

Preface

Hi no hikari
Yabu shi wakaneba
Isonokami-
Furinishi sato ni
Hana mo sakikeri

For the light of the sun
Shuns not the wild thickets,
Even in Isonokami-
This village grown ancient,
The flowers are in bloom.

-Furu Imamichi (Kokinshu).

The mysterious link between special values of complex zeta and L-functions and purely arithmetic problems was discovered by Dirichlet and Kummer in the nineteenth century, and spectacularly generalized in the twentieth century by Birch and Swinnerton-Dyer with the formulation of their celebrated conjecture on the arithmetic of elliptic curves. We owe to Iwasawa the great discovery that these problems can be attacked by p -adic methods, where p is any prime number, provided one is prepared to work with a class of infinite Galois extensions of the base field F (which is always supposed to be a finite extension of \mathbb{Q}). Iwasawa himself only considered those Galois extensions whose Galois group is isomorphic to the additive group of the ring of p -adic integers \mathbb{Z}_p and the trivial Tate motive. However, it soon became apparent that his methods ought to apply to a much wider class of infinite extensions, namely those whose Galois group over F is a p -adic Lie group of dimension ≥ 1 , and to a large class of motives defined over F . While it is still not known how to formulate it in complete generality, it is now widely believed that, in this general setting, the link between special values of complex L -functions and arithmetic should be expressed by what is known as a main conjecture. Very roughly speaking, such

a main conjecture should assert that an appropriate p -adic L -function, interpolating special values of the relevant complex L -functions, should coincide with a certain algebraically defined invariant, usually called a characteristic element, which arises naturally from the arithmetic of the motive over the p -adic Lie extension.

This book arose from a workshop held at the University of Münster from April 25–30, 2011. The principal aim of this Workshop was to present the proof of the first key example of these general ideas, namely, the case when the motive is the trivial Tate motive and the p -adic Lie extension F_∞ of F is totally real (in addition, we always assume that F_∞ contains the cyclotomic \mathbb{Z}_p -extension of \mathbb{Q}). The first important progress on this problem goes back to Iwasawa himself, although we owe to Mazur and Wiles the first complete proof of the most classical case of this main conjecture (when F_∞ is the compositum of a real abelian base field F with the cyclotomic \mathbb{Z}_p -extension of \mathbb{Q}). Subsequently, Wiles discovered a deep new method, relying heavily on the theory of automorphic forms, for attacking these problems. In this way, he succeeded in proving the main conjecture when the base field F is any totally real number field, and the Galois group of F_∞ over F is abelian. This book is concerned with the problem of how one can extend Wiles' work to establish the general non-abelian totally real main conjecture for the trivial Tate motive. Two approaches for doing this were discovered independently and simultaneously, by Kakde on the one hand, generalizing ideas of Kato, and by Ritter and Weiss on the other. Both methods do in fact require one to assume a standard conjecture of Iwasawa about the vanishing of his cyclotomic μ -invariant, and so far this has only been proven when the base field F is an abelian extension of \mathbb{Q} and the Galois group of F_∞ over F is assumed to be pro- p . Both approaches are discussed in this book, but, following the lectures at the Workshop, it is largely Kakde's method which is treated in detail here. One reason for doing this is that the remarkable set of congruences established by Kakde to describe the K_1 -group of the Iwasawa algebra of any compact p -adic Lie group should also apply to attacking the non-commutative main conjecture for other motives. Finally, for reasons of space, the book only contains a written version of the lectures at the workshop which were closely related to the proof of the main conjecture.

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