

Chapter 2

Background and Methods

2.1 Introduction

The purpose of this chapter is to collect experimental details and background material that are common to multiple chapters of this thesis. Section 2.2 discusses a model of spontaneous parametric down conversion and details various experimental down conversion setups used for multi-photon state generation: each description will refer to the chapters where they are employed. Section 2.3 reviews the underlying concepts of waveguide optics required for this thesis. Finally, Sect. 2.4 describes three different waveguide architectures that are employed in the experiments reported in this thesis. Simulation and fabrication of these architectures lies outside the scope of this thesis, and were performed by colleagues as stated in each corresponding section.

2.2 Spontaneous Parametric Down Conversion

A range of single photon sources exist. Attenuated lasers can approximate a single photon source, allowing proof of principle experiments requiring only single photonic qubits—for example quantum cryptography [1]—but are not suitable for producing more than one photon for quantum interference due to cross terms in the tensor product of two coherent states [2]. An ideal source of photons for scalable quantum technology is a truly “on-demand” source that deterministically initiates emission of one photon from either an atomic or an atom-like system (such as quantum dots or diamond colour centres). Ongoing research is currently developing two or more identical single-photon emitters and has included demonstrations of quantum interference effects using quantum dots [3, 4], trapped ions [5, 6] and trapped atoms [7, 8]. However, currently single photon emitters are yet to deliver high visibility two photon quantum interference while quantum interference of more than two photons from single emitters is yet to be demonstrated.

In the absence of fully developed on-demand multi-photon sources, we turn to post-selected multi-photon states arising from *spontaneous parametric down conversion* (SPDC) which is currently the favoured method used to produce degenerate photon pairs for quantum interference in linear optics and the process used to generate photons for all experiments reported in this thesis. Since the foundational quantum interference experiments using SPDC [9–11] there have been a vast number of quantum interference applications for down conversion including fundamental tests of quantum physics (e.g. non-locality [12, 13] and quantum teleportation [14]), proof of principle quantum computing (e.g. two-qubit gates [15], compiled quantum algorithms [16, 17] and quantum chemistry [18]) and quantum metrology (e.g. for observing of two-[19] three-[20] four-[21, 22] and five-photon [23] number-path entanglement dynamics).

SPDC generates pairs of low energy photons from an initial pump laser beam focused in a non-linear medium and is either a four-wave mixing process, as reported for example with photonic crystal fibre sources (e.g. [24]), or a three-wave mixing process. In the latter case, SPDC is understood to be the generation of two lower energy photons (referred to as the signal and idler photons) from a photon in the pump field, mediated by a material with a nonzero χ^2 value. Conservation of energy dictates the frequency of the signal and idler photons must sum to that of the pump photon ($\omega_p = \omega_s + \omega_i$). Furthermore, the momentum vectors of the signal and idler photons must sum to that of the pump photon in order for momentum to be conserved ($\mathbf{k}_p = \mathbf{k}_s + \mathbf{k}_i$).

The approximate quantum state resulting from type I SPDC is computed using the interaction Hamiltonian

$$H_I = i\hbar\chi a_s^\dagger a_i^\dagger - i\hbar\chi^* a_s a_i \quad (2.1)$$

for bosonic creation operators $a_s^\dagger$ and $a_i^\dagger$ acting on the signal s and idler i fields. Here we have adopted the approximation that the pump is a classical field whose amplitude is incorporated in the parameter χ [25]. At this point we note that SPDC can be modelled to excite different polarisation modes which categorises SPDC as being type I—where the generated photons have the same polarisation—or type II—where the polarisation of generated signal and idler photons are orthogonal. All experiments reported in this thesis use type I SPDC.

Unitary evolution of the Hamiltonian acting on the initial state of vacuum in the signal and idler modes $|\psi(0)\rangle = |0\rangle_s |0\rangle_i$ is given by

$$U(t) |\psi(0)\rangle = e^{-iH_I t/\hbar} |\psi(0)\rangle = \exp \left\{ \chi t a_s^\dagger a_i^\dagger - \chi^* t a_s a_i \right\} |0\rangle_s |0\rangle_i \quad (2.2)$$

$$\approx \exp \{ \xi a_s^\dagger a_i^\dagger \} |0\rangle_s |0\rangle_i \quad (2.3)$$

$$= \sum_{k=0}^{\infty} \frac{\xi^k}{k!} a_s^{\dagger k} a_i^{\dagger k} |0\rangle_s |0\rangle_i \quad (2.4)$$

$$= |0\rangle_s |0\rangle_i + \xi |1\rangle_s |1\rangle_i + \xi^2 |2\rangle_s |2\rangle_i + \xi^3 |3\rangle_s |3\rangle_i + \dots \quad (2.5)$$

where $\xi \equiv \chi t$ is a parameter dependent upon the interaction time t , the strength of the nonlinearity in the material and the power in the pump beam. The approximation in line 2.3 uses a normal ordering step [25] and assumes the regime of $|\xi| \ll 1$.

Direct control of the laser pump allows control over the probability amplitudes of each term in the SPDC state in Eq. (2.5). Choosing small ξ such that ξ^2 and higher order terms become negligible with respect to ξ provides an excellent approximation to a two photon state in superposition with the vacuum:

$$|0\rangle + \xi |1\rangle_s |1\rangle_i \quad (2.6)$$

Recording twofold coincidental detection events at the output from the source (or after some optical circuit), post-selects only the single pair state $|1\rangle_s |1\rangle_i$ as the input. This is the approach used for experiments reported in Chaps. 3, 4, 5, 7 and 8; experimental details of a continuous-wave pumped two photon down conversion source are given in Sect. 2.2.1.

