

# Preface

The title of this book ‘One-Dimensional Finite Elements: An Introduction to the FE Method’ stands for content and focus. Nowadays, much literature regarding the topic of the finite element method exists. The different works reflect the multifaceted perceptions and application possibilities. The basic idea of this introduction into the finite element method relies on the concept of explaining the complex method with the help of *one-dimensional* elements. It is the goal to introduce the manifold aspects of the finite element method and to enable the reader to get a methodical understanding of important subject areas. The reader learns to understand the assumptions and derivations at different physical problems in structural mechanics. Furthermore, he/she learns to critically evaluate possibilities and limitations of the finite element method. Additional comprehensive mathematical descriptions, which solely result from advanced illustrations for two- or three-dimensional problems, are omitted. Hence, the mathematical description largely remains simple and clear. The focus on one-dimensional elements, however, is not just a pure limitation on a simpler and clearer formal illustration of the necessary equations. Within structural engineering, there are various structures—for example bridges or high transmission towers—which are usually modeled via one-dimensional elements. Therefore, this work also contains a ‘set of tools’, which can also be applied in practice.

The concentration on one-dimensional elements is new for a textbook and allows the treatment of various basic and demanding physical questions of structural mechanics within one single textbook. This new concept, therefore, allows a methodical understanding of important subject areas (for example plasticity or composite materials), which occur to a prospective engineer during professional work, which however are seldom treated in this way at universities. Consequently, simple access is possible, also in supplementary areas of application of the finite element method.

This book originates from a collection of lecture notes which were developed as written material for lectures and training documents for specialized courses on the finite element method. Especially, the calculated examples and the supplementary

problems refer to typical questions which are raised by students and course participants.

The prerequisites for a good understanding are the basics in linear algebra, physics, materials science, and strength of materials, the way they are typically communicated in the basic studies of a technical subject in the field of mechanical engineering.

Within the initial chapters the one-dimensional elements will be introduced, by which the basic load cases of tension/compression, torsion, and bending can be illustrated. In each case, the differential equation as well as the basic equations from the strength of materials (this is the kinematic relationship, the constitutive relationship, and the equilibrium equation) are being derived. Subsequently, the finite elements with the usual definitions for force and displacement parameters are introduced. With the help of examples, the general procedure is illustrated. Short solutions for supplementary problems are attached in the appendix.

Chapter 6 deals with questions which are independent of the loading type and the therewith connected element formulation. A general one-dimensional finite element, which can be constructed from the combination of basic elements, the transformation of elements in the general three-dimensional space, and the numerical integration as an important tool for the implementation of the finite element method is dealt with.

The complete analysis of an entire structure is introduced in Chap. 7. The total stiffness relation results from the single stiffness relation of the basic elements under consideration of the relations to each other. A reduced system results due to the boundary conditions. Unknown parameters are derived from the reduced system. The procedure will be introduced as examples on plane and general three-dimensional structures.

Chapters 8–12 deal with topics which are usually not part of a basics book. The beam element with shear consideration is introduced in Chap. 8. The Timoshenko beam serves as a basis for this.

Within Chap. 9 a special class of material—composite materials—are introduced into a finite element formulation. First, various ways of description for direction dependent material behavior are introduced. Fiber composites are addressed briefly. A composite element is demonstrated by examples of a composite bar and a composite beam.

Chapters 10–12 deal with nonlinearities. In Chap. 10, the different types of nonlinear elasticities are introduced. The case of the nonlinear elasticity is dealt with more closeness. The problem is illustrated for bar elements. First, the principal finite element equation is derived under consideration of the strain dependency. The direct iteration as well as the complete and modified Newton–Raphson iteration is derived for the solution of a nonlinear system of equations. In addition, many examples serve as a demonstration of this issue.

Chapter 11 considers elastoplastic behavior, one of the most common forms of material nonlinearities. First, the continuum mechanics basics for plasticity in the case of the one-dimensional continuum bar are composed. The yield condition, the flow rule, the hardening law, and the elastoplastic modulus are introduced for

uniaxial, monotonic load cases. Within the hardening behavior, the description is limited to isotropic hardening. For the integration of the elastoplastic constitutive equations, the incremental predictor–corrector method is generally introduced and derived for the case of the fully implicit and the semi-implicit backward-Euler algorithm. On crucial points, the difference between one- and three-dimensional descriptions will be pointed out, to guarantee a simple transfer of the derived methods to general problems.

**Chapter 12** deals with stability, which is an issue that is especially relevant for the designing and dimensioning of lightweight components. Finite elements developed for this type of nonlinearities are used for the solving of the Euler’s buckling loads.

**Chapter 13** serves to introduce an FE formulation for dynamic problems. Stiffness matrices as well as mass matrices will be established. Different assumptions for the distribution of the masses, whether continuously or concentrated, lead to different formulations. The issue is discussed by example of axial vibrations of the bar.

As an illustration, each chapter is recessed both with precisely calculated and commented examples as well as with supplementary problems—including short solutions. Each chapter concludes with an extensive bibliography.

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Andreas Öchsner  
Markus Merkel

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Öchsner, A.; Merkel, M.

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