Preface

A classical generalization of the Fundamental Theorem of Arithmetic states that an integral domain is a principal ideal domain if and only if each of its proper ideals can be factored as a finite product of principal prime ideals. If the "principal" restriction is removed, one has a characterization of (nontrivial) Dedekind domains. The purpose of this work is to study other types of ideal factorization. Most that we consider involve writing certain types of ideals in the form $J\Pi$, where J is an ideal of some special type and Π is a (finite) product of prime ideals. For example, we say that a domain has weak factorization if each nondivisorial ideal can be factored as above with J the divisorial closure of the ideal and Π a product of maximal ideals. In a different direction, we say that a domain has pseudo-Dedekind factorization if each nonzero, noninvertible ideal can be factored as above with J invertible and Π a product of pairwise comaximal prime ideals. In each of these cases, if the domain in question is integrally closed, then it must be a Prüfer domain. While this implies, as is often the case in multiplicative ideal theory, that Prüfer domains play an important role in our study, we do provide non-integrally closed examples for each of these types of ideal factorization. On the other hand, we also consider domains in which each proper ideal can be factored as a product of radical ideals, and such domains must be almost Dedekind (hence Prüfer) domains.

This volume provides a wide-ranging survey of results on various important types of ideal factorization actively investigated by several authors in recent years, often with new or simplified proofs; it also includes many new results. It is our hope that the material contained herein will be useful to researchers and graduate students interested in commutative algebra with an emphasis on the multiplicative theory of ideals.

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