Increasing the pump power increases the amplitude of states associated to more than one pair of photons, such as the four-photon term $\xi^2 |2\rangle_s |2\rangle_i$. However, increasing ξ also increases the rate of single pairs, which is problematic for a post selection technique that cannot resolve coherently generated multi-pair states and multiple pairs of photons that are temporally distinguishable. Let us consider the following example. Suppose we have a finite coherence of photons generated—which in our case is defined spectrally via filtering ($\lambda \approx 800 \text{ nm}$ $\Delta\lambda \approx 3 \text{ nm}$) and is typically of the order 10^{-12} s . A photon pair generated in one window of time τ will be distinguishable from a second pair generated outside the coherence time of the first pair, in a second window τ' . This means two photon pairs in the state $|1\rangle_{s,\tau} |1\rangle_{i,\tau} |1\rangle_{s,\tau'} |1\rangle_{i,\tau'}$ will quantum-interfere in a different manner, in some optical circuit, to two photon pairs produced coherently together in the same time bin in the state $|2\rangle_{s,\tau} |2\rangle_{i,\tau}$. Successful post-selection of photons that have been generated within a time-frame equal to the coherence time of the generated photons would require a detector timing resolution of order 10^{-12} s , compared to the $O(10^{-9}) \text{ s}$ jitter for silicon avalanche photo-diode single photon counting modules.

For experiments requiring the state $|n\rangle_s |n\rangle_i$ for $n > 1$, an effective gating can be achieved using a pulsed laser to pump the SPDC process [25]. Making the pulse length sufficiently short ($\sim 100 \text{ fs}$) ensures down converted photons can only be produced within time-frames comparable to the coherence time of the generated photons. Separating each pump pulse by more than the detector jitter and longer than the coincidence window of the counting electronics, implies coincidental detection events arise from photons generated by a single pulse from the laser pulse. Photon counting is then used in post-selection to ignore quantum interference effects of lower photon number terms; for example counting fourfold coincidence events ignores the $|0\rangle$ and $|1\rangle_s |1\rangle_i$ terms.

Of course, increasing the laser power, such that the $\xi^2 |2\rangle_s |2\rangle_i$ becomes an appreciable event, allows higher order terms to become non-negligible. This highlights one of the major limiting factors for multi-photon experiments based on single crystal down conversion. In the absence of number resolving detectors, higher order

terms become a source of noise. For example, four photons from the six photon state $|3\rangle_s |3\rangle_i$ can be detected and counted as a data-point in a four-photon experiment. One approach is to choose a level of noise that is acceptable to the experiment and set ξ accordingly. For example in a simplified lossless case, setting the single pair generation probability $|\xi|^2 \approx 0.1$ implies an $n+2$ photon term has approximately ten times less probability amplitude than the n photon term. However each experiment must be considered individually with respect to the details of the detection scheme and the overall lumped detection efficiency of the experiment. Pulsed SPDC is used in the multi-photon experiments reported in Chaps. 3, 5 and 6. Details of the photon sources used for those experiments are given in Sect. 2.2.2.

2.2.1 Degenerate Photon Pair Sources

The two-photon experiments reported in this thesis are based on the degenerate photon pair source presented in Fig. 2.1. A vertically polarised 404 nm continuous wave (CW), diode laser (Toptica iBeam) is focused (minimum beam waist $\sim 40 \mu\text{m}$) with 60 mW of power on a 2 mm thick Type I phase matched Bismuth Borate BiB_3O_6 (BiBO). The phase matching is such that photon pairs are generated non-collinear, with signal and idler photons emitted in separate spatial modes for collection into optical fibres. Conservation of momentum dictates photon pairs of equal frequency $\omega_s = \omega_i = \omega_p/2$ are emitted in a cone structure that is symmetric around the initial pump beam, as indicated in Fig. 2.1a: this can be used to collect a single pair of photons in two spatial modes (Fig. 2.1), or to collect single pair of photons in two of four spatial modes (Fig. 2.3). The desired degenerate photons are filtered using high transmission ($>95\%$) laser line interference filters (Semrock) with centre wavelength ($\lambda_0 = 808 \text{ nm}$) matched to the degenerate photon pairs $\lambda_s = \lambda_i$ —fine tuning of wavelength below λ_0 can be achieved by tilting the filter with respect to the incoming photons.

The photons are then collected using mirrors and aspheric lenses as shown in Fig. 2.1b into polarisation maintaining fibre—the axis of each fibre is aligned to the horizontal plane. Alignment of the photon source begins with a coarse alignment (using a visible alignment laser fed back through the fibres onto the centre of the BiBO crystal) and a fine alignment based on first maximising single photon count rates detected in each mode and then on coincidental events across the two output modes. The degeneracy of the photons is then tested via a Hong–Ou–Mandel experiment (see Chap. 3). Typical two photon count rates straight from the source as described—detected with two $\sim 70\%$ efficient single photon counting modules (PerkinElmer AQR family)—is of the order 50 kHz. Typical collection efficiency of each mode is $\sim 20\%$.¹

¹ Where we define collection efficiency of one mode A of two modes A and B , as the total number coincidences detected from modes A and B , divided by the total number of single photon events detected from mode A ; note that this definition involves the detector efficiency.

