

$$R_6 = \frac{1}{2} (15|W_1|^2 - W_2 \lrcorner \bar{W}_2 - W_3 \lrcorner W_3). \quad (\text{A.3})$$

¹In this context, see also [41–45] for studies of compactifications preserving $\mathcal{N} = 1$.

²A p -form is decomposable if locally it can be written as the wedging of p complex 1-forms.

This can equivalently be expressed as

$$R_6 \text{vol}_6 = -\frac{1}{2} [dJ \wedge *dJ + d\Omega \wedge *d\bar{\Omega} - (dJ \wedge \Omega) \wedge *(dJ \wedge \bar{\Omega})], \quad (\text{A.4})$$

as it can be seen recalling (A.2) and computing

$$\begin{aligned} d\Omega \wedge *d\bar{\Omega} &= 12|W_1|^2 \text{vol}_6 - J \wedge W_2 \wedge \bar{W}_2 \\ &= (12|W_1|^2 + W_2 \lrcorner \bar{W}_2) \text{vol}_6 \\ dJ \wedge *dJ &= (9|W_1|^2 + W_3 \lrcorner W_3) \text{vol}_6 \\ (dJ \wedge \Omega) \wedge *(dJ \wedge \bar{\Omega}) &= 36|W_1|^2 \text{vol}_6. \end{aligned} \quad (\text{A.5})$$

A.1.2 The Scalar Potential from Dimensional Reduction

The 4d scalar potential receives contributions from both the NSNS and the RR sectors of type IIA supergravity. These are respectively given by

$$\begin{aligned} V_{\text{NS}} &= \frac{e^{2\varphi}}{2\mathcal{V}} \int_{M_6} \left(\frac{1}{2} H \wedge *H - R_6 *1 \right) \\ &= \frac{e^{2\varphi}}{4\mathcal{V}} \int_{M_6} \left[H \wedge *H + dJ \wedge *dJ + d\Omega \wedge *d\bar{\Omega} \right. \\ &\quad \left. - (dJ \wedge \Omega) \wedge *(dJ \wedge \bar{\Omega}) \right], \end{aligned} \quad (\text{A.6})$$

$$V_{\text{R}} = \frac{e^{4\varphi}}{2} \int_{M_6} (F_0^2 *1 + F_2 \wedge *F_2 + F_4 \wedge *F_4 + F_6 \wedge *F_6), \quad (\text{A.7})$$

and the total potential reads $V = V_{\text{NS}} + V_{\text{R}}$. In (A.6), H is the internal NSNS field-strength, $\mathcal{V} = \int_{M_6} \text{vol}_6$, and φ is the 4d dilaton $e^{-2\varphi} = e^{-2\phi} \mathcal{V}$, where we are assuming that the 10d dilaton ϕ is constant along M_6 . The k -forms F_k appearing in expression (A.7) are the internal RR field strengths, satisfying the Bianchi identity $dF_k - H \wedge F_{k-2} = 0$. The F_6 form can be seen as the Hodge-dual of the F_4 extending along spacetime, and the term $F_6 \wedge *F_6$ arises in a natural way if one considers type IIA supergravity in its democratic formulation [48].

A.1.2.1 Expansion Forms

In order to define the mode truncation, we postulate the existence of a basis of differential forms on the compact manifold in which to expand the higher

dimensional fields. For a detailed analysis of the relations that these forms need to satisfy in order that the dimensional reduction go through, see in particular [27].