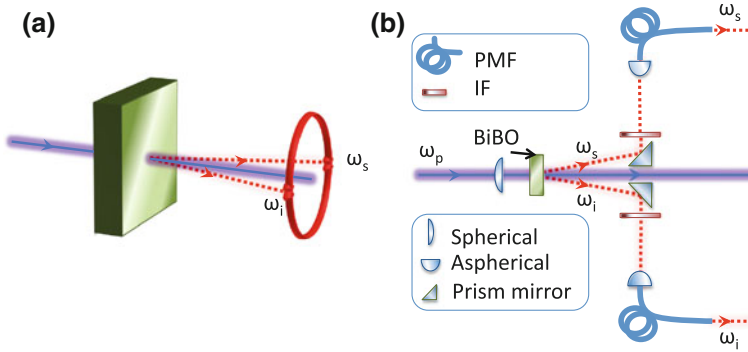


Fig. 2.1 Non-collinear SPDC generates degenerate photon pairs symmetrically in a cone structure around the initial pump. This can be used to collect a single pair of photons in two spatial modes (a) using prism mirrors and polarisation maintaining fibre (PMF) (b). Interference filters (IF) are used to ensure the desired degenerate photon pairs are collected

We also note that the polarisation state of the degenerate photon pairs emitted in the SPDC process can be manipulated with bulk optical wave plates. This is reported in more detail in Chap. 8 for post selection of polarisation entanglement in a waveguide circuit.

2.2.2 Multi-Photon Sources

The four-photon experiments reported in Chaps. 3 and 5 and the four- and six-photon experiments reported in Chap. 6 were conducted using degenerate single photon pairs produced via SPDC pumped by a pulsed laser source. Photons used in the experiment reported in Chap. 4 were generated with the same multi-photon source setup, operated with low laser pump power to approximate a two-photon source. A schematic of the typical setup is given in Fig. 2.2. The four photon states for experiments in Chaps. 3 and 5 were generated at the wavelength $\lambda_s = \lambda_i = 780$ nm and filtered with high transmission interference filters ($>95\%$, Semrock, $\lambda_0 = 780$ nm, $\Delta\lambda = 3$ nm). The four and six photon states for experiments reported in Chap. 6 were generated with at the wavelength $\lambda_s = \lambda_i = 785$ nm and filtered with high transmission interference filters (Semrock, $>95\%$, $\lambda_0 = 785$ nm, $\Delta\lambda = 3$ nm).

The nonlinear crystal used for SPDC was a 2 mm thick, Type-I phase matched Bismuth Borate BiB_3O_6 (BiBO) pumped by a pulsed (blue) $\frac{1}{2}\lambda_s$ nm beam, focused to a waist of $\omega_0 \approx 40$ μm . The blue pump was prepared using a further 2 mm thick BiBO crystal, phase matched for second harmonic generation (SHG) to double the frequency of a mode-locked 80 MHz repetition rate, 150 fs pulse length Ti:Sapphire laser (Coherent Chameleon Ultra II) focused to a waist of $\omega_0 \approx 40$ μm ; four successive dichroic mirrors (DM) are used to purify the pump beam spectrally. Degenerate

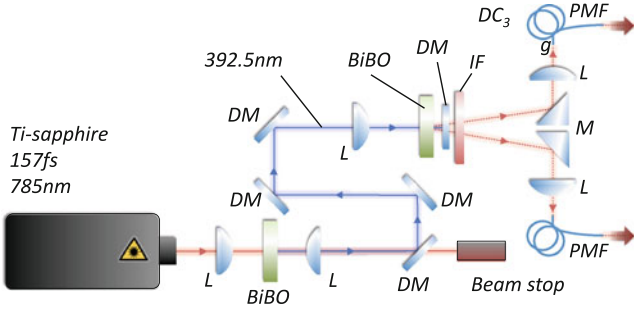


Fig. 2.2 Schematic of the two-mode pulsed SPDC multi-photon source used to generate the states $|n\rangle_s |n\rangle_i$ via post selection, for $n = 1, 2, 3$. Figure reproduced from [26]

photon pairs are created by the SPDC crystal and pass through $\Delta\lambda = 3$ nm interference filters (IF) which filter each photon to a coherence length of $l_c = \lambda^2/\Delta\lambda \approx 200$ μm . The photons are collected into two single mode polarization maintaining fibers (PMFs) coupled to two diametrically opposite points on the SPDC cone, as described in the previous sub-section. In the case of low average pump power, the state $|1\rangle_s |1\rangle_i$ is produced with a rate of 100 s^{-1} . On increasing the average pump power, the multi-photon production rate from the down-conversion process is no longer negligible such that two and three degenerate pairs of photons can be observed via the down conversion terms $|2\rangle_s |2\rangle_i$ and $|3\rangle_s |3\rangle_i$ respectively.

The cone-structure for emitting non-colinear photon pairs in type I down conversion allows for multi-mode collection as given in Fig. 2.3 for the case of collecting four spatial modes. This source was used for the Shor's algorithm experiment [27] reported in Chap. 4, which required two pairs of photonic qubits launched into a circuit that only required quantum interference between the signal and idler photons of each pair. Due to the requirements of the circuit, photons were generated at $\lambda_s = \lambda_i = 790 \text{ nm}$ using an appropriate pump ($\lambda_p = 395 \text{ nm}$) and interference filters ($\lambda_0 = 790 \text{ nm}$, $\Delta\lambda = 3 \text{ nm}$).

Detection of multiple photon states in the same optical mode is accomplished non-deterministically using cascaded non-number resolving, optical fibre-coupled single photon counting modules (see Chaps. 3, 5 and 6 for relevant details).

2.3 Single Waveguide Propagation

Optical waveguides are the main structure employed throughout this thesis for linear optical circuits, and are a thoroughly understood technology, widely used for optical telecommunications in the form of optical fibres and monolithic integrated waveguide chips. Here we present some fundamental descriptions of optical waveguides relevant to the experiments presented in this thesis.

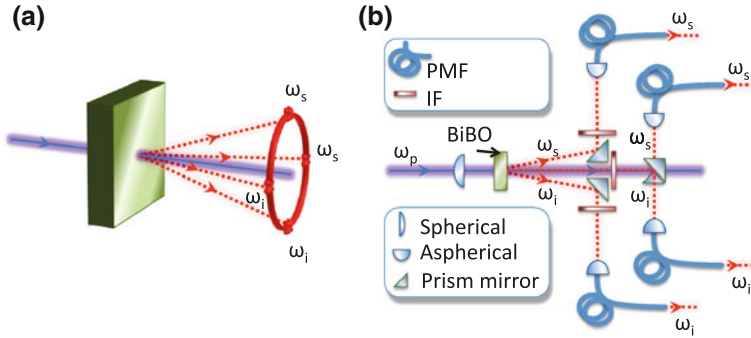


Fig. 2.3 The spontaneous parametric down conversion setup for collecting photon pairs in four spatial modes for the Shor's algorithm experiment [27] reported in Chap. 4

Light confining structures can be classified according to the number of spatial dimensions of confinement: planar waveguides or slab waveguides confine light in only one dimension; channel waveguides and optical fibres confine light in two orthogonal directions; photonic crystal structures, for example, can confine light in one-, two- or three-dimensions, depending on the structure. Channel waveguides are the structures on which our quantum optical circuits are based and the modelling of which requires numerical solution of the Maxwell's equations using commercially available beam propagation software. However, to understand some of the physics of integrated optics, in the next few sections we consider the more simple model of one-dimensional confinement in a step index waveguide, which will provide an insight into the underlying behaviour of waveguide propagation.

2.3.1 Ray Optics Description: Total Internal Reflection

There are several requirements for waveguided optics. First of all, the guiding structure must consist of a core with refractive index n_1 and cladding with lower refractive index n_0 (such that $n_1 > n_0$), as indicated in the refractive index profile in Fig. 2.4b. In a ray model of light propagation total internal reflection (TIR), illustrated in Fig. 2.4a, ensures light coupled to the end facet of a waveguide structure is confined to the core, provided the condition for TIR is satisfied:

$$n_1 \sin(\pi/2 - \phi) \geq n_0 \quad (2.7)$$

The value of ϕ for which Eq. (2.7) is an equality is known as the critical angle. Snell's law tells us that $\sin \theta = n_1 \sin \phi$, therefore we have an upper bound $\theta_{\max}$ (known as the numerical aperture) on the incident angle θ that will satisfy the TIR condition:

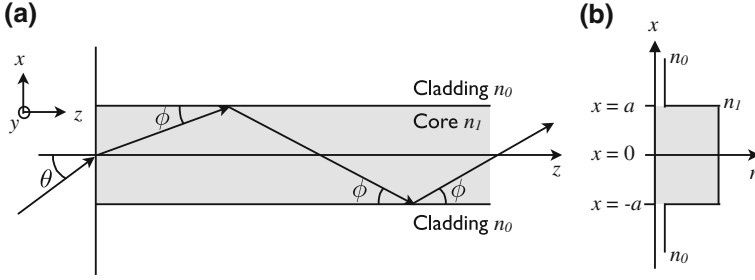


Fig. 2.4 **a** In the ray picture, light is confined in a planar waveguide by total internal reflection. **b** Illustrates the refractive index (n) profile. Figure inspired by [28]

$$\theta \leq \sin^{-1} \sqrt{n_1^2 - n_0^2} \equiv \theta_{\max} \quad (2.8)$$

TIR is a necessary, but not sufficient for guiding light in a waveguide structure. Even if ϕ is smaller than the critical angle for TIR, arbitrary values of ϕ are not permitted for light guidance. By considering phase fronts of the rays and phase changes on reflection (of the core-cladding interfaces) leads to a description of permitted solutions of ϕ_m , associated to integer values $m = 0, 1, 2, \dots$ (see [28] for example). These solutions are equivalent to the permitted guided modes that are derived from an electro-magnetic description (described in the following section). Indeed, there is good agreement between describing waveguides with ray optics and with electromagnetism. However, the simple ray optics description considered here fails in describing evanescent fields of light that lay outside of the core—a key mechanism for coupling light from one waveguide to another.

2.3.2 An Electromagnetic Description: Guided Modes and Evanescent Fields

The purpose of this section is to highlight two key features of the electromagnetic description of waveguides. The first will be that guided electro-magnetic fields have an evanescent field that lies outside of the core of a waveguide, which allows coupling between multiple waveguides and other structures. The second point is that there are discrete solutions to how light can propagate in a wave guiding structure, known as modes. As with the previous section, we shall make these two points by studying the simple model of one-dimensional confinement; the more complex situation of two-dimensional confinement of channel waveguides is solved numerically using commercial software.