We take $\omega_0 = 1$ and $\tilde{\omega}^0 = \frac{vol_6}{V}$, and we assume there exist a set of 2-forms ω_a satisfying

$$\omega_a \wedge * \omega_b = 4 g_{ab} vol_6, \quad \omega_a \wedge \omega_b = -d_{abc} \tilde{\omega}^c, \quad (\text{A.8})$$

where g_{ab} should be independent of the internal coordinates, d_{abc} should be a constant tensor, and the dual 4-forms $\tilde{\omega}^a$ are defined as

$$\tilde{\omega}^a = -\frac{1}{4V} g^{ab} * \omega_b. \quad (\text{A.9})$$

From the above relations, we see that

$$\omega_a \wedge \tilde{\omega}^b = -\delta_a^b \tilde{\omega}^0, \quad \omega_a \wedge \omega_b \wedge \omega_c = d_{abc} \tilde{\omega}^0. \quad (\text{A.10})$$

We also assume the existence of a set of 3-forms α_I, β^I , satisfying

$$\alpha_I \wedge \beta^J = \delta_I^J \tilde{\omega}^0. \quad (\text{A.11})$$

Adopting the notation $\omega^{\mathbb{A}} = (\tilde{\omega}^A, \omega_A)^T = (\tilde{\omega}^0, \tilde{\omega}^a, \omega_0, \omega_a)^T$ and $\alpha^{\mathbb{I}} = (\beta^I, \alpha_I)^T$, we see that the symplectic metrics $\mathbb{C}$ appearing in the main text are here given by

$$\mathbb{C}_1^{\mathbb{I}\mathbb{J}} = -\int \alpha^{\mathbb{I}} \wedge \alpha^{\mathbb{J}}, \quad \mathbb{C}_2^{\mathbb{A}\mathbb{B}} = -\int \langle \omega^{\mathbb{A}}, \omega^{\mathbb{B}} \rangle, \quad (\text{A.12})$$

where the antisymmetric pairing $\langle, \rangle$ is defined on even forms ρ, σ as $\langle \rho, \sigma \rangle = [\lambda(\rho) \wedge \sigma]_6$, with $\lambda(\rho_k) = (-)^{\frac{k}{2}} \rho_k$, k being the degree of ρ , and $[\]_6$ selecting the piece of degree 6.

The basis forms are used to expand Ω as

$$\Omega = Z^I \alpha_I - \mathcal{G}_I \beta^I = e^{-\frac{K_1}{2}} \Pi_1^{\mathbb{I}} \alpha_{\mathbb{I}}, \quad (\text{A.13})$$

and J together with the internal NS 2-form B as:

$$J = v^a \omega_a, \quad B = b^a \omega_a \quad \Rightarrow \quad e^{-B-iJ} = X^A \omega_A - \mathcal{F}_A \tilde{\omega}^A = e^{-\frac{K_2}{2}} \Pi_2^{\mathbb{A}} \omega_{\mathbb{A}}, \quad (\text{A.14})$$

where in the last equalities we define $\alpha_{\mathbb{I}} = \mathbb{C}_{\mathbb{I}\mathbb{J}} \alpha^{\mathbb{J}} = (\alpha_I, -\beta^I)^T$ and $\omega_{\mathbb{A}} = \mathbb{C}_{\mathbb{A}\mathbb{B}} \omega^{\mathbb{B}} = (\omega_A, -\tilde{\omega}^A)^T$, and we adopt the symplectic notation defined in (2.76). Here, $(Z^I, \mathcal{G}_I)$ and $(X^A, \mathcal{F}_A)$ represent the holomorphic sections on the moduli spaces of Ω and $B + iJ$ expanded as above, which (under some conditions [26–28]) indeed exhibit a special Kähler structure, and correspond respectively to the manifolds $\mathcal{M}_1$ and $\mathcal{M}_2$ of the main text. Notice that here $X^A \equiv (X^0, X^a) \equiv (1, x^a) = (1, -b^a - i v^a)$, while $\mathcal{F}_A = \frac{\partial \mathcal{F}}{\partial X^A}$, where the cubic holomorphic function

$\mathcal{F} = \frac{1}{6} d_{abc} \frac{X^a X^b X^c}{X^0}$ is identified with the prepotential on $\mathcal{M}_2$. The Kähler potentials on $\mathcal{M}_1$ and $\mathcal{M}_2$ are recovered from $K_1 = -\log i \int \Omega \wedge \bar{\Omega}$ and $K_2 = -\log \frac{4}{3} \int J \wedge J \wedge J$, the latter yielding the metric g_{ab} appearing in (A.8). Notice that (A.1) implies $e^{-K_1} = e^{-K_2} = 8\mathcal{V}$.