With the condition that light is propagating through a dielectric, non-magnetic, isotropic and linear medium, Maxwell's equations simplify to

$$\nabla \cdot \mathcal{E} = 0; \nabla \cdot \mathcal{H} = 0; \nabla \times \mathcal{E} = -\mu_0 \frac{\partial \mathcal{H}}{\partial t}; \nabla \times \mathcal{H} = \varepsilon_0 n^2 \mu_0 \frac{\partial \mathcal{E}}{\partial t} \quad (2.9)$$

where μ_0 is the free space permeability, ε_0 is the free space permittivity $\mathcal{E}$ and $\mathcal{H}$ are the electric and magnetic fields respectively. By definition waveguides are inhomogeneous, meaning the refractive index is position dependent: $n = n(\mathbf{r})$. The inhomogeneous wave equations [29] derived from Maxwell's equations—assuming the structure is isotropic, linear, dielectric and non-magnetic—take the form

$$\nabla^2 \mathcal{E} + \nabla \left(\frac{1}{n(\mathbf{r})^2} \nabla n(\mathbf{r})^2 \mathcal{E} \right) - \varepsilon_0 \mu_0 n(\mathbf{r})^2 \frac{\partial^2 \mathcal{E}}{\partial t^2} = 0 \quad (2.10)$$

$$\nabla^2 \mathcal{H} + \frac{1}{n(\mathbf{r})^2} \nabla n(\mathbf{r})^2 \times (\nabla \times \mathcal{H}) - \varepsilon_0 \mu_0 n(\mathbf{r})^2 \frac{\partial^2 \mathcal{H}}{\partial t^2} = 0 \quad (2.11)$$

For the step index situation described in Fig. 2.4b, the refractive index in the core, and the cladding is assumed to be respectively constant, therefore the second term for each wave equation are zero.

For plane wave propagation in two-dimensional confinement in the z direction, the solutions to the wave equations are

$$\mathcal{E}(\mathbf{r}, t) = \mathbf{E}(x, y) e^{i(\omega t - \beta z)} \quad (2.12)$$

$$\mathcal{H}(\mathbf{r}, t) = \mathbf{H}(x, y) e^{i(\omega t - \beta z)} \quad (2.13)$$

where β is the propagation constant of the wave in the structure and ω is the angular frequency of the wave. This is simplified further for one-dimensional confinement in Fig. 2.4a by realising the wave has no dependence on the y direction: $\partial E_y / \partial y = 0 = \partial H / \partial y$.

To compute the form of permitted propagation, the two separate situations of the transverse electric (TE) and the transverse magnetic (TM) waves are treated. TE propagation are defined from electric field components that are perpendicular (transverse) to the plane of incidence (Fig. 2.4 sketches the incident plane as the $x - z$). The TM propagation is defined to be comprised of electric field components that are parallel to the plane of incidence and therefore magnetic field components that are transverse to the plane of incidence. Since the derivation of the TE and TM propagation follows a similar treatment, we shall only be concerned with TE fields here.

For TE propagation only E_y , H_x and H_z are non-zero. Subject to these conditions, substituting the planar wave equation solutions Eqs. (2.12), (2.13) into the wave equations Eqs. (2.10), (2.11) yield a second order differential equation involving E_y

$$\frac{d^2 E_y}{dx^2} + \left[k_0 n^2(x) - \beta^2 \right] E_y = 0 \quad (2.14)$$

where $n(x)$ is defined by the refractive index profile in Fig. 2.4b, k_0 is related to the wavelength λ_0 of the light in free space and the angular frequency ω by $k_0 = 2\pi/\lambda_0$, $\omega = 2\pi c/\lambda_0$ and $c = 1/\sqrt{\epsilon_0\mu_0}$.

For guided mode, the propagation constant β must satisfy [29]

$$k_0 n_0 < \beta < k_0 n_1 \quad (2.15)$$

So for the core and cladding regions, we re-write Eq. (2.14) as

$$\frac{\partial^2 E_y}{\partial x^2} - \gamma^2 E_y = 0 \quad x \leq -a, \text{ or } x \geq a \quad (2.16)$$

$$\frac{\partial^2 E_y}{\partial x^2} + \kappa^2 E_y = 0 \quad -a < x < a \quad (2.17)$$

where we have defined the real parameters

$$\gamma^2 = \beta^2 - k_0^2 n_0^2; \quad \kappa^2 = k_0^2 n_1^2 - \beta^2 \quad (2.18)$$

The solution of Eq. (2.14) therefore gives the electric field of guided TE propagation:

$$E_y = \begin{cases} Ae^{\gamma x} & x \leq -a \\ Be^{i\kappa x} + Ce^{-i\kappa x} & -a < x < a \\ De^{-\gamma x} & x \geq a \end{cases} \quad (2.19)$$

For some constants A, B, C, D, γ and κ that are dependent upon the waveguide parameters (a, n_0 and n_1), the propagation constant β and the free-space wavelength λ_0 . Following a similar treatment, the TM field has a similar solution.

Examining Eq. (2.19), we arrive at our first point: The guided TE (and similarly the guided TM) field has part of its solution outside of the waveguide core in the form of an exponentially decreasing profile for $x \leq -a$ and $x \geq a$. This corresponds to the *evanescent field*, which is the mechanism that allows light to couple from one waveguide into either neighbouring waveguides (or other structures), when the core of the neighbouring waveguide intersects the evanescent field of the original. We shall look at the behaviour of two coupled waveguides in the next section.

Returning to the solution for the mode profile in Eq. (2.19), boundary conditions dictate continuity of the quantities E_y and $\partial E_y / \partial x$ at the interfaces between core and cladding ($x = -a$ and $x = a$), leading to four equations of the five unknown parameters β, A, B, C and D . Solving this system [29] leads to the so called dispersion relation

$$\tan 2a\kappa = \frac{2\gamma/\kappa}{1 - \gamma^2/\kappa^2} \quad (2.20)$$

Tangent is a periodic modulo π , therefore we have a potentially infinite set of solutions governed by

$$\tan 2\kappa a = \tan (2\kappa a + m\pi), \quad m = 0, 1, 2, \dots \quad (2.21)$$

Here we arrive at a second point of interest: The TE (and TM) field as a discrete set of permitted solutions, characterised by m , known as *modes*. Since the dispersion relation 2.20 has a dependency on the properties of the waveguide and of the light, it follows that the number of permitted modes shares the same dependency: for the case that there is only one solution ($m = 0$) the waveguide is defined to be *single mode*, for $m > 0$, the waveguide is referred to as *multi-mode*. For the purposes of all experiments reported in this thesis, we required single mode operation and the waveguides used were designed accordingly.

2.3.3 Coupled Waveguides: A Quantum Mechanical Description

We have seen in the previous section that waveguides have an evanescent field that allow coupling between neighbouring waveguides. This forms the basis for directional couplers, where two waveguides are close enough to interact with each others evanescent field, and is one of the simplest methods to create beam splitter devices in an optical waveguide. In classical optics, this is often modelled with an electromagnetic description which can be found in a multitude of classical integrated optics textbooks.² Instead, here we take the more direct approach of modelling the system with a Hamiltonian, acting on single photon Fock states.

We saw that the evanescent field is exponentially decreasing away from the core-cladding interface, therefore we can model two identical, uniform evanescently coupled waveguides (labeled 1 and 2) as two nearest neighbour coupled oscillators [30], with the Hamiltonian³

$$H = \beta a_1^\dagger a_1 + \beta a_2^\dagger a_2 + C a_2^\dagger a_1 + C a_1^\dagger a_2 \quad (2.22)$$

for propagation constants β and coupling strengths proportional to constants C dependent upon the waveguide structure and the wavelength. Here we are assuming each waveguide is single mode, and we consider H as a transformation acting on the two waveguides with the two dimensional vectors $(1, 0)^T$ and $(0, 1)^T$ respectively corresponding to the single modes of waveguide 1 and waveguide 2. Coupling is therefore represented in this basis by the matrix

$$H \doteq \begin{pmatrix} \beta & C \\ C & \beta \end{pmatrix} \quad (2.23)$$

Unitary evolution along the propagation direction z is therefore described by computing the 2×2 matrix

² For example Refs. [28, 29].

³ This approach is also used for theoretical treatment of larger systems of coupled waveguides (for example [31, 32]), which we shall return to in Chap. 7.

$$e^{iHz} \doteq \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \beta & C \\ C & \beta \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad (2.24)$$

$$= e^{iz\beta} \begin{pmatrix} \cos(zC) & i \sin(zC) \\ i \sin(zC) & \cos(zC) \end{pmatrix} \quad (2.25)$$

Interaction between the two waveguides is controlled by bending the waveguides into the evanescent coupling regime and then subsequently separating them after some interaction region z . In this manner, an appropriate reflectivity beamsplitter can be created with two waveguides by controlling z and C —dependent upon properties of each waveguide, the separation of the two waveguides and of the guided light. To see this, e^{iHz} acting on $(1, 0)^T$ for example, we see that light initially present in waveguide 1 couples sinusoidally with z to waveguide 2 with intensity $\sin^2 zC$. This is the same conclusion for modelling two coupled waveguides with an electromagnetic description.

2.4 Waveguide Architectures

Here, we list the key features of the architectures used for the quantum photonic experiments reported in this thesis. Three architectures were used: (i) directly laser written waveguides in silica, (ii) lithographically fabricated silica-on-silicon and (iii) lithographically fabricated silicon-oxynitride, all of which were designed for single mode operation for near infrared photons for which our down conversion sources emit photons (780–810 nm) and for which commercially available silicon based avalanche photo-diodes can detect photons. Furthermore, all waveguides used were designed to have a small amount of birefringence (for example, the silicon oxynitride waveguides have a total birefringence is of the order $\Delta n = n_{\text{TM}} - n_{\text{TE}} = 2 * 10^{-4}$). Small values of birefringence provides polarization preserving propagation for the TE and TM photons throughout the device, while ensuring a nearly identical mode overlap, and coupling ratio, between different waveguides in the coupled array for TM and TE modes: this is of paramount importance for the experiment reported in Chap. 8. In general, preserving polarisation states of degenerate photons throughout optical circuitry is important for high quality quantum interference and for encoding quantum information.

2.4.1 Directly Written Waveguides in Silica

Directly written waveguides are fabricated with a tightly focused femtosecond laser machining process to locally alter the refractive index inside glass to inscribe waveguide circuits (Fig. 2.5a). Using nano-positioning three-axis stages, waveguides can be written in three-dimensional architectures, without the need for lithography, yielding a technique that can offer rapid prototyping of “one-off” circuits and architectures that cannot be realised in planar architectures allowing, for example,

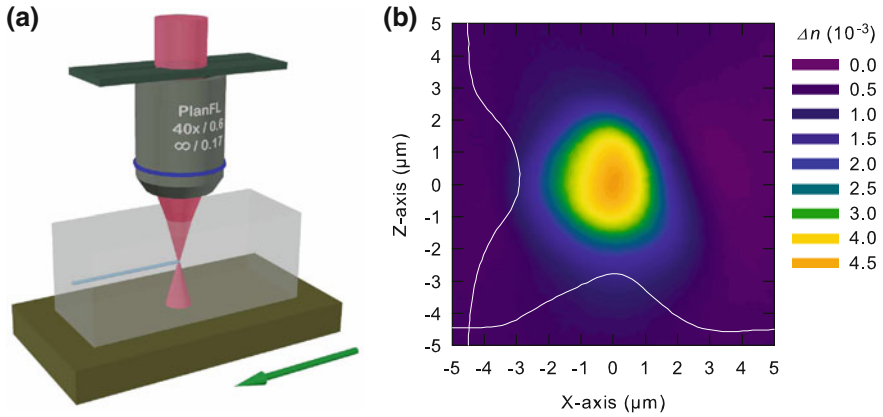


Fig. 2.5 Directly laser written waveguides. **a** Cartoon of a focused femtosecond laser writing a waveguide in pure silica. **b** Refractive index profile (Δn) of a directly written waveguide, measured by colleagues from Macquarie University. Figure reproduced from [33]

integration with micro-fluidics. Furthermore, the direct-write technique can fabricate devices with circular mode profiles, the size and ellipticity of which can be altered by controlling the laser focusing conditions. This is of particular use for producing waveguides that better match fibre modes for example, reducing optical loss.