The matrices $\mathbb{M}$ defined in (2.81) are given by

$$\mathbb{M}_{1,\mathbb{IJ}} = - \int \alpha_{\mathbb{I}} \wedge * \alpha_{\mathbb{J}}, \quad \mathbb{M}_{2,\mathbb{AB}} = - \sum_k \int (e^B \omega_{\mathbb{A}})_k \wedge * (e^B \omega_{\mathbb{B}})_k, \quad (\text{A.15})$$

and from the second relation one finds that the period matrix $\mathcal{N}_2$ on $\mathcal{M}_2$ reads

$$\text{Re} \mathcal{N}_{AB} = - \begin{pmatrix} \frac{1}{3} d_{abc} b^a b^b b^c & \frac{1}{2} d_{abc} b^b b^c \\ \frac{1}{2} d_{abc} b^b b^c & d_{abc} b^c \end{pmatrix}, \quad \text{Im} \mathcal{N}_{AB} = -4\mathcal{V} \begin{pmatrix} \frac{1}{4} + g_{ab} b^a b^b & g_{ab} b^b \\ g_{ab} b^b & g_{ab} \end{pmatrix}, \quad (\text{A.16})$$

which is in agreement with the expression derived from $\mathcal{F}$ via the standard formula [49]

$$N_{AB} = \bar{\mathcal{F}}_{AB} + 2i \frac{\text{Im} \mathcal{F}_{AD} X^D \text{Im} \mathcal{F}_{BE} X^E}{X^C \text{Im} \mathcal{F}_{CE} X^E}, \quad \mathcal{F}_{AB} \equiv \frac{\partial^2 \mathcal{F}}{\partial X^A \partial X^B}. \quad (\text{A.17})$$

Finally, we also require the following differential conditions on the basis forms:

$$d\omega_a = e_a^{\mathbb{I}} \alpha_{\mathbb{I}}, \quad d\alpha^{\mathbb{I}} = e_a^{\mathbb{I}} \tilde{\omega}^a, \quad d\tilde{\omega}^a = 0, \quad (\text{A.18})$$

where the $e_a^{\mathbb{I}} = (e_a^I, e_{aI})$ are real constants, usually called ‘geometric fluxes’. Defining the total internal NS 3-form as $H = H^{\mathbb{I}} + dB$, and expanding its flux part as

$$H^{\mathbb{I}} = -e_0^I \alpha_I + e_{0I} \beta^I \equiv -e_0^{\mathbb{I}} \alpha_{\mathbb{I}}, \quad (\text{A.19})$$

with constant $e_0^{\mathbb{I}}$, we can define $e_A^{\mathbb{I}} = (e_0^{\mathbb{I}}, e_a^{\mathbb{I}})^T$, and thus fill in half of the charge matrix $\mathcal{Q}$ introduced in (2.77):

$$\mathcal{Q}_{\mathbb{A}}^{\mathbb{I}} = \begin{pmatrix} e_A^{\mathbb{I}} \\ 0 \end{pmatrix}. \quad (\text{A.20})$$

As noticed in [28], more general matrices, involving the $m_A^{\mathbb{I}}$ charges as well, can be obtained by considering non-geometric fluxes, or $SU(3) \times SU(3)$ structure compactifications. The nilpotency condition $d^2 = 0$ applied to (A.18), together with the Bianchi identity $dH = 0$, translates into the constraint

$$e_A^{\mathbb{I}} e_{B\mathbb{I}} = 0 \quad \text{with} \quad e_{A\mathbb{I}} = \mathbb{C}_{\mathbb{IJ}} e_A^{\mathbb{J}}, \quad (\text{A.21})$$

which, taking into account (A.20), is consistent with (2.78).

In the following, by using the above relations we recast in turn expressions (A.6) and (A.7) for V_{NS} and V_{R} in terms of 4d degrees of freedom, and show their consistency with (2.79).

A.1.2.2 Derivation of V_{NS}

Recalling the expansions of J , H and Ω defined above, using the assumed properties of the basis forms, and adopting the notation introduced in (2.76), one finds

$$\begin{aligned} \int dJ \wedge *dJ &= -v^a v^b e_a^{\mathbb{I}} \mathbb{M}_{1,\mathbb{I}\mathbb{J}} e_b^{\mathbb{J}}, & \int H \wedge *H &= -b^A b^B e_A^{\mathbb{I}} \mathbb{M}_{1,\mathbb{I}\mathbb{J}} e_B^{\mathbb{J}}, \\ \int d\Omega \wedge *d\bar{\Omega} &= \frac{e^{-K_1}}{4\mathcal{V}} \Pi_1^{\mathbb{I}} e_{a\mathbb{I}} g^{ab} e_{b\mathbb{J}} \bar{\Pi}_1^{\mathbb{J}}, & \int (dJ \wedge \Omega) \wedge *(dJ \wedge \bar{\Omega}) \\ &= \frac{e^{-K_1}}{\mathcal{V}} \Pi_1^{\mathbb{I}} e_{a\mathbb{I}} v^a v^b e_{b\mathbb{J}} \bar{\Pi}_1^{\mathbb{J}}, \end{aligned}$$

where we define $b^A = (-1, b^a)$. Plugging this into (A.6), we get the NSNS contribution to V , expressed in a 4d language:

$$V_{\text{NS}} = -\frac{e^{2\varphi}}{4\mathcal{V}} \left[X^A e_A^{\mathbb{I}} \mathbb{M}_{1,\mathbb{I}\mathbb{J}} e_B^{\mathbb{J}} \bar{X}^B - \frac{e^{-K_1}}{4\mathcal{V}} \Pi_1^{\mathbb{I}} e_{a\mathbb{I}} (g^{ab} - 4v^a v^b) e_{b\mathbb{J}} \bar{\Pi}_1^{\mathbb{J}} \right]. \quad (\text{A.22})$$

Recalling (A.16), noticing that $\frac{1}{4\mathcal{V}}(g^{ab} - 4v^a v^b) = -(\text{Im}\mathcal{N}_2)^{-1ab} - 4e^{K_2}(X^a \bar{X}^b + \bar{X}^a X^b)$, and recalling that $e^{-K_1} = e^{-K_2} = 8\mathcal{V}$, we conclude that (A.22) is consistent with (2.79).

A.1.2.3 Derivation of V_{R}

We consider the modified field-strengths $G_k \equiv [e^{-B} F]_k$, which satisfy the Bianchi identity $dG_k - H^{\mathbb{I}} \wedge G_{k-2} = 0$, and we define the expansions

$$\begin{aligned} G_0 &= p^0, & G_2 &= p^a \omega_a, & A_3 &= \xi^{\mathbb{I}} \alpha_{\mathbb{I}} \\ G_4 &= G_4^{\mathbb{I}} + dA_3 = (q_a - e_{a\mathbb{I}} \xi^{\mathbb{I}}) \tilde{\omega}^a, & G_6 &= G_6^{\mathbb{I}} - H^{\mathbb{I}} \wedge A_3 = (q_0 - e_{0\mathbb{I}} \xi^{\mathbb{I}}) \tilde{\omega}^0. \end{aligned}$$

The Bianchi identities then amount just to the following constraint among the charges

$$p^A e_A^{\mathbb{I}} = 0, \quad (\text{A.23})$$

which, recalling (A.20), gives the last equality in (2.78). Then the integral in (A.7) reads

$$\sum_k \int F_k \wedge * F_k = \sum_k \int (e^B G)_k \wedge * (e^B G)_k = (c + \widetilde{Q}\xi)^T \mathbb{M}_2 (c + \widetilde{Q}\xi), \quad (\text{A.24})$$

where for the second equality we use (A.15), and here $(c + \widetilde{Q}\xi)^\mathbb{A} = (p^A, q_A - e_{A\mathbb{I}} \xi^\mathbb{I})^T$. The expression for V_R we obtain is therefore consistent with (2.79).