Our collaborators at Macquarie University used this technique [34, 35] to fabricate two chips of high purity silica with a number of direct-write quantum circuits (DWQCs) composed of 2×2 directional couplers (Fig. 2.6) and Mach Zehnder interferometers for the experiments reported in Chap. 3 and Ref. [33]. The writing process created circular waveguides with an approximately Gaussian shaped refractive index profile that supported a single transverse mode with orthogonal $1/e^2$ widths of $6 \mu\text{m} \times 6 \mu\text{m}$ measured at 806 nm. The design of directional couplers was functionally identical except for the length of the central evanescently coupled region that was varied to achieve different coupling ratios. The curved regions of the waveguides (maximum bend radius $\sim 15 \text{ mm}$) were of raised-sine form and connected the input and output waveguide pitch of $250 \mu\text{m}$ down to the closely spaced $10 \mu\text{m}$ evanescent coupling region of the waveguides. When butt-coupled to arrays of optical fibre with refractive index matching liquid, a typically 70 % transmission from facet to facet was routinely achieved.

2.4.2 Silica-on-Silicon

The second architecture used was lithographically fabricated doped silica waveguides on a silicon substrate. These waveguides were designed by my colleague Alberto Politi using the numerical simulation package Beam Prop, while fabrication was outsourced to a commercial integrated photonics fabrication company (The Centre

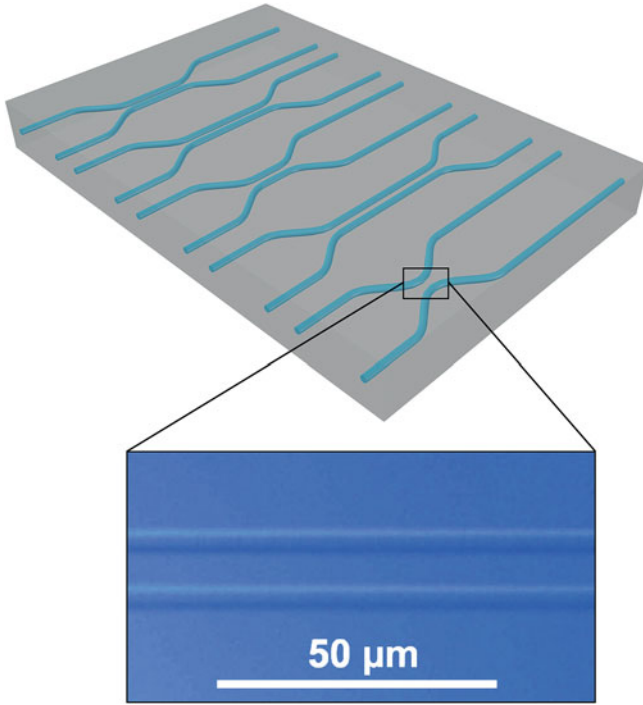
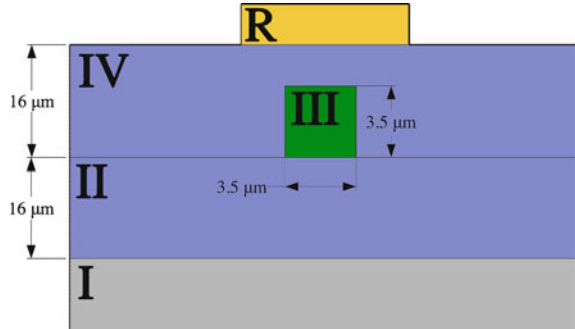


Fig. 2.6 Schematic of directional couplers fabricated by the direct-write process. The enlarged inset is an optical micrograph of the waveguides in the coupling region of one of the directional couplers, illustrating a designed 10 μm separation. Figure reproduced from [33]

Fig. 2.7 Cross-section of silica-on-silicon waveguides. Figure reproduced from [37]



for Integrated Photonics, CIP). This architecture was used for the first quantum interference experiments in monolithic waveguide circuits [36].

The optical circuits used for experiments reported in Chaps. 4, 5 and 6 all use silica-on-silicon waveguides, illustrated in cross-section in Fig. 2.7. The waveguides were fabricated on a 4" silicon wafer (material *I*), onto which a 16 μm layer of thermally grown undoped silica was deposited as a buffer to form the lower cladding

of the waveguides (II). A $3.5\text{ }\mu\text{m}$ layer of silica doped with germanium and boron oxides was then deposited by flame hydrolysis; the material of this layer constitutes the core of the structure and was patterned into $3.5\text{ }\mu\text{m}$ wide waveguides via standard optical lithographic techniques (III). The $16\text{ }\mu\text{m}$ upper cladding (IV) is phosphorus and boron doped silica with a refractive index matched to that of the buffer. Simulations indicated single mode operation at 780 nm and 800 nm . A refractive index contrast of $\Delta = (n_{core}^2 - n_{cladding}^2)/2n_{core}^2 = 0.5\%$ yields a minimum bend radius of $\sim 15\text{ mm}$. A final metal layer was lithographically patterned on the top of the devices to form resistive elements (R) used for controlling optical phase shifts (reported in Chaps. 5 and 6) and metal contact pads. When butt-coupled to arrays of optical fibre with refractive index matching liquid, $\sim 70\%$ transmission from facet to facet was routinely achieved.