References

1. S. Ferrara, R. Kallosh, A. Strominger, N=2 extremal black holes. Phys. Rev. **D52**, 5412–5416 (1995). <http://arxiv.org/abs/hep-th/9508072> (hep-th/9508072)
2. A. Strominger, Macroscopic entropy of $N = 2$ extremal black holes. Phys. Lett. **B383**, 39–43 (1996). <http://arxiv.org/abs/hep-th/9602111> (hep-th/9602111)
3. S. Ferrara, R. Kallosh, Supersymmetry and attractors. Phys. Rev. **D54**, 1514–1524 (1996). <http://arxiv.org/abs/hep-th/9602136> (hep-th/9602136)
4. S. Ferrara, R. Kallosh, Universality of supersymmetric attractors. Phys. Rev. **D54**, 1525–1534 (1996). <http://arxiv.org/abs/hep-th/9603090> (hep-th/9603090)
5. R. Kallosh, New attractors. J. High Energy Phys. **12**, 022 (2005). <http://arxiv.org/abs/hep-th/0510024> (hep-th/0510024)
6. P.K. Tripathy, S.P. Trivedi, Non-supersymmetric attractors in string theory. J. High Energy Phys. **03**, 022 (2006). <http://arxiv.org/abs/hep-th/0511117> (hep-th/0511117)
7. A. Sen, Black hole entropy function and the attractor mechanism in higher derivative gravity. J. High Energy Phys. **09**, 038 (2005). <http://arxiv.org/abs/hep-th/0506177> (hep-th/0506177)
8. J.F. Morales, H. Samtleben, Entropy function and attractors for AdS black holes. J. High Energy Phys. **10**, 074 (2006). <http://arxiv.org/abs/hep-th/0608044> (hep-th/0608044)
9. S. Ferrara, A. Marrani, J.F. Morales, H. Samtleben, Intersecting attractors. Phys. Rev. **D79**, 065031 (2009). <http://arxiv.org/abs/0812.0050> (arXiv:0812.0050)
10. D. Cassani, S. Ferrara, A. Marrani, J. F. Morales, H. Samtleben, A Special road to AdS vacua. J. High Energy Phys. **1002** (2010) 027. <http://arxiv.org/abs/0911.2708> (arXiv:0911.2708)
11. S. Bellucci, S. Ferrara, R. Kallosh, A. Marrani, Extremal black hole and flux vacua attractors. Lect. Notes Phys. **755**, 115–191 (2008). <http://arxiv.org/abs/0711.4547> (arXiv:0711.4547)
12. A. Sen, Black hole entropy function, attractors and precision counting of microstates. Gen. Relat. Gravity **40**, 2249–2431 (2008). <http://arxiv.org/abs/0708.1270> (arXiv:0708.1270)
13. D.Z. Freedman, S.S. Gubser, K. Pilch, N.P. Warner, Renormalization group flows from holography supersymmetry and a c-theorem. Adv. Theor. Math. Phys. **3**, 363–417 (1999). <http://arxiv.org/abs/hep-th/9904017> (hep-th/9904017)
14. K. Goldstein, R.P. Jena, G. Mandal, S.P. Trivedi, A c-function for non-supersymmetric attractors. J. High Energy Phys. **02**, 053 (2006). <http://arxiv.org/abs/hep-th/0512138> (hep-th/0512138)
15. A. Sen, Quantum entropy function from AdS(2)/CFT(1) correspondence. Int. J. Mod. Phys. **A24**, 4225–4244 (2009). <http://arxiv.org/abs/0809.3304> (arXiv:0809.3304)
16. J.D. Brown, M. Henneaux, Central charges in the canonical realization of asymptotic symmetries: an example from three-dimensional gravity. Commun. Math. Phys. **104**, 207–226 (1986)
17. J.A. Strathdee, Extended poincare supersymmetry. Int. J. Mod. Phys. **A2**, 273 (1987)
18. A. Van Proeyen, Tools for supersymmetry. <http://arxiv.org/abs/hep-th/9910030> (hep-th/9910030)
19. P.K. Townsend, P-brane democracy, <http://arxiv.org/abs/hep-th/9507048> (hep-th/9507048)

20. S. Cecotti, S. Ferrara, L. Girardello, Geometry of type II superstrings and the moduli of superconformal field theories. *Int. J. Mod. Phys. A* **4**, 2475 (1989)
21. S. Ferrara, S. Sabharwal, Quaternionic manifolds for type II superstring vacua of calabi-yau spaces. *Nucl. Phys. B* **332**, 317 (1990)
22. B. de Wit, F. Vanderseypen, A. Van Proeyen, Symmetry structure of special geometries. *Nucl. Phys. B* **400**, 463–524 (1993). <http://arxiv.org/abs/hep-th/9210068> (hep-th/9210068)
23. R. D'Auria, S. Ferrara, M. Trigiante, S. Vaula, Gauging the heisenberg algebra of special quaternionic manifolds. *Phys. Lett. B* **610**, 147–151 (2005). <http://arxiv.org/abs/hep-th/0410290> (hep-th/0410290)
24. R. D'Auria, S. Ferrara, M. Trigiante, On the supergravity formulation of mirror symmetry in generalized Calabi-Yau manifolds. *Nucl. Phys. B* **780**, 28–39 (2007). <http://arxiv.org/abs/hep-th/0701247> (hep-th/0701247)
25. S. Gurrieri, J. Louis, A. Micu, D. Waldram, Mirror symmetry in generalized Calabi-Yau compactifications. *Nucl. Phys. B* **654**, 61–113 (2003). <http://arxiv.org/abs/hep-th/0211102> (hep-th/0211102)
26. M. Grana, J. Louis, D. Waldram, Hitchin functionals in $N = 2$ supergravity. *J. High Energy Phys.* **01**, 008 (2006). <http://arxiv.org/abs/hep-th/0505264> (hep-th/0505264)
27. A.-K. Kashani-Poor, R. Minasian, Towards reduction of type II theories on $SU(3)$ structure manifolds. *J. High Energy Phys.* **03**, 109 (2007). <http://arxiv.org/abs/hep-th/0611106> (hep-th/0611106)
28. M. Grana, J. Louis, D. Waldram, $SU(3) \times SU(3)$ compactification and mirror duals of magnetic fluxes. *J. High Energy Phys.* **04**, 101 (2007). <http://arxiv.org/abs/hep-th/0612237> (hep-th/0612237)
29. D. Cassani, A. Bilal, Effective actions and $N=1$ vacuum conditions from $SU(3) \times SU(3)$ compactifications. *J. High Energy Phys.* **09**, 076 (2007). <http://arxiv.org/abs/0707.3125> (arXiv:0707.3125)
30. D. Cassani, Reducing democratic type II supergravity on $SU(3) \times SU(3)$ structures. *J. High Energy Phys.* **06**, 027 (2008). <http://arxiv.org/abs/0804.0595> (arXiv:0804.0595)
31. D. Cassani, A.-K. Kashani-Poor, Exploiting $N=2$ in consistent coset reductions of type IIA. *Nucl. Phys. B* **817**, 25–57 (2009). <http://arxiv.org/abs/0901.4251> (arXiv:0901.4251)
32. L. Andrianopoli, R. D'Auria, S. Ferrara, M. Trigiante, Extremal black holes in supergravity. *Lect. Notes Phys.* **737**, 661–727 (2008). <http://arxiv.org/abs/hep-th/0611345> (hep-th/0611345)
33. L. Andrianopoli et al., $N = 2$ supergravity and $N = 2$ super yang-mills theory on general scalar manifolds: symplectic covariance, gaugings and the momentum map. *J. Geom. Phys.* **23**, 111–189 (1997). <http://arxiv.org/abs/hep-th/9605032> (hep-th/9605032)
34. M.K. Gaillard, B. Zumino, Duality rotations for interacting fields. *Nucl. Phys. B* **193**, 221 (1981)
35. E. Cremmer, A. Van Proeyen, Classification of Kahler manifolds in $N=2$ vector multiplet supergravity couplings. *Class. Quantum Gravity* **2**, 445 (1985)
36. S. Bellucci, A. Marrani, R. Roychowdhury, On quantum special Kaehler geometry, <http://arxiv.org/abs/0910.4249> (arXiv:0910.4249)
37. A. Ceresole, S. Ferrara, A. Marrani, 4d/5d Correspondence for the black hole potential and its critical points. *Class. Quantum Gravity* **24** (2007) 5651–5666. <http://arxiv.org/abs/0707.0964> (arXiv:0707.0964)
38. A. Tomasiello, Topological mirror symmetry with fluxes. *J. High Energy Phys.* **06**, 067 (2005). <http://arxiv.org/abs/hep-th/0502148> (hep-th/0502148)
39. T. House, E. Palti, Effective action of (massive) IIA on manifolds with $SU(3)$ structure. *Phys. Rev. D* **72**, 026004 (2005). <http://arxiv.org/abs/hep-th/0505177> (hep-th/0505177)
40. M. Grana, J. Louis, A. Sim, D. Waldram, $E7(7)$ formulation of $N=2$ backgrounds. *J. High Energy Phys.* **07**, 104 (2009). <http://arxiv.org/abs/0904.2333> (arXiv:0904.2333)
41. I. Benmachiche, T.W. Grimm, Generalized $N = 1$ orientifold compactifications and the Hitchin functionals. *Nucl. Phys. B* **748**, 200–252 (2006). <http://arxiv.org/abs/hep-th/0602241> (hep-th/0602241)