2.4.3 Silicon Oxynitride Waveguide Arrays

The third architecture used in this thesis is silicon oxynitride (SiON) waveguides lithographically fabricated on a silicon substrate, which were designed with numerical beam propagation software (Beam Prop) by colleagues in the University of Bristol, and fabricated by colleagues at the University of Twente.

The role of this material was to realise arrays of O(10) evanescently coupled waveguide, single mode for $\sim 800\text{ nm}$, that could quickly bend the waveguides from the coupling region to a separation suitable for butt-coupling to arrays of optical fibre while realising a compact optical circuit. Silica-on-silicon is not a suitable architecture for this purpose. However, the high refractive index contrast $\Delta = (n_{core}^2 - n_{cladding}^2)/2n_{core}^2 = 4.4\%$ of silicon oxynitride does provide a suitable platform for doing quantum experiments using waveguide arrays, as reported in Chaps. 7 and 8.

Thin 600 nm films of SiON were deposited on a silicon substrate to create the core of the waveguide by standard lithography and reactive ion etching; a $5\text{ }\mu\text{m}$ thick layer of SiO_2 was then deposited to complete the cladding of the structure. The waveguides were $1.8\text{ }\mu\text{m}$ wide and were separated by a distance of $2.8\text{ }\mu\text{m}$ between waveguides in the coupled array to define the tunnelling rate of light between adjacent waveguides (Fig. 2.8). The waveguides at the end of the array spread out, at an equal rate of relative separation between adjacent waveguides, to a pitch $125\text{ }\mu\text{m}$ for coupling to optical fibre external to the chip. The overall coupling efficiency through the chip (from input fibre to output fibre) is $\sim 10\%$, which is attributed to the mode-mismatch between circular input/output optical fibres and the rectangular waveguides.

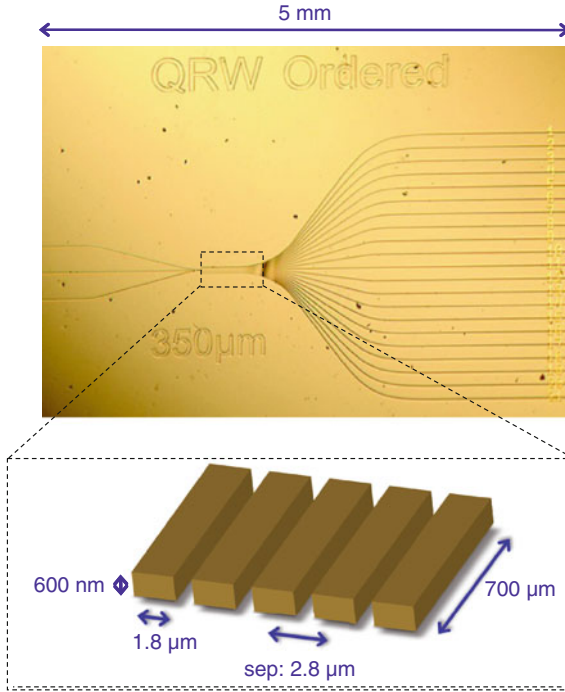


Fig. 2.8 Optical micrograph of a silicon oxynitride waveguide array chip, illustrating the coupling region (*highlighted*) and input and output bends. The enlarged section illustrates the dimensions of the waveguides and the evanescently coupled region

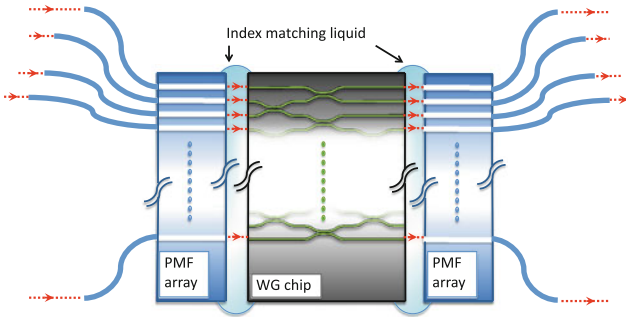


Fig. 2.9 Illustration butt-coupling arrays of optical fibre to waveguide chips. Figure reproduced from [37]

2.4.4 Chip-Fibre Coupling

For all experiments reported in this thesis, single photons were launched into, and collected from, waveguide chips using v-groove arrays of optical fibre. Each waveguide

chip has a fixed spacing of input and output waveguides. For the direct-write circuits and for the silica-on-silicon circuits, this separation was $250\text{ }\mu\text{m}$ to match the standard $250\text{ }\mu\text{m}$ spacing of commercially available arrays of optical fibre (OZ optics). The input for the silicon oxynitride circuits also had a $250\text{ }\mu\text{m}$ spacing, while the output had a $125\text{ }\mu\text{m}$ spacing. Since this spacing is non standard,⁴ a $250\text{ }\mu\text{m}$ array was used to access respectively all even and then all odd outputs of the array. Input fibre arrays were of polarisation maintaining fibre in all cases. The output fibre arrays used include polarisation maintaining fibre, non-polarisation maintaining single mode fibre and arrays of multimode fibre. Nano-positioning six-axis stages (X – Y – Z , pitch, roll and yaw) were used to align the arrays to the waveguide circuits. Refractive index matching liquid was then used to reduce loss due to reflection at the facet. Fig. 2.9 illustrates a typical setup.

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⁴ The closest standard fibre-array separation is $127\text{ }\mu\text{m}$.

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