42. P. Koerber, L. Martucci, From ten to four and back again: how to generalize the geometry. *J. High Energy Phys.* **08**, 059 (2007). <http://arxiv.org/abs/0707.1038> (arXiv:0707.1038)
43. L. Martucci, On moduli and effective theory of $N = 1$ warped flux compactifications. *J. High Energy Phys.* **05**, 027 (2009). <http://arxiv.org/abs/0902.4031> (arXiv:0902.4031)
44. J.-P. Derendinger, C. Kounnas, P.M. Petropoulos, F. Zwirner, Superpotentials in IIA compactifications with general fluxes. *Nucl. Phys.* **B715**, 211–233 (2005). <http://arxiv.org/abs/hep-th/0411276> (hep-th/0411276)
45. G. Villadoro, F. Zwirner, $N = 1$ effective potential from dual type-IIA D6/O6 orientifolds with general fluxes. *J. High Energy Phys.* **06**, 047 (2005). <http://arxiv.org/abs/hep-th/0503169> (hep-th/0503169)
46. S. Chiossi, S. Salamon, The intrinsic torsion of $SU(3)$ and G_2 structures. <http://arxiv.org/abs/math/0202282> (math/0202282)
47. L. Bedulli, L. Vezzoni, The Ricci tensor of $SU(3)$ -manifolds. *J. Geom. Phys.* **57**, 1125 (2007). <http://arxiv.org/abs/math/0606786> (math/0606786)
48. E. Bergshoeff, R. Kallosh, T. Ortin, D. Roest, A. Van Proeyen, New formulations of $D=10$ supersymmetry and D8-O8 domain walls. *Class. Quantum. Gravity* **18**, 3359–3382 (2001). <http://arxiv.org/abs/hep-th/0103233> (hep-th/0103233)
49. B. Craps, F. Roose, W. Troost, A. Van Proeyen, What is special Kaehler geometry? *Nucl. Phys.* **B503**, 565–613 (1997). <http://arxiv.org/abs/hep-th/9703082> (hep-th/9703082)

Supersymmetric Gravity and Black Holes

Proceedings of the INFN-Laboratori Nazionali di Frascati

School on the Attractor Mechanism 2009

Bellucci, S. (Ed.)

2013, IX, 204 p., Hardcover

ISBN: 978-3-642-31379-0